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A Note On: Direct Methods for Finding Optimal Solution of a Transportation Problem Are Not Always Reliable

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ABSTRACT: *Abdul Quddooset al. (July 2012) developed and published ASM-Method for obtaining the optimal solution for transportation problems (TP) directly in a lesser number of iterations with very easy computations. Mohammad KamrulKasan (October 2012) has revealed that the ASM-Method for finding optimal solution of a transportation problem do not present optimal solution at all times. The author has given one illustration as Problem 2 and showed that the minimum transportation cost generated by the ASM-Method for the said problem is $114 against the optimal solution of $112. In this paper, I have tried to expose that the ASM-Method has produced the optimal solution of $112 for that problem. Also, I admit the statement of the author that the ASM-Method does not reflect optimal solution continuously. This statement is established bysolving two classical benchmark problems.*
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I. INTRODUCTION

In Operations Research, transportation problem (TP) is famous for its wide application in real life. TP is a special class of the linear programming problem, which deals with the situation in which a commodity is shipped from a set of sources to a set of destinations, subject to the supply and demand of the source and destination respectively, such that the total cost of shipping is minimized. The model assumes that the shipping cost on a given route is directly proportional to the number of units shipped on that route. In general, the transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling and personnel assignment. In the literature, several methods have been developed to find IBFS and optimal solution to a TP. Among them one method has been introduced which directly produces the optimal solution namely the ASM-Method due to Abdul Quddoos et al [1], [2]. This method requires least number of iterations with very easy computations to reach optimality, compared to the existing methods available in the literature. But for certain classical benchmark instances, the optimal solution found by the method are not actually optimal. Mohammad KamrulKasan [3]has discovered that the ASM-Method for finding optimal solution of a TP does not present optimal solution at all times. He has given one numerical example as Problem 2 and showed that the minimum transportation cost generated by the ASM-Method for the said problem is $114 against the optimal solution of $112. In this paper, I have tried to make representation that the ASM-Method has produced the optimal solution of $112 for that problem. Also, I confess the statement of the author that the ASM-Method does not produce optimal solution constantly. This statement has beenrecognized by tryingclassical benchmark instances.

Initial Basic Feasible Solution (IBFS):
A set of non-negative values X_{ij}, i = 1,2,3, …,m and j = 1,2,3,…,n that satisfies the row and column restrictions of a given TP is known as IBFS to thatTP. The IBFS may or may not be optimal.

Optimal Solution:
An IBFS is said to be optimal if it **minimizes** the total transportation cost.

Non-degenerate Basic Feasible Solution:
A basic feasible solution to a (m×n) TP that contains exactly m+n-1 allocations in independent positions is said to be non-degenerate.

Degenerate Basic Feasible Solution:
A basic feasible solution that contains less than m+n-1 non-negative allocations is said to be degenerate.

Balanced and Unbalanced TP:

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in the destinations, and is called unbalanced otherwise.

Optimality Test:

Optimality test can be performed only if "the number of allocated cells in a basic feasible solution exactly equals m+n-1, where m = No. of rows and n = No. of columns". The object of optimality test is that, if we put an allocation in a vacant cell then whether the total transportation cost decreased. Two methods for optimality test namely "Stepping Stone Method" and "MODI Method" areusually used, whereas "MODI Method"is mostly used.

Modified Distribution Method (MODI Method) or u-v Method:

This method involves the following steps:

Step-1: Take the costs only that cells where allocations have. It is called cost matrix for allocated cells.

Step-2: On the above of each column we put v_1, v_2, v_3,…,v_n and at the same time on the left of each row we put u_1, u_2, u_3, …, u_m so that the sum of corresponding u's and v's in every allocated cell is equal to above cost. That is, $u_i + v_j = c_{ij}$ .Then by algebraic calculations, the values of each u's and v's are to be found out.It is called $u_i + v_j$ matrix for allocated cells.

Step-3: The empty cells are filled up by the sum results of corresponding u's and v's. It is called $u_i + v_j$ matrix for vacant cells..

Step-4: Subtract the above matrix's cells from the corresponding cells of original matrix. It is called cell evaluation matrix.

Step-5: If the above cell evaluation matrix contains only non-negative cells , then the basic feasible solution is optimal.

On the other hand, if the above cell evaluation matrix contains any –vecell , then the basic feasible solution is not optimal. For optimal solution the following iteration should be run:

Step-1: Select the most negative cell from the above cell evaluation matrix. If there have more than one equal cell, then any one can be chosen.

