International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 6, Issue 3 (March 2017), PP.01-09

A New Double Numerical Integration Formula Based On The First Order Derivative

Guo Ye
(School Of Mechanical Engineering,Shanghai University Of Engineering Science,Shanghai 201600, P. R. China)

ABSTRACT: A new double numerical integration formula based on the value of integrated function and first order derivative of the integrable function was proposed. Different from the traditional mechanical quadrature formula, contibuted integral function and first order derivative of the integral function. Four nodes are selected appropriately in the integral interval. Used the value of function and first order derivative, constructed a new numerical integration formula that achieve seven order algebraic precision. Then analysed the algebraic precision、remainder、stypticity and stability of the formula. Then generalized the formula into double integral. In the end, according to the two typical examples vertified the formula's validity and fesibility. This formula enrich the content of numerical calculation. Provided a new method for solving double numerical integrations.
Keywords: derivative;numerical integral formula; remainder; stypticity; stability

I. INTRODUCTION

With the rapid development of computer technology and science and technology, people not only in the field of signal processing, automatic control, finite element analysis and wavelet analysis of science and engineering will often encounter integral computational problems, and theoretical research in the field of mechanics also often encounter integral computational problems, for example, is based on the absolute nodal coordinate beam element internal force matrix and the stiffness matrix and solve the bending torsional buckling load and solving problems in particle mechanics. But now, when people encounter a unary function integral problem, often will first think of Newton - Leibniz:

$$\int_a^b f(x)\mathrm{d}x=\mathrm{F}(b)-\mathrm{F}(a).$$

However, in reality the Newton Leibniz formula has obvious limitations, because there are a lot of the integrand can be expressed by elementary functions, for example,e^{-x^2}、$sinx/x(x\neq 0)$ and so on.Even if the original function of the integrand is obtained, the calculation is very difficult, for example, $f(x)=1/(1+x^6)$.The original function is

$$\mathrm{F}(x)=\frac{1}{3}arctan(x)+\frac{1}{6}arctan(x-\frac{1}{x})+\frac{1}{4\sqrt{3}}In\frac{x^2+x\sqrt{3}+1}{x^2-x\sqrt{3}+1}+\mathrm{C}.$$

Calculate $\mathrm{F}(a)$ and $\mathrm{F}(b)$ is very difficult. In addition, when the function $f(x)$ in the integration interval in the discontinuous point, can not be directly used Newton Leibniz formula. Therefore, the numerical integration method of [1-3]. such as Newton Cotes formula, Rhomberg Gauss, quadrature quadrature formula. It can be said that the problem of numerical integral of a function have been solved. But so far, the numerical calculation of multiple integrals is still not a particularly effective solution. The main reason is that the multi-dimensional interpolation problem is an unsolved problem, and the integral region of multiple integral than the integral area of the single integral complex, so in the calculation problem of dealing with multiple numerical integration, to construct a numerical integration formula of a high accuracy then, it is extended to multiple numerical integral is a more reasonable method. In reference [4-10], a number of articles are used by the method, but they have some problems. For example, reference [4] first uses four nodes to construct a numerical integration formula, the formula of the coefficient is complex. In addition, the author will not be directly extended to double the formula but the first integral, the formula of complex, then the complex numerical integration formula of generalized to the double integral. Indeed, it can effectively improve the accuracy of double numerical integration formula, but it will also result in a substantial increase in the amount of calculation, it is difficult to manually calculated results, and complex the formula is not easy programming. Therefore, high precision calculation method in reference [4], but practicality is questionable. For example Ref. [10], the integrand two derivative to construct a formula of numerical integration, and then directly to the The formula is generalized to the double integral method, the numerical structure of double integral formula is feasible, but the formula form is complex, large amount of calculation. Some theoretical literature is too strong, it's hard to understand, which had a negative impact on the formula is practical. The main purpose is to construct the numerical integral formula the actual application, if practical formula is not strong, so the value of the formula will be greatly diminished. And the practical formula depends on the calculation quantity and calculation precision, is easy to program, easy to understand and so on. Therefore, this paper aims to construct a simple, less calculation and high precision, easy programming and practical double integral formula.

