

Possible limits of accuracy in measurement of fundamental physical constants

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Abstract:- The measurement uncertainties of Fundamental Physical Constants should take into account all possible and most influencing factors. One from them is the finiteness of the model that causes the existence of a-priori error. The proposed formula for calculation of this error provides a comparison of its value with the actual experimental measurement error that cannot be done an arbitrarily small. According to the suggested approach, the error of the researched Fundamental Physical Constant, measured in conventional field studies, will always be higher than the error caused by the finite number of dimensional recorded variables of physical-mathematical models. Examples of practical application of the considered concept for measurement of fine structure constant, speed of light and Newtonian constant of gravitation are discussed.

Keywords: - A priori error analysis, information theory, theory of similarity, theory of measurement, fundamental physical constants

I. INTRODUCTION

Starting from Newton's law of universal gravitation and all without excluding the basic equations and proportions of the theory, along with the variables, there are appeared allocated physical values - the fundamental physical constants (FPC). It is important, that the interest is focused on the selected values, the fundamental physical numbers. The approach is based on the idea of constant primacy of constants (more exactly - dimensionless FPC) that do not depend on the choice of the measuring system in a physical theory. With this approach, the dimensionless (DS) constant, being both physical quantity and the required mathematical number is the primaries object of physical theory and selected number of its mathematical apparatus, because the problem of the theoretical calculation of the DS constants is the considered basic [1].

At its core, FPC is the cornerstone of any general theory. Therefore, scientists tend to calculate it with the greatest possible precision. However, along the way there are pitfalls - objective and subjective errors of the physical-mathematical model and the methods of calculation associated with it.

Many inferences and assumptions can be justified on the basis of experience (and sometimes uncertainties can be estimated), but the degree to which our assumptions hold in a study of FPC can never be established.

Our goal in this paper is to quantify possible estimates of the expedient level of the applicable accuracy for calculation of FPC, including evidence of two conditions: a. there is a pre-specified list of recorded variables used for FPC measurement; b. any factors that ignored in making predictions *actually have been taken into account* for calculating an error of the physical-mathematical model that is applied to verify the exact value of FPC.

II. BACKGROUND

The definition of FPC depends on, to a given point in time, the formed physical theory. This theory is now the standard model of three fundamental interactions - strong, electromagnetic and weak - with the theory of the gravitational interaction (GTR - the general theory of relativity), which is not combined with other fundamental interactions at this moment [2].

Recently there is conducted an intense study of the possibility of temporal and spatial changes of FPC, both within the grand unified theories, and on the phenomenological level. The experimental data from which you can get the restrictions on temporary changes constants of interactions are nucleon-synthesis of elements in the big bang, the electromagnetic spectrum of the quasar, the laboratory search for changes of FPC.

Analysis of data on the possible variation of the fine structure constant is associated with both the latest observations and with the impending reform in fundamental metrology - the introduction of new definitions of basic SI units.

Discovered in 1998, the accelerated expansion of the universe has led to the study of cosmological models that predict a changing of the Newtonian constant of gravitation with time.

Possible temporal variations of FPCs should be considered when interpreting the existing and planning future experiments.

The rapid development of measuring techniques based on the use of quantum physical phenomena, allows achieving the highest accuracy of the determination of many FPC. To improve the accuracy and stability of measurements it is necessary moving to the quantum standards. This transition is the main way of improving the standard base metrology organizations of many countries [2].

Development, implementation and use of quantum standards of physical units with a high accuracy are based on the use of values of FPC, such as the speed of light, Planck's constant, Boltzmann's constant, mass and charge of elementary particles - electrons, protons, and so on.

At the same time, the universe in which we live - it is a unique object, and therefore it is not clear that is accidentally, and that is natural [3].

In this paper, the principles of the information theory of thermodynamic processes are applied in order to formulate a value of the "firstborn" comparative error that limits any future accuracy of FPC measurements. It is closed only to finite number of recorded variables taken into account by the development of experimental schemes and the physical-mathematical models.

III. PROBLEM FORMULATION

What value of accuracy can be achieved, or what is the smallest achievable error of FPC measurement?

Fundamental limits on the maximum accuracy with which we can determine the physical variables are defined by the principle of Heisenberg's uncertainty. However, Planck's constant is vanishing small with respect to macro bodies. That is why this uncertainty in the macroscopic measurements cannot be used for practical application. Uncertainties of position and momentum, calculated in accordance with the Heisenberg's principle, do not show themselves in practice and lie far beyond the achievable accuracy of experiments.

In [4] the approach for calculating the lowest error of the researched variable, in our case, it is FPC, based on principles of information theory of thermodynamic processes is formulated. Following it, the certain error exists before starting experiment if it is known the amount of variables that are taken into account.

