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On The Structure Equation $F^7 + F = 0$

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Abstract: In this paper, we have studied various properties of the *F*- sturcture manifold satisfying $F^7 + F = 0$. Nijenhuis tensor and metric *F*-structures have also been discussed.
Keywords: Differnetiable manifold, projection operators, Nijenhuis tensor and metric.

I. INTRODUCTION

Let M^n be a differentiable manifold of class C^∞ and *F* be a (1,1) tensor of class C^∞, satisfying

(1.1) $F^7 + F = 0$

we define the operators *l* and *m* on M^n by

(1.2) $l = -F^6, \qquad m = I + F^6$

From (1.1) and (1.2), we get

(1.3) $l + m = I, \quad l^2 = l, \; m^2 = m, \quad lm = ml = 0$
$lF = Fl = F, \quad Fm = mF = 0,$

where *I* denotes the identify operator.

Theorem (1.1): Let the (1,1) tensors *p* and *q* be defined by

(1.4) $p = m + F^3, \quad q = m - F^3,$ then

p and *q* are invertible operators satisfying

(1.5) $p^{-1} = q = p^3, \; q^{-1} = p = q^3, \quad p^2 = q^2, \quad p^2 - p - q + I = 0$
$pl = -ql = F^3, \; p^2 l = q^2 l = -l, \; pm = qm = p^2 m = q^2 m = m.$

Proof: Using (1.2), (1.3) and (1.4), we have

(1.6) $pq = qp = I$, Thus

(1.7) $p^{-1} = q, \quad q^{-1} = p$

Also, using (1.1),(1.3) and (1.4), we get

(1.8) $p^3 = q, \quad q^3 = p$

From (1.7) and (1.8) we have $p^{-1} = q = p^3$. Other results follow similarly.

Theorem (1.2): Let the (1,1) tensors α and β be defined by

(1.9) $\alpha = l + F^3, \quad \beta = l - F^3,$ then

(1.10) $\alpha^2 + \beta^2 = 0, \quad \alpha^5 + 4\alpha = 0, \; \beta^5 + 4\beta = 0$

Proof: Using (1.2), (1.3) and (1.9), we get

$\alpha^2 = 2F^3, \quad \beta^2 = -2F^3$ Thus we get $\alpha^2 + \beta^2 = 0$

The other results follow similarly.

Theorem (1.3): Define the (1,1) tensors γ and δ by

(1.11) $\gamma = m + F^6, \; \delta = m - F^6,$ then

(1.12) $\gamma^{-1} = \gamma$ and $\delta = I$

Proof: Using (1.2), (1.3) and (1.11), we get

(1.13) $\gamma = m - l, \quad \gamma^2 = I$ thus $\gamma^{-1} = \gamma$ and $\delta = m + l = I$

II. NIJENHUIS TENSOR

The Nijenhuis tensors corresponding to the operators *F, l, m* be defined as

(2.1) $N(X,Y) = [FX, FY] + F^2[X,Y] - F[FX,Y] - F[X,FY]$

(2.2) $N_l(X,Y) = [lX, lY] + l^2[X,Y] - l[lX,Y] - l[X,lY]$

(2.3) $N_m(X,Y) = [mX, mY] + m^2[X,Y] - m[mX,Y] - m[X,mY]$

Theorem (2.1): Let *F, l, m* satisfy (1.1) and (1.2), then

(2.4) (i) $N(mX, mY) = F^2[mX, mY]$

(ii) $mN(mX, mY) = 0$

(iii) $N_l(mX, mY) = l[mX, mY]$

(iv) $N_m(lX, lY) = m[lX, lY]$

(v) $N_l(lX, mY) = 0$

(vi) $N_m(mX, lY) = 0$

Proof: With proper replacements of *X* and *Y* in (2.1), (2.2) and (2.3), and using (1.3) we get the reuslts.

III. METRIC F-STRUCTURE

Let the Riemannian metric *g* be such that

(3.1) $`F(X,Y) = g(FX,Y)$ is skew- symmetric.

Then

(3.2) $g(FX,Y) = -g(X,FY),$ and

$\{F, g\}$ is called metric *F*-structrure.

Theorem (3.1): On the metric structrure-*F*, satisfying (1.1) we have

(3.3) $g(F^3X, F^3Y) = g(X,Y) - `m(X,Y),$ where

(3.4) $`m(X,Y) = g(mX,Y) = g(X,mY).$

Proof: From (1.2), (1.3) and (3.2), (3.4)

$$g(F^3X, F^3Y) = (-1)^3 g(X, F^6Y)$$
$$= -g(X, -lY)$$
$$= g(X, lY)$$
$$= g(X,Y) - g(X,mY)$$
$$= g(X,Y) - `m(X,Y)$$

REFERENCES

[1]. A Bejancu: On semi-invariant submanifolds of an almost contact metric manifold. An Stiint Univ., "A.I.I. Cuza" Lasi Sec. Ia Mat. (Supplement) 1981, 17-21.

[2]. B. Prasad: Semi-invariant submanifolds of a Lorentzian Para-sasakian manifold, Bull Malaysian Math. Soc. (Second Series) 21 (1988), 21-26.

[3]. F. Careres: Linear invairant of Riemannian product manifold, Math Proc. Cambridge Phil. Soc. 91 (1982), 99-106.

[4]. Endo Hiroshi: On invariant submanifolds of connect metric manifolds, Indian J. Pure Appl. Math 22 (6) (June-1991), 449-453.

[5]. H.B. Pandey & A. Kumar: Anti-invariant submanifold of almost para contact manifold. Prog. of Maths Volume 21(1): 1987.

[6]. K. Yano : On a structure defined by a tensor field *f* of the type (1,1) satisfying f^3+f=0. Tensor N.S., 14 (1963), 99-109.

[7]. R. Nivas & S. Yadav: On CR-structures and $F_\lambda(2\nu+3,2)$ - HSU - structure satisfying $F^{2\nu+3} + \lambda^r F^2 = 0$, Acta Ciencia Indica, Vol. XXXVII M, No. 4, 645 (2012).

[8]. Abhisek Singh, : On horizontal and complete lifts Ramesh Kumar Pandey of (1,1) tensor fields F satisfying & Sachin Khare the structure equation $F(2k+S,S)$=0. International Journal of Mathematics and soft computing. Vol. 6, No. 1 (2016), 143-152, ISSN 2249-3328