II. STRUCTURAL FORMULA

2.1 Construction of one dimensional numerical integration formula

It is assumed that the integrand $f(x)$ in the interval [a, b] is smooth enough and the first derivative values at any point in the interval exist. In the integral interval [a, b] node selection $a, \frac{2a+b}{3}, \frac{a+2b}{3}, b$. Known integrand $f(x)$ on the a, $\frac{2a+b}{3}$, $\frac{a+2b}{3}$, b .function value on $f(a)$, $f(\frac{2a+b}{3})$, $f(\frac{a+2b}{3})$, $f(b)$. And first order derivative $f'(a), f'(\frac{2a+b}{3}), f'(\frac{a+2b}{3}), f'(b)$.The numerical integral formula is constructed

$$\int_a^b f(x)dx \approx h[A_0 f(a)+A_1 f(\frac{2a+b}{3})+A_2 f(\frac{a+2b}{3})+A_3 f(b)]+h^2[B_0 f'(a)+B_1 f'(\frac{2a+b}{3}) +B_2 f'(\frac{a+2b}{3})+B_3 f'(b)] \quad (1)$$

Among $h=(b-a)/2, A_i$、$B_i (i=0,1,2,3)$

According to the concept of algebraic precision, the value of the coefficient is determined.And the numerical quadrature formula (1) with the algebraic precision as high as possible,Just make formula (1) $f(x)=1,x,x^2 \ldots x^7$ can accurately set up, you can obtain.

$$A_0=\frac{31}{112}, A_1=\frac{81}{112}, A_2=\frac{81}{112}, A_3=\frac{31}{112}, B_0=\frac{19}{840}, B_1=-\frac{9}{280}, B_2=\frac{9}{280}, B_3=-\frac{19}{840}$$

$$\int_a^b f(x)dx \approx \frac{h}{112}[31f(a)+81f(\frac{2a+b}{3})+81f(\frac{a+2b}{3})+31f(b)]+\frac{h^2}{280}[\frac{19}{3}f'(a)-9f'(\frac{2a+b}{3})+9f'(\frac{a+2b}{3}) -\frac{19}{3}f'(b)] \quad (2)$$

The formula (2) has a precision of 7 algebra.

The following formula (2) to analyze and study the remainder, convergence and stability.

It is known that the formula (2) the algebraic precision of 7, through the remainder of quadrature formula expression [1], the formula (2) the remainder is expressed as follows:

$$R[f]=\int_a^b f(x)\,dx-h\sum_{i=0}^{3}A_i f(x_i)-h^2\sum_{i=0}^{3}B_i f'(x_i)=Kf^{(8)}(\eta)\qquad (a<\eta<b)\tag{3}$$

Where K is an undetermined parameter, the formula (3) shows that when the integrand $f(x)$ is not more than 7 polynomial,so $f^{(8)}(x)=0$ .Here $R[f]=0$ .The structure of the quadrature formula (2) holds exactly.When $f(x)=x^8$, $f^{(8)}(x)=8!$.Type (3) is not equal to 0, so it can be obtained

$$K=\frac{1}{8!}\left[\frac{1}{9}(b^9-a^9)-h\sum_{i=0}^{3}A_i x_i^8-8h^2\sum_{i=0}^{3}B_i x_i^7\right]\tag{4}$$

The type (4) into equation (3) can be obtained by the formula (2) of the remainder.
For quadrature formula (2), there is

$$\lim_{\substack{n\to\infty\\ h\to 0}}\sum_{i=0}^{n}\left[hA_i f(x_i)+h^2 B_i f'(x_i)\right]=\int_a^b f(x)\,dx\quad (5)$$

According to the definition of convergence of the mechanical quadrature formula, the quadrature formula (2) is convergent.

