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Active Control and Dynamical Analysis of two Coupled Parametrically Excited Van Der Pol Oscillators

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Abstract: The dynamical behavior of two coupled parametrically excited van der pol oscillator is investigated by using perturbation method. Resonance cases were obtained, the worst one has been chosen to be discussed. The stability of the obtained numerical solution is investigated using both phase plane methods and frequency response equations. Effect of the different parameters on the system behavior is studied numerically. Comparison between the approximate solution and numerical solution is obtained.

Keywords: Vibration control, nonlinear oscillation, perturbation technique, Resonance cases, Frequency response curves.

I. INTRODUCTION

Vibrations at most time are non-desirable, humans suffered from these bad vibrations, so it must be eliminating or at least controlled. The dynamic absorber is the most common methods for reduced the vibrations. Its importance tends to as it is need low coast, and it is a simple operation at one modal frequency. El-Badawy and Nayfeh [1] adopted linear velocity feedback and cubic velocity feedback control laws.Yang,Cao and Morris[2] use Mat lab for applying numerical methods. Amer [3] investigation the coupling of two nonlinear oscillators of the main system and absorber representing ultrasonic cutting process subjected to parametric excitation force. Non-linearities necessary introduce a whole range of phenomena that are not found in linear system [4], including jump phenomena, occurrence of multiple solutions, modulation, shift in natural frequencies, the generation of combination resonances, evidence of period multiplying bifurcations and chaotic motion [5-8]. In these systems the vibrations are needed to be controlled to minimize or eliminating the hazard of damage or destruction. There are two types for vibration control, active and passive control.

Pinto and Goncalves [9] investigated the active control of the nonlinear vibration of a simply-supported buckled beam under lateral loading. One of most effective tools of passive control is dynamic absorber or the neutralizer [10]. Nabergoj et al [11] studied the stability of auto- parametric resonance in an external excited system. Abdel Hafz and Eissa [12] study the effect of nonlinear elastomeric torsion absorber to control the vibrations of the crank shaft in internal combustion engines, when subject to external excitation torque. Fuller, Elliot and Nelson [13] investigate the active control of vibrations which give many ideas and approaches for controlling chaos. Abe et al. [14] investigate the nonlinear responses of clamped laminated shallow shells with 1:1 internal resonance. Eissa et al. [15-16] investigated saturation phenomena in non-linear oscillating systems subjected to multi-parametric and external excitation. Gerald [17] apply the numerical analysis to find out the solutions for the vibrations problems.

Sayed and Kamel [18-19] investigated the effect of different controllers on the vibrating system and saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. Kamel et al [20] studied the vibration suppression in ultrasonic machining described by non-linear differential equations via passive controller. Elena et al. [21] studied the formal analysis and description of the steady-state behavior of an electrostatic vibration energy harvester operating in constant-charge mode and using different types of electromechanical transducers .Orhan and Peter [22] investigate the effect of excitation and damping parameters on the super harmonic and primary resonance responses of a slender cantilever beam undergoing flapping motion.

In this paper we studied vibration control of a nonlinear system under tuned excitation force. The method of multiple scale method is applied to obtain the approximate solution of the system. Vibration method is used to reduce the amplitude of vibration at the worst resonance case. The effect of different parameters are investigated, the comparison between the numerical solution and approximation solution obtained.

II. MATHEMATICAL MODELING

The considered system is described by the equations:

$$\ddot{X} + (\omega_1^2 - 2\varepsilon f_1 \cos(\Omega_1 t))X - \varepsilon(X^2 + Y^2)X + \varepsilon(\mu_1 + X^2 + aY^2)\dot{X} = -\varepsilon G_1 \dot{X}^3 \quad (1)$$

$$\ddot{Y}+(\omega_2^2-2\varepsilon f_2\cos(\Omega_2 t))Y-\varepsilon(X^2+Y^2)Y+\varepsilon(\mu_2+bX^2+Y^2)\dot{Y}=-\varepsilon G_2\dot{Y}^3 \qquad (2)$$

where the dots indicate differentiation with respect to t, X and Y are the generalized coordinate of the plant (main system) and the controller. ω_1 and ω_2 are incommensurate fundamental frequencies, the parametric excitation frequencies are Ω_1 and Ω_2, the constants a and b are of order 1, and ε is a small parameter, f_1, f_2 are the external excitation forces, G_1, G_2 are the controller of the main system and the absorber.

We can solve equations (1)&(2) analytically using multiple time scale perturbation technique as:

$$X(t,\varepsilon)=x_\circ(T_\circ,T_1)+\varepsilon x_1(T_\circ,T_1)+\varepsilon^2 x_2(T_\circ,T_1)+O(\varepsilon^3) \qquad (3)$$

$$Y(t,\varepsilon)=y_\circ(T_\circ,T_1)+\varepsilon y_1(T_\circ,T_1)+\varepsilon^2 y_2(T_\circ,T_1)+O(\varepsilon^3) \qquad (4)$$

where $T_\circ=t$ represents a fast time scale characterizing motions with the natural and excitation frequencies, and $T_1=\varepsilon t$ represents a slow time scales characterizing modulation and phases of both modes of vibration. The times derivatives transform are reacted in terms of the new time scales as:

$$\frac{d}{dt}=D_\circ+\varepsilon D_1+.............. \qquad (5)$$

$$\frac{d^2}{dt^2}=D_\circ^2+2\varepsilon D_\circ D_1+\varepsilon^2 D_1^2+........ \qquad (6)$$

From equations (3) to (6) we have:

$$\dot{X}(t;\varepsilon)=\sum_{n=0}^{2}\varepsilon^n(D_\circ+\varepsilon D_1)x_n+O(\varepsilon^3) \qquad (7)$$

$$\ddot{X}(t;\varepsilon)=\sum_{n=0}^{2}\varepsilon^n(D_\circ^2+2\varepsilon D_\circ D_1+\varepsilon^2 D_1^2)x_n+O(\varepsilon^3) \qquad (8)$$

$$\dot{Y}(t;\varepsilon)=\sum_{n=0}^{2}\varepsilon^n(D_\circ+\varepsilon D_1)y_n+O(\varepsilon^3) \qquad (9)$$

$$\ddot{Y}(t;\varepsilon)=\sum_{n=0}^{2}\varepsilon^n(D_\circ^2+2\varepsilon D_\circ D_1+\varepsilon^2 D_1^2)y_n+O(\varepsilon^3) \qquad (10)$$

