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N-Dimensional Kaluza-Klein String Cosmological Model In Lyra's Manifold

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Abstract:N- dimensional Kaluza-Klein space time is considered in presence of cosmic string in the frame work of Lyra Geometry. A relation between metric potential is assumed to get a determinate solution of the field equations of this theory. Some physical and Kinematical properties of the model are also discussed.
Key words: Kaluza-Klein space time, cosmic string, Lyra's Geometry.

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I. INTRODUCTION:

The study of cosmologies with more than four dimensions is all the more important in these days. These higher dimensional scenarios are based on various Kaluza-Kelin theories. Kaluza-Kelin theories have been used as a way of unifying all gauge interactions with gravity. Here the extra dimensions play a physical role and its unobservability is usually explained by the assumption that they are restricted to compact space with a very small length in scale. The detection of time variation of fundamental constraints may also be a strong evidence for the existence of extra dimensions (Weinberg 1986). Chodos and Detweller(1980),Ibanez and Verdaguer (1986), Gleiser and Diaz (1988), Banerjee and Bhui (1990), Reddy and Venkateswara Rao(2001), Khadekar and Gaikwad(2001),Adhav *et.al*(2008) have studied the multidimensional cosmological models in Einstein's general relativity theory.

Lyra (1951) proposed a modification of Riemannian geometry by introducing gauge function into the structure less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's (1918) geometry. In subsequent investigations Sen (1957) and Sen and Dunn (1971) formulated a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry. Halford (1972) has shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity.

Rahaman *et al* (2002), Pradhan and Pandy (2003),Bhowmik and Rajput (2004), Reddy DRK (2005),Nimkar *et.al.*(2014) are some of the authors who have investigated different cosmological models in Lyra's geometry.Recently Tade *et.al.*(2016),Nimkar *et.al.(*2016) and Pund *et.al*(2016) have been studied different cosmological model in Lyra Geometry.
In this paper we have considered N-dimensional Kaluza-Klein string cosmological model in Lyra geometry. Exact solutions of the field equations are obtained. Some physical and geometrical properties of the models are also discussed.

II. METRIC AND FIELD EQUATION :

We consider n- dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 + a^2 \sum_{i=1}^{n-2} dx_i^{\ 2} + b^2 dx_{n-1}^{\ \ 2}, \tag{1}$$

Where R and A are function of time *t*- only.

The relativistic field equations in normal gauge in Lyra's manifold are as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij}\phi_k \phi^k = -T_{ij} \tag{2}$$

Where ϕ_i is a displacement vector field and the other symbols have their usual meaning as in Riemmanian geometry. We now assume the vector displacement field β to be the time like constant vector.

$\phi_i = (0,0,0,\beta)$

Where $\beta = \beta(t)$ is a function of time alone. (3)

The energy momentum tensor for Cosmic String is given by

$$T_i^{\,j} = \rho\, u_i u^j - \lambda\, x_i x^j \quad , \tag{4}$$

where ρ is the rest energy density of cloud of strings with particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative, u^i describes the system four-velocity and x^i represents a direction of anisotropy, i.e. the direction of the strings.

We have $u^i u_i = -x^i x_i = 1$, (5)

and $u^i x_i = 0$. (6)

Using the comoving coordinate system, the non-vanishing components T_j^i can be obtained as

$$T_0^0 = -\rho \quad T_1^1 = T_2^2 = ------= T_{n-2}^{n-2} = 0, T_{n-1}^{n-1} = -\lambda \quad . \tag{7}$$

The Einstein's field equations in general Relativity are

$$R_i^j - \frac{1}{2}R = -8\pi G T_i^j \tag{8}$$

Here we consider geometrized units so that $8\pi G = C = 1$.

The field equation (8) for the metric (1) with the help of (2) to (5) can be written as

$$(n-2)\frac{\dot{a}\dot{b}}{ab} + \left(\frac{n^2-5n+6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 + \frac{3}{4}\beta^2 = \rho \tag{9}$$

$$(n-3)\frac{\ddot{a}}{a} + \left(\frac{n^2-7n+12}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 + (n-3)\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} - \frac{3}{4}\beta^2 = 0 \tag{10}$$

$$(n-2)\frac{\ddot{a}}{a} + \left(\frac{n^2-5n+6}{2}\right)\left(\frac{\dot{a}}{a}\right)^2 - \frac{3}{4}\beta^2 = \lambda \tag{11}$$

III. Solution and the model:

Here we have three independent field equations (9)-(11) connecting five unknown a, b, λ, ρ and β .Therefore in order to obtain exact solutions, we must need one more relation connecting the unknown quantities. We assume the relation between metric coefficients

$$b = a^n .$$

Using this relation, the field equations (9)-(11) admit the exact solution.

$$a = (p+1)^{\frac{1}{p+1}}(k_1 t + k_2)^{\frac{1}{p+1}} \tag{12}$$

$$b = (p+1)^{\frac{n}{p+1}}(k_1 t + k_2)^{\frac{n}{p+1}} \tag{13}$$

With the help of equation (12) and (13), the metric (1) can be expressed as

$$ds^2 = -dt^2 + (p+1)^{\frac{2}{p+1}}(k_1 t + k_2)^{\frac{2}{p+1_1}}\sum_{i=1}^{n-2} dx_l^{\ 2} + (p+1)^{\frac{2n}{p+1}}(k_1 t + k_2)^{\frac{2n}{p+1_1}} dx^2{}_{n-1} \tag{14}$$

Through a proper choice of coordinates and constants of integration, the above equation (14) reduces to

$$ds^2 = -\frac{dT^2}{k_1^{\ 2}} + [(p+1)T]^{\frac{2}{p+1}}\sum_{i=1}^{n-2} dx_i^{\ 2} + [(p+1)\ T]^{\frac{2n}{p+1_1}} dx^2{}_{n-1} \tag{15}$$

Where $T = (k_1 t + k_2)$

The Physical and Kinematical Properties:

The energy density ρ and tension density λ for the model (15) are given by

$$\rho = \lambda = \frac{L}{T^2} \tag{16}$$

where

$$L = \left[n(n-2)\left(\frac{k_1}{p+1}\right)^2 - \frac{(n^2-5n+6)}{2}\left(\frac{k_1}{p+1}\right)^2 + M \right]$$

$$\frac{3}{4}\beta^2 = M\,\frac{1}{(k_1 t + k_2)^2} \tag{17}$$

where $M = (2n-3)\left(\frac{-k_1^2 p}{(p+1)^2}\right) + \left(\frac{5n^2 - 15n + 12}{2}\right)\frac{k_1^2}{(p+1)^2}$

Also,

The spatial Volume , $V = (p+1)^{\frac{-2}{p+1}}\frac{1}{(T)^{\frac{2}{p+1}}}$ (18)

The Scalar expansion, $\theta = \frac{2k_1(n+1)}{3(p+1)}\frac{1}{T}$ (19)

Shear scalar, $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = Const\,\frac{1}{T^2}$ (20)

The model (15) has no initial singularities while the energy density and tension density given by (16) and β given by (17) possess initial singularities. However as T increases these singularities vanish. Spatial volume of the model given by (18) shows anisotropic expansion of the universe (15) with time. For the model the Scalar expansion θ and shear scalar σ tend to zero as $T \to \infty$.

IV Conclusion:

In this paper, we have obtained Kaluza-Kelin string cosmological model in the frame work of Lyra Manifold. For finding the exact solution the relation between the metric potential is used. The model is free from initial singularities and they are expanding, shearing and non- rotating in the standard way. Our model throws some light on the understanding of structure formation of the universe in Lyra's manifold.

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