In the quadrature formula (2), because in the calculation of $f(x_i)$ may produce error δ_i, which leads to the actual values obtained for $\tilde{f}_i$, i.e. $f(x_i)=\tilde{f}_i+\delta_i$.

$$I_n(f)=h\sum_{i=0}^{n}A_i f(x_i)+h^2\sum_{i=0}^{n}B_i f'(x_i)\,,\quad I_n(\tilde{f})=h\sum_{i=0}^{n}A_i\tilde{f}_i+h^2\sum_{i=0}^{n}B_i\tilde{f}'_i$$

For a given arbitrarily small positive ε, have

$$\left|I_n(f)-I_n(\tilde{f})\right|=\left|h\sum_{i=0}^{n}A_i\left[f(x_i)-\tilde{f}_i\right]+h^2\sum_{i=0}^{n}B_i\left[f'(x_i)-\tilde{f}'_i\right]\right|\le\varepsilon\tag{6}$$

Type (6) shows that the error of delta I is sufficiently small, the quadrature formula (2) is stable.

2.2 The construction of double numerical integration formula

The formula (2) is generalized to the double integral

$$II=\iint_D f(x,y)\,dx\,dy$$

The integrand $f(x,y)$ on the integral region D is sufficiently smooth to obtain.

$$II=\iint_D f(x,y)\,dx\,dy=\int_a^b dx\int_{c(x)}^{d(x)} f(x,y)\,dy$$

Among

$$g(x)=\int_{c(x)}^{d(x)} f(x,y)dy$$

If the x value is determined, then $g(x)$ is a definite integral, and it is smooth in interval $[a,b]$.

$$II=\int_a^b dx\int_{c(x)}^{d(x)} f(x,y)\,dy=\int_a^b g(x)dx\qquad (7)$$

Import mark $g_i=g(x_i),g'_i=g'(x_i)(i=0,1,2,3)$.Then apply the formula (2) to formula (7)

$$II=\int_a^b g(x)dx\approx\frac{h}{112}\left[31g_0+81g_1+81g_2+31g_3\right]+\frac{h^2}{280}\left[\frac{19}{3}g'_0-9g'_1+9g'_2-\frac{19}{3}g'_3\right]\tag{8}$$

Among

$$h=\frac{b-a}{2},x_i=a+ih_0,h_0=\frac{b-a}{3},i=0,1,2,3.$$

The formula (8) for the remainder: $R[g]=Kg^{(8)}(\eta)$, $a<\eta<b$.

The following solutions for g_0,g_1,g_2,g_3 and g'_0,g'_1,g'_2,g'_3.

Application of formula (2) on interval $[c(x_i),d(x_i)]$.

$$g_i=g(x_i)=\int_{c(x_i)}^{d(x_i)} f(x_i,y)\,dy\approx\frac{h_i}{112}[31f(x_i,c(x_i))+81f(x_i,\frac{2c(x_i)+d(x_i)}{3})+81f(x_i,\frac{c(x_i)+2d(x_i)}{3})+$$

$$31f(x_i,d(x_i))]+\frac{h_i^2}{280}[\frac{19}{3}f_y(x_i,c(x_i))-9f_y(x_i,\frac{2c(x_i)+d(x_i)}{3})+9f_y(x_i,\frac{c(x_i)+2d(x_i)}{3})-$$

$$\frac{19}{3}f_y(x_i,d(x_i))] \qquad (9)$$

Among

$$h_i=\frac{d(x_i)-c(x_i)}{2},i=0,1,2,3.$$

The formula (9) for the remainder: $R[f(x_i,y)]=Kf_y^{(8)}(\eta_1),a<\eta_1<b$.

$$g'_i=g'(x_i)=\frac{d}{dx}\left[\int_{c(x_i)}^{d(x_i)} f(x_i,y)\,dy\right]=\int_{c(x_i)}^{d(x_i)} f_x(x_i,y)dy+f(x_i,d(x_i))d'(x_i)-f(x_i,c(x_i))c'(x_i)$$

$$\approx\frac{h_i}{112}\left[31f_x(x_i,c(x_i))+81f_x(x_i,\frac{2c(x_i)+d(x_i)}{3})+81f_x(x_i,\frac{c(x_i)+2d(x_i)}{3})+31f_x(x_i,d(x_i))\right]$$

$$+\frac{h_i^2}{280}\left[\frac{19}{3}f_{xy}(x_i,c(x_i))-9f_{xy}(x_i,\frac{2c(x_i)+d(x_i)}{3})+9f_{xy}(x_i,\frac{c(x_i)+2d(x_i)}{3})-\frac{19}{3}f_{xy}(x_i,d(x_i))\right]$$

$$+f(x_i,d(x_i))d'(x_i)-f(x_i,c(x_i))c'(x_i) \qquad (10)$$

The formula (10) for the remainder: $R\left[f_x(x_i,y)\right]=Kf_{xy}^{(8)}(\eta_2)$, $a<\eta_2<b$.