Substituting from equations (3), (4) and (7)-(10) into equations (1) and (2) and equating the same power of ε we have:

$$O(\varepsilon^0):(D_\circ^2+\omega_1^2)x_\circ=0 \qquad (11)$$

$$(D_\circ^2+\omega_2^2)y_\circ=0 \qquad (12)$$

$$\begin{aligned}O(\varepsilon):(D_\circ^2+\omega_1^2)x_1=&-2D_\circ D_1 x_\circ+2f_1 x_\circ\cos(\Omega_1 t)+x_\circ^3+x_\circ y_\circ^2-\mu_1(D_\circ x_\circ)\\&-x_\circ^2(D_\circ x_\circ)-ay_\circ^2(D_\circ x_\circ)+G_1(D_\circ x_\circ)^3\end{aligned} \qquad (13)$$

$$\begin{aligned}(D_\circ^2+\omega_2^2)y_1=&-2D_\circ D_1 y_\circ+2f_2 y_\circ\cos(\Omega_2 t)+y_\circ^3+x_\circ^2 y_\circ-\mu_2(D_\circ y_\circ)\\&-y_\circ^2(D_\circ y_\circ)-bx_\circ^2(D_\circ y_\circ)+G_2(D_\circ y_\circ)^3\end{aligned} \qquad (14)$$

$$\begin{aligned}O(\varepsilon^2):(D_\circ^2+\omega_1^2)x_2=&-D_1^2 x_\circ-2D_\circ D_1 x_1+2x_1 f_1\cos(\Omega_1 t)+3x_\circ^2 x_1+2x_\circ y_\circ y_1\\&+x_1 y_\circ^2-\mu_1(D_1 x_\circ)-\mu_1(D_\circ x_1)-x_\circ^2(D_1 x_\circ)-x_\circ^2(D_\circ x_1)\\&-2x_\circ x_1(D_\circ x_\circ)-ay_\circ^2(D_1 x_\circ)-ay_\circ^2(D_\circ x_1)-2ay_\circ y_1(D_\circ x_\circ)\\&-3G_1(D_\circ x_\circ)^2(D_\circ x_1)-3G_1(D_\circ x_\circ)^2(D_1 x_\circ)\end{aligned} \qquad (15)$$

$$\begin{aligned}(D_\circ^2+\omega_2^2)y_2=&-D_1^2 y_\circ-2D_\circ D_1 y_1+2y_1 f_2\cos(\Omega_2 t)+2x_\circ y_\circ x_1+x_\circ^2 y_1\\&+3y_\circ^2 y_1-\mu_2(D_1 y_\circ)-\mu_2(D_\circ y_1)-b\,x_\circ^2(D_1 y_\circ)-b\,x_\circ^2(D_\circ y_1)\\&-2bx_\circ x_1(D_\circ y_\circ)-y_\circ^2(D_1 y_\circ)-y_\circ^2(D_\circ y_1)-2y_\circ y_1(D_\circ y_\circ)\end{aligned}$$

$$-3G_2(D_\circ y_\circ)^2(D_\circ y_1)-3G_2(D_\circ y_\circ)^2(D_1 y_\circ) \tag{16}$$

The general solution for equations (11) & (12) can be written in the form:

$$x_\circ(T_\circ,T_1)=A(T_\circ,T_1)\,e^{i\omega_1 T_\circ}+\bar{A}(T_\circ,T_1)\,e^{-i\omega_1 T_\circ} \tag{17}$$

$$y_\circ(T_\circ,T_1)=B(T_\circ,T_1)\,e^{i\omega_2 T_\circ}+\bar{B}(T_\circ,T_1)\,e^{-i\omega_2 T_\circ} \tag{18}$$

where A and B are unknown complex functions, which can be determined by imposing the solvability conditions at the next approximation order by eliminating the secular terms, and solving resulting equation gives:

$$x_1=H_1e^{3i\omega_1T_\circ}+H_2e^{i(\omega_1+\Omega_1)T_\circ}+H_3e^{i(\omega_1-\Omega_1)T_\circ}+H_4e^{i(\omega_1-2\omega_2)T_\circ}+H_5e^{i(\omega_1+2\omega_2)T_\circ}+cc \tag{19}$$

$$y_1=H_6e^{3i\omega_2T_\circ}+H_7e^{i(\omega_2+\Omega_2)T_\circ}+H_8e^{i(\omega_2-\Omega_2)T_\circ}+H_9e^{i(2\omega_1+\omega_2)T_\circ}+H_{10}e^{i(2\omega_1-\omega_2)T_\circ}+cc \tag{20}$$

where $(H_n, n=1...10)$ are complex function in T_1 and cc denotes the complex conjugate terms. Substituting equations (17),(18), (19), and (20) into equations (15) and (16) the following are obtained, after eliminating the secular term and solve it :

$$\begin{aligned}x_2=&H_{11}e^{3i\omega_1T_\circ}+H_{12}e^{5i\omega_1T_\circ}+H_{13}e^{i(\omega_1+\Omega_1)T_\circ}+H_{14}e^{i(\omega_1-\Omega_1)T_\circ}+H_{15}e^{i(\omega_1+\Omega_2)T_\circ}+H_{16}e^{i(\omega_1-\Omega_2)T_\circ}\\&+H_{17}e^{i(\omega_1+2\omega_2)T_\circ}+H_{18}e^{i(\omega_1-2\omega_2)T_\circ}+H_{19}e^{i(\omega_1+2\Omega_1)T_\circ}+H_{20}e^{i(\omega_1-2\Omega_1)T_\circ}+H_{21}e^{i(\omega_1+4\omega_2)T_\circ}\\&+H_{22}e^{i(\omega_1-4\omega_2)T_\circ}+H_{23}e^{i(3\omega_1+\Omega_1)T_\circ}+H_{24}e^{i(3\omega_1-\Omega_1)T_\circ}+H_{25}e^{i(3\omega_1+2\omega_2)T_\circ}+H_{26}e^{i(3\omega_1-2\omega_2)T_\circ}\\&+H_{27}e^{i(\omega_1+2\omega_2+\Omega_1)T_\circ}+H_{28}e^{i(\omega_1+2\omega_2-\Omega_1)T_\circ}+H_{29}e^{i(\omega_1-2\omega_2+\Omega_1)T_\circ}+H_{30}e^{i(\omega_1-2\omega_2-\Omega_1)T_\circ}\\&+H_{31}e^{i(\omega_1+2\omega_2+\Omega_2)T_\circ}+H_{32}e^{i(\omega_1+2\omega_2-\Omega_2)T_\circ}+H_{33}e^{i(\omega_1-2\omega_2+\Omega_2)T_\circ}+H_{34}e^{i(\omega_1-2\omega_2-\Omega_2)T_\circ}+cc\end{aligned} \tag{21}$$

$$\begin{aligned}y_2=&H_{35}e^{3i\omega_2T_\circ}+H_{36}e^{5i\omega_2T_\circ}+H_{37}e^{i(\omega_2+\Omega_1)T_\circ}+H_{38}e^{i(\omega_2-\Omega_1)T_\circ}+H_{39}e^{i(\omega_2+\Omega_2)T_\circ}+H_{40}e^{i(\omega_2-\Omega_2)T_\circ}\\&+H_{41}e^{i(\omega_2+2\Omega_1)T_\circ}+H_{42}e^{i(\omega_2-2\Omega_1)T_\circ}+H_{43}e^{i(2\omega_1+\omega_2)T_\circ}+H_{44}e^{i(2\omega_1-\omega_2)T_\circ}+H_{45}e^{i(4\omega_1+\omega_2)T_\circ}\\&+H_{50}e^{i(3\omega_2-\Omega_2)T_\circ}+H_{46}e^{i(4\omega_1-\omega_2)T_\circ}+H_{47}e^{i(2\omega_1+3\omega_2)T_\circ}+H_{48}e^{i(2\omega_1-3\omega_2)T_\circ}+H_{49}e^{i(3\omega_2+\Omega_2)T_\circ}\\&+H_{51}e^{i(2\omega_1+\omega_2+\Omega_1)T_\circ}+H_{52}e^{i(2\omega_1+\omega_2-\Omega_1)T_\circ}+H_{53}e^{i(2\omega_1-\omega_2+\Omega_1)T_\circ}+H_{54}e^{i(2\omega_1-\omega_2-\Omega_1)T_\circ}\\&+H_{55}e^{i(2\omega_1+\omega_2+\Omega_2)T_\circ}+H_{56}e^{i(2\omega_1+\omega_2-\Omega_2)T_\circ}+H_{57}e^{i(2\omega_1-\omega_2+\Omega_2)T_\circ}+H_{58}e^{i(2\omega_1-\omega_2-\Omega_1)T_\circ}+cc\end{aligned} \tag{22}$$