Step-2: Write the initial basic feasible solution. Give a tick (√) at the most negative entry cell. It is called identified cell.

Step-3: Trace or draw a path in this matrix consisting of a series of alternatively horizontal and vertical lines. The path begins and terminates in the identified cell. All corners of the path lie in the cells for which allocations have been made. The path may skip over any number of occupied or vacant cells.

Step-4: Mark the identified cell as +ve and each occupied cell at the corners of the path alternatively –ve , +ve , -ve and so on.

Step-5: Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a –vesign. Add and subtract this new allocation from the cells at the corners of the path, maintaining the row and column requirements. This causes one basic cell to become zero and other cells remain non-negative. The basic cell whose allocation has been made zero, leaves the solution.

II ASM-Method

Step 1: Construct the transportation table from given transportation problem.

Step 2: Subtract each row entries of the transportation table from the respective row minimum and then subtract each column entries of the resulting transportation table from respective column minimum.

Step 3: Now there will be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero (row-wise) occurring in the cost matrix. Suppose (i, j)th zero is selected, count the total number of zeros (excluding the selected one) in the ith row and jth column. Now select the next zero and count the total number of zeros in the corresponding row and column in the same manner. Continue it for all zeros in the cost matrix.

Step 4: Now choose a zero for which the number of zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in step 3 then choose a (k,l)th zero breaking tie such that the total sum of all the elements in the kth row and lth column is maximum. Allocate maximum possible amount to that cell.

Step 5: After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 6: Check whether the resultant matrix possesses at least one zero in each row and in each column. If not, repeat step 2, otherwise go to step 7.

Step 7: Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are exhausted.

II. REFLECTING THE OPTIMAL SOLUTION

Numerical Example 2.1 (Problem 2as in [3]):

Consider the following cost minimizing TP with four sources and six destinations:

Table 2.1: The given TP

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	9	12	9	6	9	10	**5**
S2	7	3	7	7	5	5	**6**
S3	6	5	9	11	3	11	**2**
S4	6	8	11	2	2	10	**9**
Demand	**4**	**4**	**6**	**2**	**4**	**2**	

Constructing the Reduced Cost Matrix:

(a) Perform Row Minimum Subtraction

Table 2.2: The Resultant Matrix after Row Minimum Subtraction

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	3	6	3	0	3	4	**5**
S2	4	0	4	4	2	2	**6**
S3	3	2	6	8	0	8	**2**
S4	4	6	9	0	0	8	**9**
Demand	**4**	**4**	**6**	**2**	**4**	**2**	

(b) Perform Column Minimum Subtraction

Table 2.3: The Resultant Matrix after Column Minimum Subtraction

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	0	6	0	0	3	2	**5**
S2	1	0	1	4	2	0	**6**
S3	0	2	3	8	0	6	**2**
S4	1	6	6	0	0	6	**9**
Demand	**4**	**4**	**6**	**2**	**4**	**2**	

The Reduced Cost Matrix (RCM-1)

Making the Allocations one by one

Making the First Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	3	
(1, 3)	2	
(1, 4)	3	
(2, 2)	**1***	**22†**
(2, 6)	1*	**22†**
(3, 1)	2	
(3, 5)	2	
(4, 4)	2	
(4, 5)	2	

Note: The minimum entry in column (ii) is marked with the symbol * and the maximum entry in column (iii) is marked with the symbol †.

Since tie occurs in column (iii), we can choose any cell. We arbitrary choose the cell (2, 2). [The optimal solution will not change if we choose the cell (2, 6) instead of (2, 2) also]. In the identified cell (2, 2), the maximum possible allocation value of 4 is allocated. Now delete the 2rdcolumn of the RCM-1 and adjust the supply of the 2rdrow. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Second Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	3	
(1, 3)	2	
(1, 4)	3	
(2, 6)	**0***	
(3, 1)	2	
(3, 5)	2	
(4, 4)	2	
(4, 5)	2	

In the identified cell (2, 6), the maximum possible allocation value of 2 is allocated. Now delete the 2nd row and 5th column of the RCM-1as at a time supply is exhausted as well as demand is satisfied. (Note that this result in a degenerate BFS). Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Third Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	3	
(1, 3)	2*	12
(1, 4)	3	
(3, 1)	2*	12
(3, 5)	2*	14
(4, 4)	**2***	**15†**
(4, 5)	2*	10

In the identified cell (4, 4), the maximum possible allocation value of 2 is allocated. Now delete the 4th column of the RCM-1and adjust the supply of 4th row. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Fourth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	2	
(1, 3)	**1***	**12†**
(3, 1)	2	
(3, 5)	2	
(4, 5)	1*	10

In the identified cell (1, 3), the maximum possible allocation value of 5 is allocated. Now delete the 1st row of the RCM-1 and adjust the demand of 3rd column. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM is shown in Table 2.4.