Put the g_i,g_i' (i=0,1,2,3) back to the formula (8) can get double integral formula, the formula for the remainder term.

$$R[f]=R[g]+\frac{h}{112}\{31R[f(x_0,y)]+81R[f(x_1,y)]+81R[f(x_2,y)]+31R[f(x_3,y)]\}$$

$$+\frac{h^2}{280}\left\{\frac{19}{3}R[f_x(x_0,y)]-9R[f_x(x_1,y)]+9R[f_x(x_2,y)]-\frac{19}{3}R[f_x(x_3,y)]\right\} \qquad (11)$$

III. CONCLUSION

The numerical integration method not only in many engineering fields has important applications and theoretical research in the field of mechanics also have a wide range of applications. So the research and construct new numerical integration formula is of great significance. The first derivative of the integrand function values and the integrand function based on structure using a 4 node, to achieve 7 numerical integration formula of algebraic precision. And the formula of algebraic precision and error, convergence and stability are analyzed, then the formula is generalized to the double integral. Finally, by solving two examples can be seen, double integral formula used in this paper to to solve the double integral, the solution accuracy is very high, can be used as the integral value of the approximate solution. And this paper method in calculation precision and calculation, all method is better than in the reference [10]. Don't is when the integral area is rectangular, the use of the method in this paper is more convenient. We assume that the integral area is X-, but for Y- type integral region can also through similar method to find out if the integral, integral area for other complex region, this region could be divided into n X- region and Y- type area, using the method in this paper can be one quadrature.

Double integral formula of the structure of the advantages to construct the formula calculation, less conditions with high accuracy and low computation complexity and easy to program. Therefore, I believe the theory research, methods in mechanical and engineering fields will have extensive application. In addition, the compound formula can also be obtained, a more accurate numerical integration formula, but the computation will be increased.

REFERENCE

[1]. Li Qingyang, Wang Nengchao, Yi. Numerical analysis [M]. Beijing: Tsinghua University press, 2008

[2]. Chen Gongning, Shen Jiaji. Calculation method of guide [M]. Beijing: Beijing Normal University press, 2009

[3]. Yang Tao, Wang Airu, et al. Calculation method [M]. Beijing: China Water Conservancy and Hydropower Press, 2005 (in Chinese)

[4]. Xu Jiang Hao, Chen Zhikun, Liu Bin. A high precision numerical integration formula [J]. Journal of Sichuan University of Science and Engineering, 2011,24 (2):168-170.

[5]. Lou Aifang, Hu Junhao. A new theory and application of numerical integral formula of [J]. mathematics, 2010,30 (4): 72-74.

[6]. Xu Wei, Zheng Huasheng, et al. Construction and application of a kind of high precision numerical integration formula [J]. mathematics practice, 2012,42 (18):207-215. ()

[7]. Zhu Zhenguang. A method for numerical calculation of double integral in complex region [J]. Journal of Liaoning Institute of technology, 2004,24 (2):52-54.

[8]. Liu Haifeng, Chen Minge. Research and implementation of the double trapezoidal recursive method of double numerical integration [J]. computer application and engineering, 2006,42 (5):94-96.

[9]. Nam Quoc Ngo, Le Nguyen Binh. Optical realization of Newton-Cotes-based integrators for dark soliton generation[J]. Journal of Lightwave Techology, 2006,24(1):563-572.

[10]. Chen Yating, Hao Fenxia,, et al. The construction of the double numerical integration formula [J]. Journal of Hebei University, 2015,35 (2):118-121.