III. STABILITY ANALSIS

After studying the different resonance numerically to see the worst resonance; one of the worst cases has been chosen to study the system stability. The selected resonance case $\Omega=\omega_1$ and $\omega_2=\omega_1$ .in this case we introduce the detuning parameter σ according to:

$$\Omega=\omega_1+\varepsilon\sigma_1 \; and \; \omega_2=\omega_1+\varepsilon\sigma_2 \quad (23)$$

where σ_1 and σ_2 are called detuning parameters. Also for stability investigation, the analysis is limited to the second approximation. So our solution is only depend on $T_\circ$ and T_1. Substituting equation (23) into equations (13) and (14) and eliminating the leads to the solvability conditions:

$$-2i\omega_1\dot{A}+3A^2\bar{A}+2AB\bar{B}-i\omega_1\mu_1A-i\omega_1A^2\bar{A}-2ia\omega_1AB\bar{B}+3iG_1\omega_1^3A^2\bar{A}+f_1\bar{A}e^{i\sigma_1T_1}=0 \tag{24}$$

$$-2i\omega_1\dot{B}+3B^2\bar{B}+2A\bar{A}B-2i\omega_2\mu_2B-i\omega_2B^2\bar{B}-2ib\omega_2A\bar{A}B+3iG_2\omega_2^3B^2\bar{B}+f_2\bar{B}e^{i\sigma_2T_1}=0 \tag{25}$$

To analyze the solution of equations (5.30) and (5.31), it is convenient to express A and B in the polar form as:

$$A(T_1)=\frac{1}{2}a_1(T_1)e^{i\beta_1(T_1)} \qquad \text{and} \qquad B(T_1)=\frac{1}{2}b_1(T_1)e^{i\beta_2(T_1)} \tag{26}$$

where A , B and β_1,β_2 are the steady state amplitudes and phases of the motions respectively. Inserting equation (26) into equations (24) and (25) and equating real and imaginary parts, we obtain:

$$a_1' = -\frac{1}{2}\left[\mu_1 + \frac{1}{2}ab_1^2\right]a_1 - \frac{1}{8}\left[1 + 3G_1\omega_1^2\right]a_1^3 - \frac{f_1}{2\omega_1}a_1 \sin\varphi_1 \tag{27}$$

$$\frac{1}{2}a_1(\sigma_1 - \varphi_1') = -\frac{3}{4\omega_1}a_1^3 - \left[\frac{1}{2\omega_1}b_1^2\right]a_1 - \frac{f_1}{2\omega_1}a_1 \cos\varphi_1 \tag{28}$$

$$b_1' = -\left[\mu_2 + \frac{1}{4}a_1^2 b\right]b_1 - \frac{1}{8}\left[-1 + 3G_2\omega_2^2\right]b_1^3 - \frac{f_2}{2\omega_2}b_1 \sin\varphi_2$$

(29)

$$\frac{1}{2}\mathrm{b}_1(\sigma_2 - \varphi_2') = -\frac{3}{4\omega_2}b_1^3 - \left[\frac{1}{2\omega_2}a_1^2\right]b_1 - \frac{f_2}{2\omega_2}b_1 \cos\varphi_2 \tag{30}$$

where $\varphi_1 = \sigma_1 T_1 - 2\beta_1$ and $\varphi_2 = \sigma_2 T_1 - 2\beta_2$.

For steady state solutions, $a_1' = b_1' = \varphi_1' = \varphi_2' = 0$ and the periodic solution at the fixed points corresponding to equations (27)-(30) is given by:

$$\frac{1}{4}\left[1 + 3G_1\omega_1^2\right]a_1^2 + \left[\mu_1 + \frac{1}{2}ab_1^2\right] = -\frac{f_1}{\omega_1}\sin\varphi_1 \quad (31)$$

$$\sigma_1 + \frac{3}{4\omega_1}a_1^2 + \frac{1}{2\omega_1}b_1^2 = -\frac{f_1}{\omega_1}\cos\varphi_1 \quad (32)$$

$$\frac{1}{8}\left[-1 + 3G_2\omega_2^2\right]b_1^2 + \left[\mu_2 + \frac{1}{4}a_1^2 b\right] = -\frac{f_2}{2\omega_2}\sin\varphi_2 \quad (33)$$

$$\sigma_2 + \frac{3}{4\omega_2}b_1^2 + \frac{1}{2\omega_2}a_1^2 = -\frac{f_2}{2\omega_2}\cos\varphi_2 \quad (34)$$

Squaring equations (5.41) and (5.42) and summation, yields:

$$\left[\frac{1}{4}\left(1 + 3G_1\omega_1^2\right)a_1^2 + \left(\mu_1 - \frac{2}{3}a\omega_2\left(\sigma_2 + \frac{1}{2\omega_2}a_1^2 + \frac{f_2}{\omega_2}\cos\varphi_2\right)\right)\right]^2 + \left[\sigma_1 + \frac{3}{4\omega_1}a_1^2 - \frac{2\omega_2}{3\omega_1}\left(\sigma_2 + \frac{1}{2\omega_2}a_1^2 + \frac{f_2}{\omega_2}\cos\varphi_2\right)\right]^2 = \frac{f_1^2}{\omega_1^2}$$

$$b_1^2 = \frac{-4\omega_2}{3}\left(\sigma_2 + \frac{1}{2\omega_2}a_1^2 + \frac{f_2}{\omega_2}\cos\varphi_2\right) \quad (35)$$

Similarly, from equations (33) and (34), we get:

$$\left[\frac{1}{8}\left(-1 + 3G_2\omega_2^2\right)b_1^2 + \left(\mu_2 + \frac{1}{4}a_1^2 b\right)\right]^2 + \left[\sigma_2 + \frac{3}{4\omega_2}b_1^2 + \frac{1}{2\omega_2}a_1^2\right]^2 = \frac{f_2^2}{4\omega_2^2} \quad (36)$$

From equations (35) and (36), we have the following cases:

Case1: $a = b = 0$ (the trivial solution).

Case2: $a \neq 0, b = 0$, in this case, the frequency response equation (35) is given by:

$$\left[\frac{1}{4}\left(1 + 3G_1\omega_1^2\right)a_1^2\right]^2 + \left[\sigma_1 + \frac{3}{4\omega_1}a_1^2\right]^2 - \frac{f_1^2}{\omega_1^2} = 0 \quad (37)$$

After that we have:

$$\sigma_1^2 + \left[\frac{3a_1^2}{2\omega_1}\right]\sigma_1 + \frac{1}{16}\left[1 + 6G_1\omega_1^2 + 9G_1^2\omega_1^4 - \frac{16f_1}{\omega_1^2 a_1^4} + \frac{9}{\omega_1^2}\right] = 0 \quad (38)$$

The solution of algebraic equation (38) has two roots, given by:

$$\sigma_1 = \frac{1}{2}\left[\frac{3a_1^{\;2}}{2\omega_1} \pm \frac{1}{2}\left[1+6G_1\omega_1^2+9G_1^{\;2}\omega_1^4-\frac{16f_1}{\omega_1^2 a_1^{\;4}}+\frac{18}{\omega_1^2}\right]^{\frac{1}{2}} a_1^2\right] \quad (39)$$