Table 2.4: Further Reduced Cost Matrix (RCM-2)

Sources	**D2**	**D3**	**D5**	**Supply**
S3	0	0	0	**2**
S4	1	3	0	**7**
Demand	**4**	**1**	**4**	

Making the Fifth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(3, 1)	2	
(3, 3)	2	
(3, 5)	3	
(4, 5)	**1***	

In the identified cell (4, 5), the maximum possible allocation value of 4 is allocated. Now delete the 5th column of the RCM-2 and adjust the supply of 4th row. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The resulting RCM is given in Table 2.5.

Table 2.5: Further Reduced Cost Matrix (RCM-3)

Sources	**D2**	**D3**	**Supply**
S3	0	0	**2**
S4	0	2	**3**
Demand	**4**	**1**	

Making the Sixth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(3, 1)	2	
(3, 3)	**1***	**2†**
(4, 1)	**1***	**2†**

Since tie occurs in column (iii), we can choose any cell. We choose the cell (3, 3). In the identified cell (3, 3), the maximum possible allocation value of 1 is allocated. Now delete the 3rd column of the RCM-3 and adjust the supply of the 3rd row. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Seventh Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(3, 1)	**1***	**0†**
(4, 1)	**1***	**0†**

Since tie occurs in column (iii), we can choose any cell. We choose the cell (3, 1). In the identified cell (3, 1), the maximum possible allocation value of 1 is allocated. Now delete the 3rd row of the RCM-3 and adjust the demand of the 1st column.

Making the Eight Allocation

Since only one cell (4, 1) is remaining, we make the last allocation in the cell (4, 1) with the possible and remaining allocation value of 3. Now the allocation process is complete. The final allocation table obtained through ASM-Method is shown in Table 2.6.

Writing the Allocation Values:

$X_{13} = 5$, $X_{22} = 4$, $X_{26} = 2$, $X_{31} = 1$, $X_{33} = 1$, $X_{41} = 3$, $X_{44} = 2$, $X_{45} = 4$, and all other $X_{ij} = 0$. Note that the generated solution is a degenerate one as it contains only eight allocations instead of nine (m+n-1 = 6+4-1= 9) allocations.

Computing the Total Transportation Cost:

Z = (9 × 5) + (3 × 4) + (5 × 2) + (6 × 1) + (9 × 1) + (6 × 3) + (2 × 2) + (2 × 4)
= 45 + 12 + 10 + 6 + 9 + 18 + 4 + 8
= $112.

It is noted that the IBFS generated by ASM-Method is the optimal solution to the given TP.

Table 2.6: Allocation table due to the ASM-Method

	Destinations						
Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**D6**	**Supply**
S1	9	12	[5] 9	6	9	10	**5**
S2	7	[4] 3	7	7	5	[2] 5	**6**
S3	[1] 6	5	[1] 9	11	3	11	**2**
S4	[3] 6	8	11	[2] 2	[4] 2	10	**9**
Demand**4 462 4 2**							

III. ESTABLISHING THE CLAIM

Numerical Example 3.1 (A. Mahlanga et al. 2014 [4]):

Consider the following cost minimizing balanced TP with four sources and five destinations:

Table 3.1: The given TP

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	4	9	8	10	12	**24**
S2	6	10	3	2	3	**18**
S3	3	2	7	10	3	**20**
S4	3	5	5	4	8	**16**
Demand	**10 20 10 18 20**					

Constructing the Reduced Cost Matrix:

(a) Perform Row Minimum Subtraction

Table 3.2: The Resultant Matrix after Row Minimum Subtraction

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	0	5	4	6	8	**24**
S2	4	8	1	0	1	**18**
S3	1	0	5	8	1	**20**
S4	0	2	2	1	5	**16**
Demand	**10 20 10 18 20**					

(b) Perform Column Minimum Subtraction

Table 3.3: The Resultant Matrix after Column Minimum Subtraction

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	0	5	3	6	7	**24**
S2	4	8	0	0	0	**18**
S3	1	0	4	8	0	**20**
S4	0	2	1	1	4	**16**
Demand	**10 20 10 18 20**					

The Reduced Cost Matrix (RCM-1)

Making the Allocations one by one

Making the First Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	1*	26
(2, 3)	2	
(2, 4)	2	
(2, 5)	3	
(3, 2)	**1***	**28†**
(3, 5)	2	
(4, 1)	1*	13

Note: The minimum entry in column (ii) is marked with the symbol * and the maximum entry in column (iii) is marked with the symbol †.