DS comparative error $\Delta u_{\text{FPC}}/S$ of the DS variable u (FPC), which varies in a predetermined DS range of values S , for a given number of selected physical dimensional (DL) variables z'' , and β'' (the number of the recorded primary physical variables) can be determined from the relation:

$$\Delta u_{\text{FPC}}/S \leq [(z' - \beta')/(\Psi - \xi) + (z'' - \beta'')/(z' - \beta')] \quad (1)$$

where Δu_{FPC} - DS error of physical-mathematical model describing the experiment of measurement of FPC;

Ψ - total number of DL physical variables; according to [4], for the system of primary variables (SPV) like International system of units (SI), $\Psi = 38,272$;

ξ - number of primary physical variables with independent dimension; SI, includes the following seven ($\xi = 7$) basic primary variables: L -length, M -weight, T -time, I -powered by electric current, Θ -thermodynamic temperature, J -force of light, F -number of substances; At this moment, the numerical value of $\Psi - \xi$ can be calculated by use of a heuristic approach and main fundamental constants (with a relative error of $4.4098 \cdot 10^{-7}$), as

$$\Psi - \xi = (\alpha/\beta^2) * (e - 0.3 + \alpha/10)^{1/2} \quad (2)$$

where $e = 2.7182818284$, β - electron-to-proton ratio, 0.0005446170 , α - fine structure constant, 0.0072973526 ;

z' - total number of DL physical variables in the chosen class of phenomena (COP); in SI frames, every researcher selects a particular COP to study material object. COP is a set of physical phenomena and processes described by a finite number of primary and secondary variables that characterize certain features of MO from the position with qualitative and quantitative aspects [5]. In studying mechanics, for example, the base units of SI are typically used: L, M, T (LMT). In studying the phenomena of electromagnetism, the basic set often includes L, M, \square and I ($LMTI$);

β'' - the number of primary physical variables in the chosen COP.

Equation (1) quantifies $\Delta u_{\text{FPC}}/S$ caused by the limited number of variables taken into account in the theoretical/experimental analysis of FPC value. On the other hand, it also sets a limit on the expedient increasing of the measurement accuracy in conducting experimental studies.

Equating the derivative of $\Delta u_{\text{FPC}}/S$ (1) to zero, we obtain the condition for achieving the minimum comparative error for a particular COP:

$$(z' - \beta')^2 / (\Psi - \xi) = (z'' - \beta'') \quad (3)$$

We apply the above mentioned results for the mechanics application ($COP \equiv LMT$) in order to show the frames of this approach usage.

Consider the motion of a simple pendulum - the ball of mass m , suspended in a gravitational field on a weightless rod of length l . We also assume that the pendulum is moving in the same plane. Let the pendulum is under impact of the friction force R_{fr} , which is, it turn, proportional to the velocity of the sinker v , $R_{fr} = -A \cdot v$, where A - proportionality factor, which is determined by the properties of the medium and the shape of the body. The angle of deviation of the pendulum from the vertical direction is x .

The dependence of the dimensionless amplitude x_{max} of the ball can be represented by the following dimensionless equation [6]:

$$x_{max} = \varphi(a = (A/m) \cdot (l/g)^{1/2}, p = R_{fr}/(mg)) \quad (4)$$

where, g - acceleration of gravity.

Such transformation shows some similarity laws: the dependence x (for given boundary conditions) is the same for different values of m , l , g , A , if the DS combinations of a and p , composed from them, are the same. The numerical values of these complexes do not have to depend on SPV. The form of these functions can be determined either by solving the equation of motion of the pendulum, or experimental method. This fact allows us to reduce the amount of full investigations on the problem, since it suffices to consider different values of the two parameters instead of four. In other words, the results of a pendulum can be transferred to other simple change of scale.

In addition, at the numerical solution of the DS equations of motion of the pendulum, we usually do not have to deal with the values that differ from each other by many orders of magnitude, while the size of the equations of motion of the pendulum it could well happen with failure choice of units.

Taking into account that $z' - \beta' = 189$ [4] and $z'' - \beta'' = 2$, for $COP_{SI} \equiv LMT$

$$\begin{aligned} (A_{pmm}/S)_1 &= [(z' - \beta') / (\Psi - \xi) + (z'' - \beta'') / (z' - \beta')] = \\ &= 189/38,265 + 2/189 = 0.0049 + 0.0106 = 0.0155 \end{aligned} \quad (5)$$

If in this model we neglect the effect of friction ($p = R_{fr}/(mg) = 0$), i.e. $z'' - \beta'' = 1$, the apriority comparative error of the mathematical model due to its dimension, will be

$$(A_{pmm}/S)_2 = 189/38,265 + 1/189 = 0.0049 + 0.0054 = 0.0103 \quad (6)$$

i.e., it reduced by the value 0.0052.