Case3: $a \neq 0, b \neq 0$, in this case the frequency response equations (35) and (36) are given by:

$$\left[\frac{1}{4}\left(1+3G_1\omega_1^{\;2}\right)a_1^{\;2}+\left(\mu_1-\frac{2}{3}a\omega_2\left(\sigma_2+\frac{1}{2\omega_2}a_1^2+\frac{f_2}{\omega_2}\cos\varphi_2\right)\right)\right]^2$$

$$+\left[\sigma_1+\frac{3}{4\omega_1}a_1^2-\frac{2\omega_2}{3\omega_1}\left(\sigma_2+\frac{1}{2\omega_2}a_1^2+\frac{f_2}{\omega_2}\cos\varphi_2\right)\right]^2=\frac{f_1^{\;2}}{\omega_1^{\;2}} \tag{40}$$

2.1 Linear Solution:

To study the stability of the linear solution of the obtained fixed points, let us consider A and B in the polar forms:

$$A(T_1)=\frac{1}{2}\left(p_1-iq_1\right)e^{i\delta_1 T_1}, \qquad B(T_1)=\frac{1}{2}\left(p_2-iq_2\right)e^{i\delta_2 T_1}$$

(41)

where p_1, q_1, p_2 and q_2 are real functions in T_1.

Substituting equation (41) into the linear parts of equations (24) and (25) and equating the imaginary and real parts, we have the following cases:

Case1: $(a \neq 0, b = 0)$

$$p_1'+\frac{1}{2}\mu_1 p_1-\frac{1}{2}\left(-\sigma_1+\frac{f_1}{\omega_1}\right)q_1=0 \tag{42}$$

$$q_1'-\frac{1}{2}\left(\sigma_1-\frac{f_1}{\omega_1}\right)p_1+\frac{1}{2}\mu_1 q_1=0 \tag{43}$$

The stability of the linear solution is obtained from the zero characteristic equation:

$$\begin{vmatrix} -\frac{1}{2}\mu_1-\lambda & \frac{1}{2}\left(-\sigma_1+\frac{f_1}{2\omega_1}\right) \\ \frac{1}{2}\left(\sigma_1+\frac{f_1}{2\omega_1}\right) & -\frac{1}{2}\mu_1-\lambda \end{vmatrix}=0 \tag{44}$$

Then, we have that:

$$\lambda_{1,2}=\frac{1}{2}\left[\mu_1 \pm \sqrt{\frac{f_1^{\;2}}{\omega_1^2}-\sigma_1}\right] \tag{45}$$

Case2: $(a \neq 0, b \neq 0)$

$$p_1'+\frac{1}{2}\mu_1 p_1-\frac{1}{2}\left(-\sigma_1+\frac{f_1}{\omega_1}\right)q_1=0 \tag{46}$$

$$q_1'-\frac{1}{2}\left(\sigma_1-\frac{f_1}{\omega_1}\right)p_1+\frac{1}{2}\mu_1 q_1=0 \tag{47}$$

$$p_2' + \mu_2 p_2 + \frac{1}{2}\left(-\sigma_2 + \frac{f_2}{\omega_2}\right) q_2 = 0 \tag{48}$$

$$q_2' - \frac{1}{2}\left(\sigma_2 + \frac{f_2}{\omega_2}\right) p_2 - \mu_2 q_2 = 0 \tag{49}$$

Equations (46)-(49) can be written in the matrix form:

$$\begin{bmatrix} p_1' \\ q_1' \\ p_2' \\ q_2' \end{bmatrix} = \begin{pmatrix} -\frac{1}{2}\mu_1 & \frac{1}{2}\left[-\sigma_1 + \frac{f_1}{\omega_1}\right] & 0 & 0 \\ \frac{1}{2}\left[\sigma_1 + \frac{f_1}{\omega_1}\right] & -\frac{1}{2}\mu_1 & 0 & 0 \\ 0 & 0 & -\mu_2 & \frac{1}{2}\left[-\sigma_2 + \frac{f_2}{\omega_2}\right] \\ 0 & 0 & \frac{1}{2}\left[\sigma_1 + \frac{f_2}{\omega_2}\right] & -\mu_2 \end{pmatrix} \begin{bmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{bmatrix} \tag{50}$$

The stability of a particular fixed point with respect to perturbation proportional to $\exp(\lambda T_1)$ is determined by zeros of characteristic equation:

$$\begin{vmatrix} -\frac{1}{2}\mu_1 - \lambda & \frac{1}{2}\left[-\sigma_1 + \frac{f_1}{\omega_1}\right] & 0 & 0 \\ \frac{1}{2}\left[\sigma_1 + \frac{f_1}{\omega_1}\right] & -\frac{1}{2}\mu_1 - \lambda & 0 & 0 \\ 0 & 0 & -\mu_2 - \lambda & \frac{1}{2}\left[-\sigma_2 + \frac{f_2}{\omega_2}\right] \\ 0 & 0 & \frac{1}{2}\left[\sigma_1 + \frac{f_2}{\omega_2}\right] & -\mu_2 - \lambda \end{vmatrix} = 0 \tag{51}$$

After extract we obtain that:

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0 \tag{52}$$

According to Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied:

$$r_1 > 0, r_1 r_2 - r_3 > 0, r_3(r_1 r_2 - r_3) - r_1^2 r_4 > 0, r_4 > 0. \tag{53}$$

2.2 Non-Linear Solution:

To determine the stability of the fixed points, one lets:

$$a = a_\circ + a_1, b = b_\circ + b_1, \varphi_m = \varphi_{m0} + \varphi_{m1} (m = 1,2) \tag{54}$$

where $a_\circ, b_\circ$ and φ_{m0} are solutions of equations (27)-(30) and a_1, b_1, φ_{m1} are perturbations which are assumed to be small comparing to $a_\circ, b_\circ$ and φ_{m0}. Substituting equation (54) in to equations (27)-(30) and keeping only the linear terms in $a_{11}, b_{11}, \varphi_{m1}$ we obtain:

1. For the case $(a \neq 0, b = 0)$, we have:

$$a'_{11} = [\frac{-1}{2}\mu_1 - \frac{3}{8}a_{10}^2 - \frac{9}{8}\omega_1{}^2 G_1 a_{10}^2 + \frac{1}{2\omega_1}f_1 \sin\varphi_{10}]a_{11} + [\frac{1}{2\omega_1} f_1 \mathrm{a}_{10}\cos\varphi_{10}]\varphi_{11} \tag{55}$$

$$\varphi'_{11} = [\frac{\sigma_1}{a_{10}} + \frac{9}{4\omega_1}a_{10} + \frac{1}{\omega_1 a_{10}}f_1\cos\varphi_{11}]a_{11} - [\frac{1}{\omega_1}f_1\sin\varphi_{10}]\varphi_{11} \tag{56}$$

The stability of a given fixed point to a disturbance proportional to $\exp(\lambda t)$ is determined by the roots of:

$$\begin{bmatrix} \frac{-1}{2}\mu_1 - \frac{3}{8}a_{10}^2 - \frac{9}{8}\omega_1{}^2 G_1 a_{10}^2 + \frac{1}{2\omega_1}f_1\sin\varphi_{10} & \frac{1}{2\omega_1}f_1 \mathrm{a}_{10}\cos\varphi_{10} \\ \frac{\sigma_1}{a_{10}} + \frac{9}{4\omega_1}a_{10} + \frac{1}{\omega_1 a_{10}}f_1\cos\varphi_{11} & \frac{-1}{\omega_1}f_1\sin\varphi_{10} \end{bmatrix} = 0 \tag{57}$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (57) are less than zero.