In the identified cell (3, 2), the maximum possible allocation value of 20 is allocated. Since the supply is exhausted as well as demand is satisfied for this cell, we can delete either the 3rd row or the 2nd column of the RCM-1. We delete the 3rd row and adjust the demand of the 2nd column as zero (20 – 20 = 0). Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM-2 is shown in Table 3.4.

Table 3.4: Further Reduced Cost Matrix (RCM-2)

Sources	D1	D2	D3	D4	D5	Supply
S1	0	3	3	6	7	**24**
S2	4	6	0	0	0	**18**
S4	0	0	1	1	4	**16**
Demand	**10**	**0**	**10**	**18**	**20**	

Making the Second Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 1)	**1***	**23†**
(2, 3)	2	
(2, 4)	2	
(2, 5)	2	
(4, 1)	2	
(4, 2)	1	15

In the identified cell (1, 1), the maximum possible allocation value of 10 is allocated. Now delete the 1st column and 5thcolumn of the RCM-2 and adjust the supply of the 1st row as 24 – 10 = 14. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM-3 is shown in Table 3.5.

Table 3.5: Further Reduced Cost Matrix (RCM-3)

Sources	D2	D3	D4	D5	Supply
S1	0	0	3	4	**14**
S2	6	0	0	0	**18**
S4	0	1	1	4	**16**
Demand	**0**	**10**	**18**	**20**	

Making the Third Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 2)	2	
(1, 3)	2	
(2, 3)	3	
(2, 4)	2	
(2, 5)	2	
(4, 2)	**1***	

In the identified cell (4, 2), the maximum possible allocation value of 0 is allocated. Now delete the 2nd column of the RCM-3 and adjust the supply of 4th row as 16 – 0 = 0. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM-4 is shown in Table 3.6.

Table 3.6: Further Reduced Cost Matrix (RCM-4)

Sources	D3	D4	D5	Supply
S1	0	3	4	**14**
S2	0	0	0	**18**
S4	0	0	3	**16**
Demand	**10**	**18**	**20**	

Making the Fourth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 3)	**2***	**7†**
(2, 3)	4	
(2, 4)	3	
(2, 5)	2*	7†
(4, 3)	3	
(4,4)	2*	6

Since tie occurs in column (iii), we can choose any cell. We arbitrary choose the cell (1, 3). [The optimal solution will not change if we choose the cell (2, 5) instead of (1, 3) also]. In the identified cell (1, 3), the maximum possible allocation value of 10 is allocated. Now delete the D3 column of the RCM-4 and adjust the demand of S1 row as 14 – 10 = 4. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM-5 is shown in Table 3.7.

Table 3.7: Further Reduced Cost Matrix (RCM-5)

Sources	**D4**	**D5**	**Supply**
S1	0	1	**4**
S2	0	0	**18**
S4	0	3	**16**
Demand	**18**	**20**	

Making the Fifth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 4)	2	
(2, 4)	3	
(2, 5)	**1***	
(4, 4)	2	

In the identified cell (2, 5), the maximum possible allocation value of 18 is allocated. Now delete the S2 row of the RCM-5 and adjust the demand of the D5 column as 20 – 18 = 2. Observe that the resultant cost matrix does not possess at least one zero in each row and in each column. So, we go for constructing the RCM further. The further RCM-6 is shown in Table 3.8.

Table 3.8: Further Reduced Cost Matrix (RCM-6)

Sources	**D2**	**D3**	**Supply**
S1	0	0	**4**
S4	0	2	**16**
Demand	**18**	**2**	

Making the Sixth Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 4)	2	
(1, 5)	**1***	**2†**
(4, 4)	1	2†

Since tie occurs in column (iii), we can choose any cell. We arbitrary choose the cell (1, 5). [The optimality will not be affected if we choose the cell (4, 4) instead of (1, 5) also]. In the identified cell (1, 5), the maximum possible allocation value of 2 is allocated. Now delete the D5column of the RCM-6 and adjust the supply of the S1 row as 4 – 2 = 2. Observe that the resultant cost matrix possesses at least one zero in each row and in each column. So, we go for the next allocation.