And yet, it is well known that the neglect of friction, on the contrary, increases the error of mathematical model, and this increase is not constant, but depends on the size of the complexes a and p . It is the smaller than less p and the value of a is far from a resonance region.

The apparent contradiction is explained by the fact that, if we ignore friction, mathematical model worse describes the studied material object. Therefore, to obtain reliable experimental data and verify eligibility of the selected MM, it requires increase of the accuracy of the measuring instruments. Then, the DS error A_{exp} (the estimated experimental absolute error in the determination of the dimensionless amplitude of sinker x_{max}) will be smaller, and ratio of A_{pmm}/A_{exp} will be closer to 1, where A_{pmm} is the DS absolute error of mathematical model depended on only amount of recorded variables. In this case, if the spread of the experimental data in comparison with the results of computer simulation is in the range allowed by the researcher, it can be assumed that the selected mathematical model adequately describes the observed process.

As a second example, we use (5) for the comparison of comparative errors of mathematical models describing the same material object, but with different COP. We consider a thin metal plate, moving in a viscous and elastic medium that is under the influence of an external force distributed on one surface of the plate. Suppose that in the first case, the pressure of the mechanical force affects on the plate $P_{mech} = p_0 \cdot \exp(-t/\tau)$, where τ - time constant of the process, p_0 - initial constant value of P_{mech} . Consequently, material object can be represented as a mechanical system ($COP_{PMM} \equiv LMT$). In the second case, the magnetic field pressure affects on the side of the plate $P_{mag} = 0.5 \cdot \mu \cdot H^2$, $H = h_0 \cdot \exp(-t/(2 \cdot \tau))$, where μ is the magnetic permeability, h_0 - initial constant value of the magnetic field. In this case, material object is represented in the form of electro-mechanical system ($COP_{PMM} \equiv LMTI$).

Find out the relationship between the required number of dimensionless complexes in LMT (K_{LMT}) and $LMTI$ (K_{LMTI}), in which the comparative errors are equal.

For $COP_{SI} \equiv LMT$

$$(\Delta_{\text{pmm}}/S)_{\text{LMT}} = 189/38,265 + K_{\text{LMT}}/189 \quad (7)$$

for $\text{COP}_{\text{SI}} \equiv \text{LMTI}$

$$(\Delta_{\text{pmm}}/S)_{\text{LMTI}} = 945/38,265 + K_{\text{LMTI}}/945 \quad (8)$$

Equating (7) and (8), we obtain

$$K_{\text{LMTI}} \approx 5 \cdot (K_{\text{LMT}} - 4) \quad (9)$$

Obviously, although the compared processes are described by the same equation form, the difference of modeled objects and statements of research problems leads to a difference in the values of the comparative errors of mathematical models and to differences in the requirements for verifying the accuracy of the experiments.

Thus, within the proposed approach, to achieve the equal comparative errors of mathematical models describing the same material object, but with different COP, requires a distinctive number of dimensionless complexes used in mathematical model.

It should be noted two remarks.

For the mechanics processes ($\text{COP}_{\text{SI}} \equiv \text{LMT}$), taking into account [4], the lowest comparative error can be reached at the following conditions:

$$(z' - \beta') = (7 \cdot 3 \cdot 9 - 1)/2 = 94 \quad (10)$$

$$(z'' - \beta'') = (z' - \beta')^2 / (\Psi - \xi) = 94^2 / 38,265 = 0.2309 < 1 \quad (11)$$

And it equals

$$(\Delta_{\text{pmm}}/S)_{\text{LMT}} = 94/38,265 + 0.2309/94 = 0.0049 \quad (12)$$

In other words, according to (11), even one DS main variable for $\text{COP}_{\text{SI}} \equiv \text{LMT}$ is not enough to reach the lowest comparative error. So in the frame of the suggested approach, any mechanistic model ($\text{COP}_{\text{SI}} \equiv \text{LMT}$) contains the original error depending on the number of parameters taken into account. Moreover, the greater the amount of mechanical parameters, the greater the firstborn embedded error.

Such statements seem very, very controversial, trivial and one might even say, very unprofessional, not credible and far from the current reality. However, as we shall see below, the proposed approach allows making the not obvious conclusions.