2. For the practical solution $(a \neq 0, b \neq 0)$, we have:

$$a'_{11} = [\frac{-1}{2}\mu_1 - \frac{3}{8}a_{10}^2 - \frac{9}{8}\omega_1{}^2 G_1 a_{10}^2 + \frac{1}{2\omega_1} f_1\sin\varphi_{10}]a_{11} - [\frac{1}{2}a_{10}b_{10}]b_{11} + [\frac{1}{2\omega_1} f_1 \mathrm{a}_{10}\cos\varphi_{10}]\varphi_{11} \tag{58}$$

$$\varphi'_{11} = [\frac{\sigma_1}{a_{10}} + \frac{9}{4\omega_1}a_{10} - \frac{1}{2\omega_1 a_{10}}b_{10}{}^2 + \frac{1}{\omega_1 a_{10}} f_1\cos\varphi_{11}]a_{11} - [\frac{1}{\omega_1}b_{10}]b_{11} - [\frac{1}{\omega_1} f_1\sin\varphi_{10}]\varphi_{11} \tag{59}$$

$$b_{11}{}' = [\frac{-1}{2}ba_{10}b_{10}]a_{11} + [\frac{-1}{2}\mu_2 - \frac{3}{8}b_{10}^2 - \frac{1}{4}ba_{10}^2 - \frac{9}{8}\omega_2^2 G_2 b_{10} + \frac{1}{2\omega_2} f_2\sin\varphi_{20}]b_{11}\varphi'_{11} + [\frac{1}{2\omega_2} f_2 b_{10}\cos\varphi_{20}]\varphi_{21} \tag{60}$$

$$\varphi'_{21} = [\frac{1}{\omega_2}a_{10}]a_{11} + [\frac{\sigma_2}{b_{10}} + \frac{9}{4\omega_2}b_{10} + \frac{1}{\omega_2 b_{10}}f_2\cos\varphi_{20}]b_{11} - [\frac{1}{\omega_2}f_2\sin\varphi_{20}]\varphi_{21} \tag{61}$$

The stability of a particular fixed point with respect to perturbations proportional to $\exp(\lambda t)$ depends on the real parts of the roots of the matrix. Thus, a fixed point given by equations (58)-(61) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

IV. NUMERICAL RESULTS:

The nonlinear dynamical system is solved numerically using Rung -Kutta fourth order method by using Maple 16 software. At non resonance case (basic case) as shown in Fig.1, we can see that the steady state amplitude without controller is about 0.08 and with controller is about0.01.

3.1) Resonance cases

Sub harmonic resonance, $\Omega_1 = 2\omega_1$, for the main system the phase plane is a limit cycle and its steady state amplitude which is the largest without controller is about 0.9 and about 0.17, which appears in Fig.(3). In Fig (4), Sub harmonic resonance $\Omega_2 = 2\omega_2$, for the main system the phase plane is a limit cycle and its steady state amplitude without controller is about 0.0004 and about 0.02.

3.2) Effect of control

The effect of controller appears at Fig.(5) and Fig.(6), in Fig.(5), the amplitude of the main system at Sub harmonic resonance: when $\Omega_1 = 2\omega_1$, is decreasing to 0.05. Similarly for the Fig.(6), the amplitude of the main system Sub harmonic resonance: when $\Omega_2 = 2\omega_2$ decreasing to 0.01.

3.3) Effect of parameters

For the damping coefficient μ_1, Fig(7) (a) shows that the steady state amplitude of the main system is monotonic increasing function. For the parameter ω_1, Fig.(7) (b) shows that the steady state amplitude of the main system is monotonic decreasing function. But that the steady state amplitude of the main system is monotonic increasing function of the excitation forces f_1, f_2, Fig (7) (c),(f) shows.

3.4) Frequency response curves

The frequency response in the second case $(a \neq 0, b = 0)$ which represented by equation (37) is a nonlinear algebraic equation solved numerically of the amplitude against the detuning parameter σ_1. Fig.(8)(a) shows that the steady state amplitude is monotonic decreasing function on the natural frequency. Fig.(8)(b) shows that the steady state amplitude is a monotonic increasing function in the non-linear parameter μ_1. The excitation force f_1 of the main system is a monotonic increasing function at the steady state amplitude which appeared on the Fig.(8)(c). The steady state amplitude is a monotonic decreasing function on the gain G_1.

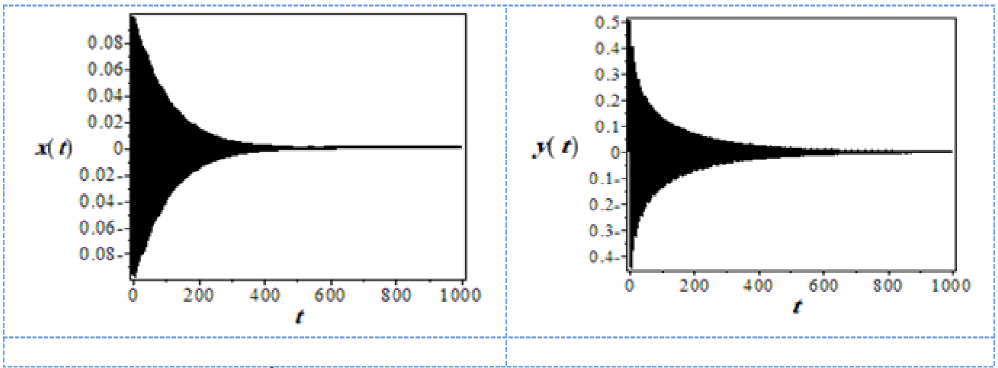


Fig.(1) Non-resonase case (basic case) of the system without controller.