Making the Seventh Allocation

(i) Zero entry cells in order (row-wise)	(ii) No. of zeros in its row and col. (excluding the selected zero) [Minimum]	(iii) Sum of all the elements in the row and col. [Maximum]
(1, 4)	**1***	**0†**
(4, 4)	1*	0†

Since tie occurs in column (iii), we can choose any cell. We arbitrary choose the cell (1, 4). [The optimality will not be affected if we choose the cell (4, 4) instead of (1, 4) also] In the identified cell (1, 4), the maximum possible allocation value of 2 is allocated. Now delete the S1row of the RCM-6 and adjust the demand of the D4 column 18 – 2 = 16.

Making the Eight Allocation

Since only one cell (4, 4) is remaining, we make the last allocation in the cell (4, 4) with the possible and remaining allocation value of 16. Now the allocation process is over. The final allocation table obtained through ASM-Method is shown in Table 3.9.

Table 3.9: Allocation table due to the ASM-Method

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	10 4	9	10 8	2 10	2 12	**24**
S2	6	10	3	2	18 3	**18**
S3	6	20 5	9	11	3	**20**
S4	3	0 5	5	16 4	8	**16**
Demand	**10 20 10 18 20**					**78**

Writing the Allocation Values:

X_{11} = 10, X_{13} = 10, X_{14} = 2, X_{15} = 2, X_{25} = 180, X_{32} = 20, X_{42} = 0, X_{44} = 16, and all other X_{ij} = 0. Note that the solution generated due to ASM-Method is a degenerate one as it contains only eight (positive) allocations instead of nine (m+n-1 = 6+4-1= 9) allocations.

Computing the Total Transportation Cost:

Z = (10 × 4) + (10 × 8) + (2 × 10) + (2 × 12) + (18× 3) + (20 × 2) + (0 × 5) + (16 × 4)
= 40 + 80 + 20 + 24 + 54 + 40 + 0 + 64
= $322.

It is noted that the IBFS generated by the ASM-Method is not the optimal solution to the given TP. In the obtained IBFS, by applying the MODI method for optimality check, we are able to get the optimal allocations with minimum total transportation cost of 316 in four iterations. The optimal allocation table established through MODI method is shown in Table 3.10.

Writing the Optimal Allocation Values:

X_{11} = 10, X_{12} = 4, X_{13} = 10, X_{24} = 2, X_{25} = 16, X_{32} = 16, X_{35} = 4, X_{44} = 16, and all other X_{ij} = 0. Note that the optimal solution is a non-degenerate one as it contains exactly eight (m+n-1 = 4+5-1= 8) positive allocation.

Computing the Total Transportation Cost:

Z = (10 × 4) + (4 × 9) + (10 × 8) + (2 × 2) + (16× 3) + (16 × 2) + (4 × 3) + (16 × 4)
= 40 + 36 + 80 + 4 + 48 + 32 + 12 + 64
= $316.

Table 3.10: Optimal allocation table due to the MODI Method

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	10 4	4 9	10 8	10	12	**24**
S2	6	10	3	2 2	16 3	**18**
S3	6	16 5	9	11	4 3	**20**
S4	3	5	5	16 4	8	**16**
Demand	**10**	**20**	**10**	**18**	**20**	78

Numerical Example3.2 (K. Karagul and Y. Sahin, 2019 [5])
Consider the following cost minimizing problem with four sources and six destinations:

Table 3.11: The given transportation problem

Sources	**D1**	**D2**	**D3**	**D4**	**D5**	**Supply**
S1	73	40	9	79	20	**8**
S2	62	93	96	8	13	**7**
S3	96	65	80	50	65	**9**
S4	57	58	29	12	87	**3**
S5	56	23	87	18	12	**5**
Demand	**6**	**8**	**10**	**4**	**4**	

For this problem, the total transportation cost produced by the ASM-Method is $1103, whereas the minimum total transportation cost established due to MODI method is $1102.

IV. CONCLUSION

Mohammad KamrulKasan (October 2012) has revealed that the ASM-Method for finding optimal solution of a TP does not provide optimal solution at all times. Hehas tried to provide evidence for his claim all the way by means of a numerical example and showed that the minimum transportation cost generated by the ASM-Method for the said problem is $114 against the optimal solution of $112. But actually the ASM-Method has produced the optimal solution of $112 for that problem, which has been demonstrated and confirmed. Further, the statement that the ASM-Method does not produce optimal solution constantly has been recognized by testing twoclassicalbenchmark instances.

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