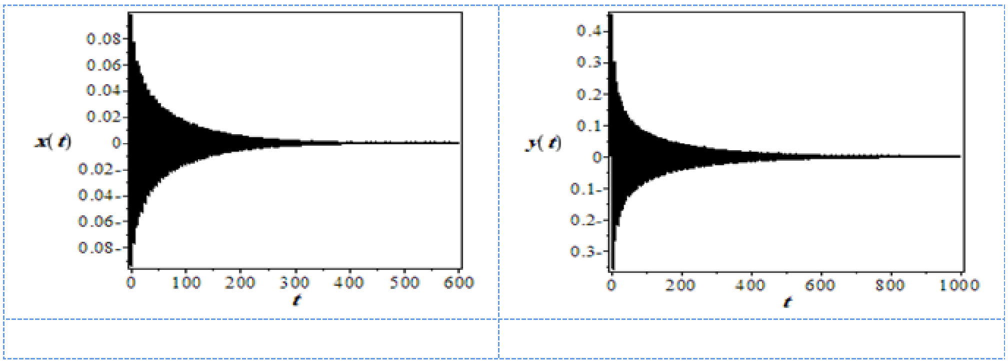
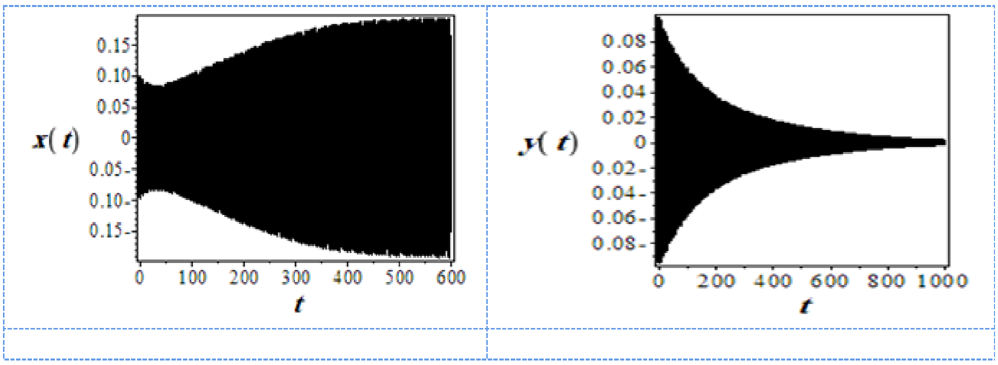
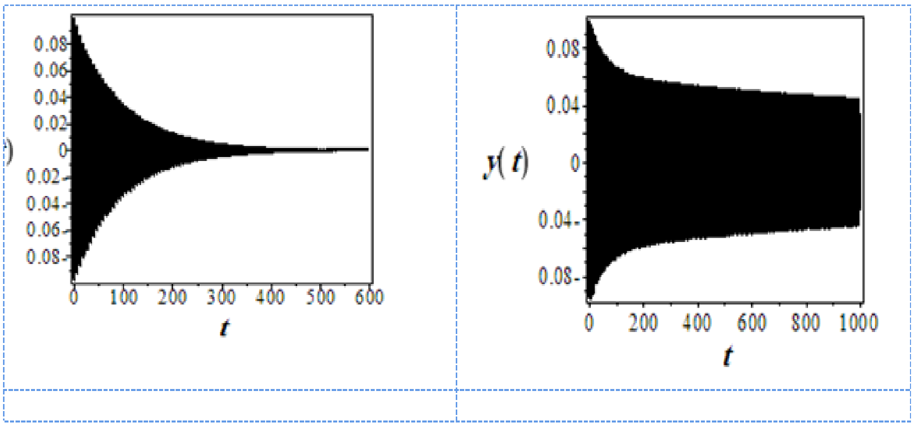


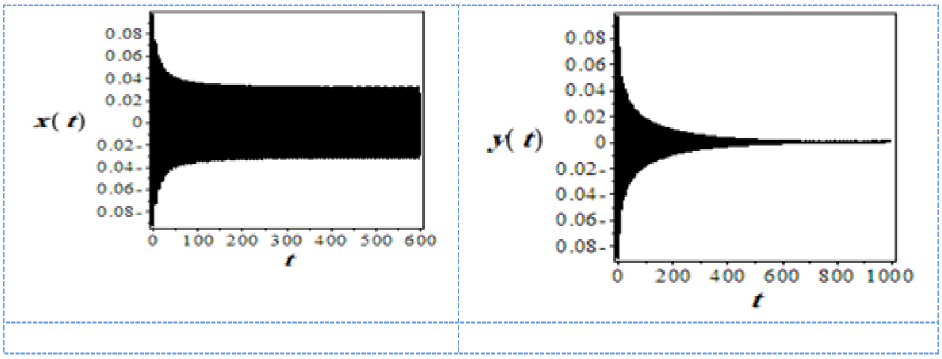
Fig.(2) : Non-resonase case (basic case) of the system with controller.



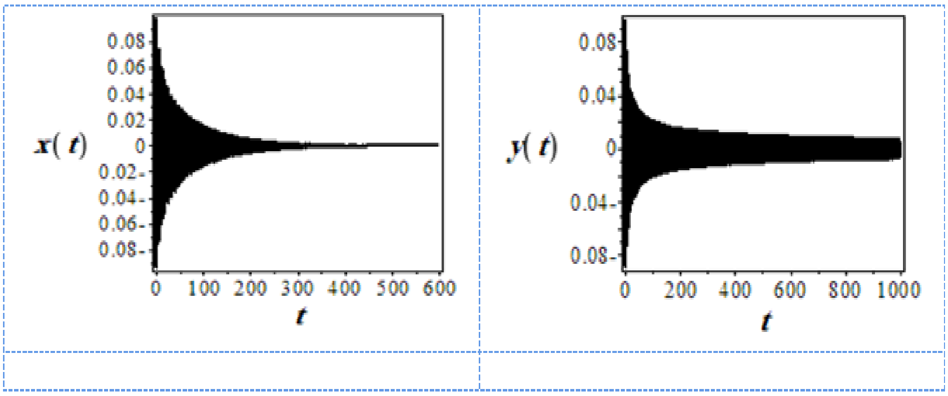
Fig(3): System behavior without controller at sub-harmonic resonance case $\Omega_1 = 2\omega_1$



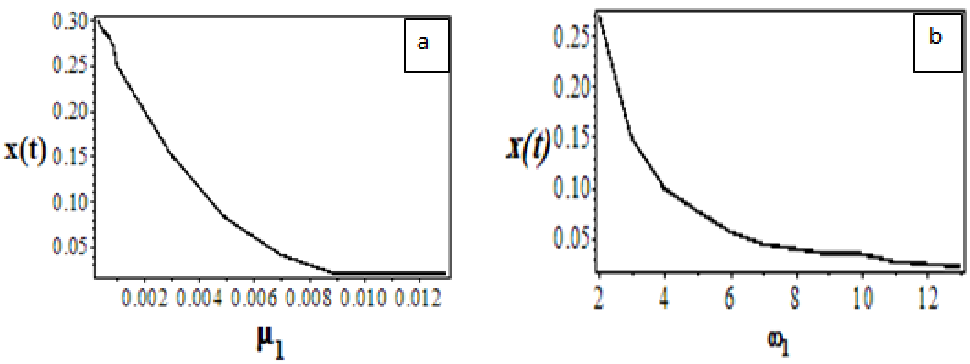
Fig(4): System behavior without controller at sub-harmonicresonance case $\Omega_2 = 2\omega_2$



Fig(5): system behavior with controller at sub-harmonicresonance case $\Omega_1 = 2\omega_1$



Fig(6): system behavior with controller at sub-harmonicresonance case $\Omega_2 = 2\omega_2$



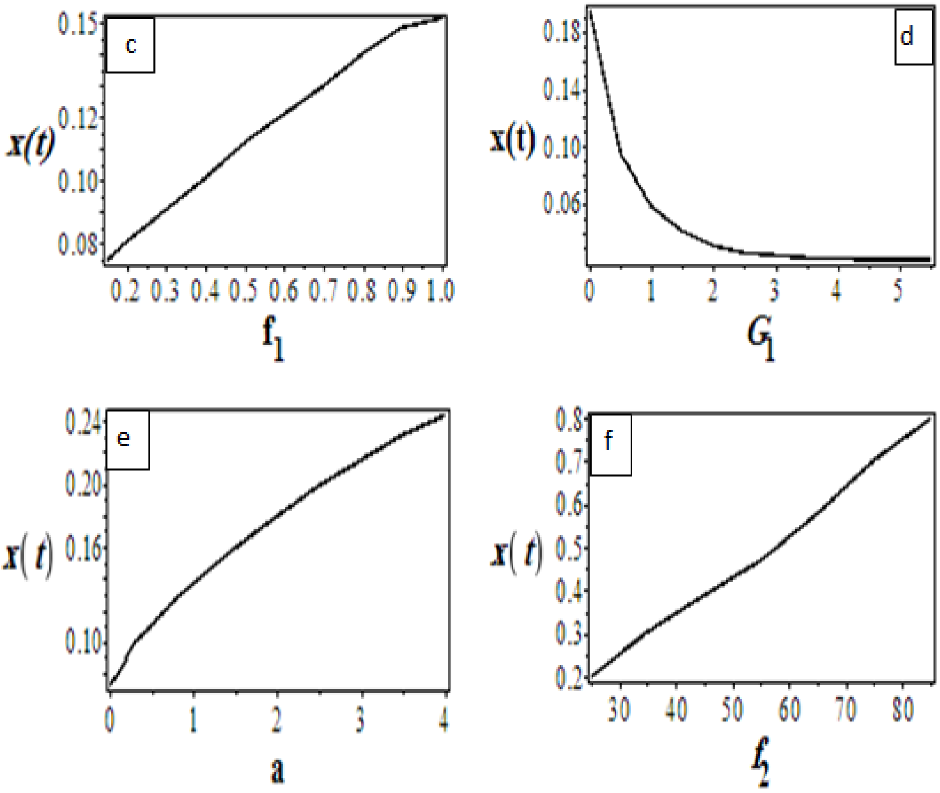
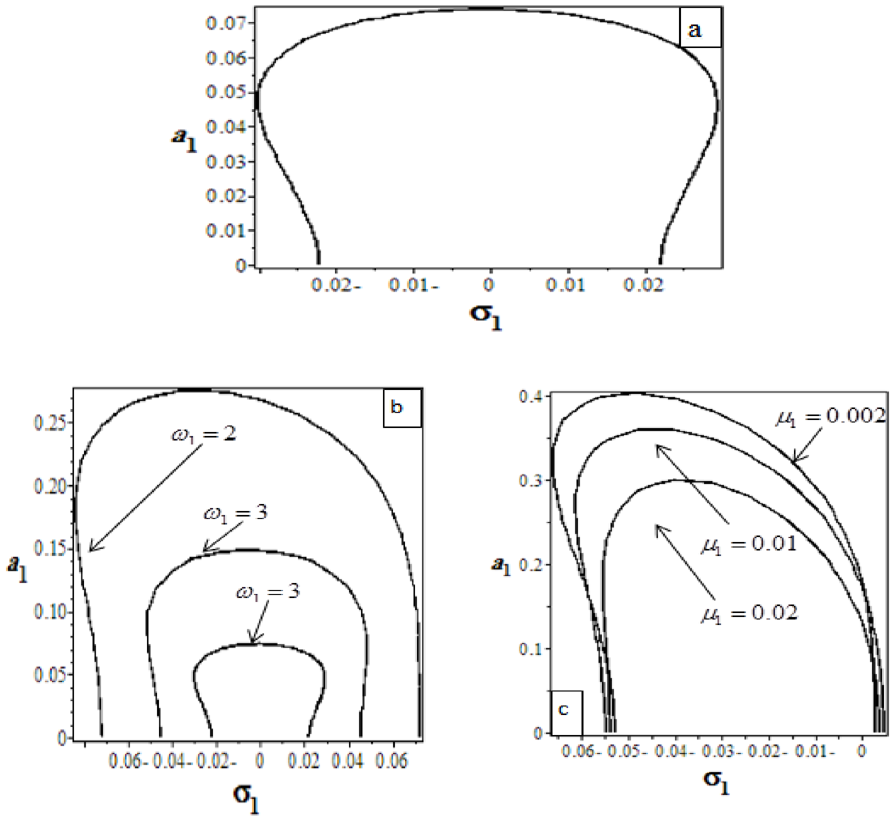


Fig.(7), Effects of different parameters.



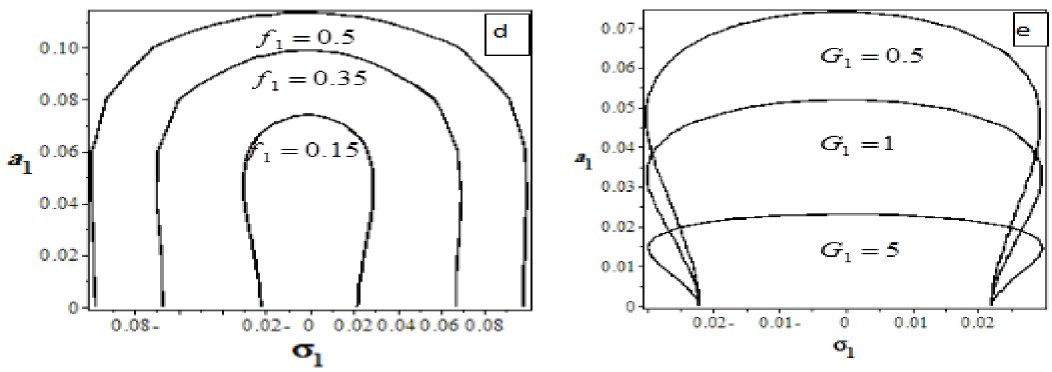


Fig.(8): Response curves at $(a \neq 0, b = 0)$.

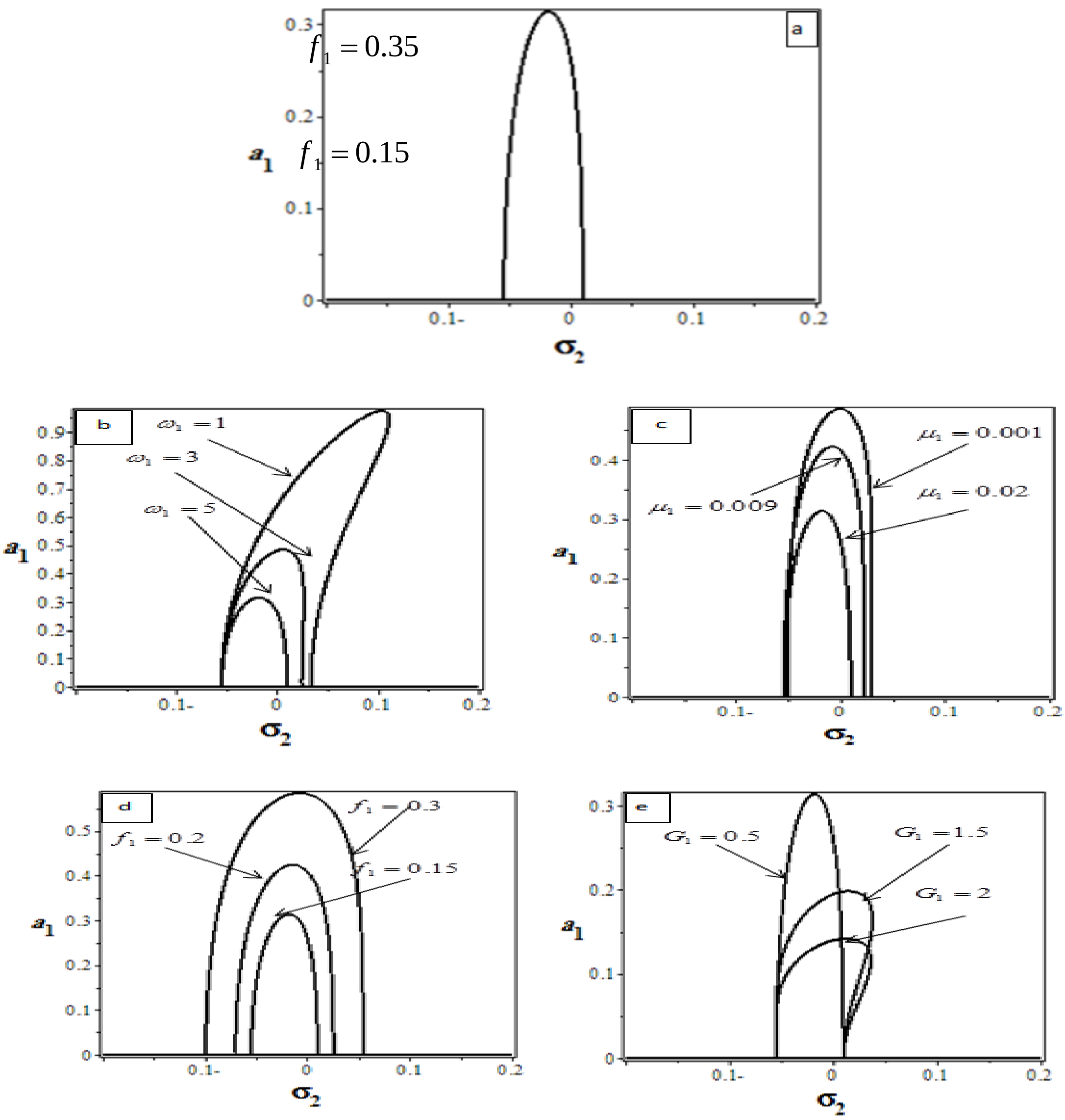


Fig.(9): Response curves at $(a \neq 0, b \neq 0)$

4) Comparison between approximation solution and numerical solution

In this subsection we compare between the numerical solutions (which we obtained by using Maple 16 software) and the approximation solution (which we obtained by using equations (58) to (61)). All this comparison done in the resonance sub-harmonic case which we choose to be the worst we obtained that there is a good agreement between the two solutions.

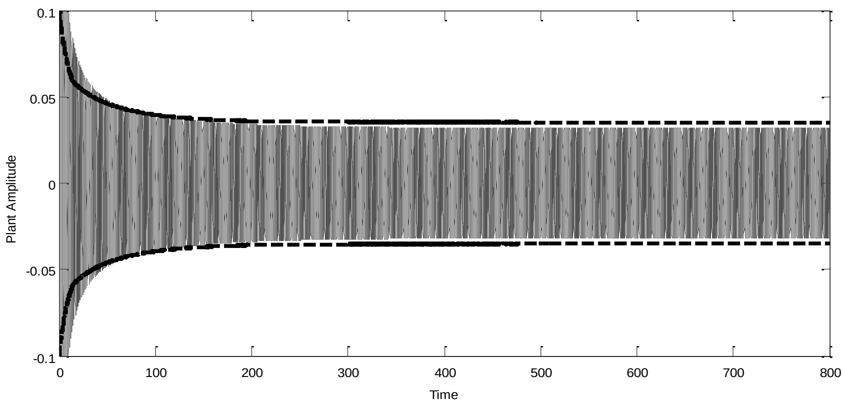


Fig.(10): the comparison between the numerical solution (______)and approximation solution(----------)

V. CONCLUSION

The dynamical behavior of two coupled parametrically excited van der pol oscillator is investigated by using perturbation method. Resonance cases were obtained, the worst one has been chosen to be discussed which is $\Omega_1 = 2\omega_1$, .Hence the stability of the system and controller is studied using the frequency response functions from the above study, the following results are concluded:

1. The steady state of the system without controller is about 0.08 which is considering basic case.
2. The damping coefficient μ_1 is monotonic decreasing function.
3. The natural frequency ω_1 is monotonic decreasing function.
4. The forces f_1, f_2 are monotonic increasing functions.
5. The numerical solution has a good agreement with the approximation solution.

REFERENCES:

[1]. El-BadawyA A and Nayfeh,A H,Control of a directly excited structural dynamic model of an F-15 tail section, *journal of the Franklin Institute* 338 (2001),133-147.
[2]. Yang,WYY, Cao, W, Chung,T-SMorris,J, Applied Numerical Methods Using Matlab. John Wiley & Sons, Inc; (2005).
[3]. Amer YA, Hamed YS, Stability and response of ultrasonic cutting via dynamic absorber, Chaos SolitonsFract. 34 (2) (2007)1328-1345.
[4]. Elnaschie MS. Stress, Stability and Chaos. Mc Graw-Hill International Editions (1992). Singapore. Copyright Mc Graw Hill United Kingdom (1990).
[5]. Nayfeh AH, Mook RT. Nonlinear oscillations. New York: John Wiley; (1979).
[6]. Cartmell MP, Lawson J, Performance enhancement of an auto-parametric vibration absorber by means of computer control. J. sound Vib. 177(2)(1994)173-195.
[7]. Woafa P, Fotsin HB, Chedjou JC, Dynamics of two nonlinear coupled oscillators. PhysScr 57(1998)195-200.
[8]. Nayfeh AH, Sanchez NE. Bifurcations in a forced softening doffing oscillators. Int j Non-linear Mech 24(6)(1989):483-497.
[9]. Pinto OC and P.B. Goncalves, Active non-linear control of buckling and vibrations of a flexible buckled beam. Choas, solitons and Fractals 14 (2002), 227-239.
[10]. Shen, WeiliGuo TY, Torsional vibration control of a shaft through active constrained layer damping treatments, j. Vib. Acoust. 119(1997) 504-511.
[11]. Nabergoj R, Tondl A, Ving Z. Auto parametric resonance in an externally excited system. Chaos, Solitons&Franctals 4(1994) 263-273.
[12]. Abdel Hafez HM, Eissa M, Stability and control of non-linear torsional vibrating systems, Faculty of Engineering Alexandria University, Egypt 41 (2)(2002):343-353.
[13]. Fuller CR, Elliott,SJ,Nelson PA, Active Control of Vibration, Academic press, New York,1997.
[14]. Abe,A,.Kobayashi,YYamada,Y, Nonlinear dynamic behaviors of clamped laminated shallow shells with one-to-one internal resonance. J Sound Vibrat. 304 (2007) 957-58.
[15]. Eissa M, El-Ganaini W, Hamed YS, Saturation, stability and resonance of nonlinear systems, Phys. A 356(2005) 341-358).

[16]. Eissa M, El-Ganaini W, Hamed YS, On the saturation phenomena and resonance of non-linear differential equations, Minnufiya J. Electron. Eng. Res MJEER 15 (1)(2005)73-84.
[17]. Gerald,GF, Applied Numerical Analysis. Reading, MA: Addison-Wesley; (1980).
[18]. Sayed M, Kamel M, Stability study and control of helicopter blade flapping vibrations, Appl. Math. Model.35:2820-2873; (2011).
[19]. Sayed M, Kamel M, 1:2 and 1:3 internal resonance active absorber for non-linear vibrating system. 36(2012) 310-332.
[20]. Kamel M, El-Ganaini W, Hamed YS, Vibration suppression in ultrasonic machining described by non-linear differential equations via passive controller I Appl Math Comput. 219(2013) 4692-4701.
[21]. B. Elena, G. Dimitri, B. Philippe and F. Orla, Steady-State Oscillations in Resonant Electrostatic Vibration Energy Harvesters, IEEE Transactions on Circuits and Systems I, 60 (2015) 875-884.
[22]. O. Orhan and J. A. Peter, Nonlinear response of flapping beam to resonant excitations under nonlinear damping, ActaMech (2015) 1-27.

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