IV. EXAMPLES OF APPLICATION

For the following comparison of the suggested approach and results of realized measurements of different FPC we need to note that comparative errors of the DL variable \mathbf{U} and the DS variable \mathbf{u} are equaled

$$(\Delta \mathbf{u}/S) = (\Delta \mathbf{U}/r^*) / (S^*/r^*) = (\Delta \mathbf{U}/S^*) \quad (13)$$

where S , $\Delta \mathbf{u}$ - DS variables, respectively, range of variations and total error in determining the DS variable \mathbf{u} ; S^* , $\Delta \mathbf{U}$ - DL variables, respectively, range of variations and total error in determining the DL variable \mathbf{U} ; r^* - DL scale parameter with the same dimension that \mathbf{U} and S^* have.

4.1 Fine structure constant

4.1.1 In [7] authors reported a new experimental scheme which combines atom interferometry with Bloch oscillations that leading to a new determination of the fine structure constant $\alpha^{-1}_1 = 137.03599945(62)$ with a relative uncertainty r_1 of $4.6 \cdot 10^{-9}$. It means that absolute uncertainty is $\Delta_1 = \alpha^{-1}_1 \cdot r_1 = 6.3037 \cdot 10^{-7}$. The declared range S_1 of α^{-1}_1 variations is $0.14 \cdot 10^{-5}$ (see Fig. 4, [7]). Research is organized into the frame of $\text{COP}_{\text{SI}} \equiv \text{LMT}$. One can calculate the achieved comparative error

$$\varepsilon_1 = \Delta_1/S_1 = 6.3037 \cdot 10^{-7} / 0.14 \cdot 10^{-5} = 0.4503 \quad (14)$$

The obtained comparative error is far (much higher) from the recommended (12) by the discussed approach. So, the above mentioned method and apparatus need to verify the fine structure constant value with a better accuracy.

4.1.2 There is presented a recoil-velocity measurement of Rubidium and obtained a new determination of the fine structure constant $\alpha^{-1}_2 = 137.035999037(91)$ with a relative uncertainty r_2 = of $6.6 \cdot 10^{-10}$ [8]. It means

that absolute uncertainty is $\Delta_2 = \alpha^{-1} \cdot r_2 = 9.0444 \cdot 10^{-8}$. Following the description of the experimental unit and methods, $\text{COP}_{\text{SI}} \equiv \text{LMT}$. According to (12), the lowest comparative error equals 0.0049. The range of variations S_2 of α^{-1} is $6.0 \cdot 10^{-7}$ (see Fig.1, [8]). In this case, the comparative error of the introduced method will be

$$\varepsilon_2 = \Delta_2/S_2 = 9.0444 \cdot 10^{-8} / 6.0 \cdot 10^{-7} = 0.1507 \quad (15)$$

This value is larger than the lowest comparative error for $\text{COP}_{\text{SI}} \equiv \text{LMT}$ calculated according to (12). That is why the research team can try to find more perspective method for reaching the best results.

4.1.3 The achieved value of α^{-1} is 137.035999044(90) with a relative uncertainty r^* of $6.6 \cdot 10^{-10}$ [9]. It means that absolute uncertainty is $\Delta^* = \alpha^{-1} \cdot r^* = 9.044375934 \cdot 10^{-8}$. The range of variations S^* of α^{-1} is $(5,999.15-5,9998.9) \cdot 10^{-6} = 2.5 \cdot 10^{-7}$ (see Fig.1, [9]).

In this case, the comparative error for $\text{COP}_{\text{SI}} \equiv \text{LMT}$ will be

$$\varepsilon^* = \Delta^*/S^* = 9.044375934 \cdot 10^{-8} / (2.5 \cdot 10^{-7}) = 0.3618 \quad (16)$$

Comparing (12) and (16), one should make decision to continue experiments with improved measuring devices and methods in order to reach smaller comparative error.

All three above-stated studies differ from each other by the design of experimental facilities and methods of measurement. However, in the framework of the suggested approach it can be argued that the greatest accuracy in the measurement of the fine structure constant is achieved in [8]. This was possible due the comparison of the value of the comparative errors made in these studies, with comparative error that chosen in accordance with the recommended approach and calculated for the particular class of phenomena $\text{COP}_{\text{SI}} \equiv \text{LMT}$.

4.2 Speed of light

Impressed review of uncertainty in speed of light is introduced in [10]. Only by the end of the 80s of the twentieth century, after a long hectic history, the value of the light speed is settled down into fairly satisfactory “steady” state.

The speed of light was measured using the Foucault method of reflecting a beam of light from a rotating mirror to a fixed mirror and back creating two separate reflected beams with an angular displacement [11].

By taking measurements relating the displacement of the two light beams and the angular speed of the rotating mirror, the speed of light was found to be $(3.09 \pm 0.204) \cdot 10^8$ m/s, which is within 2.7% of the defined value for the speed of light.

The amount of variables recorded in experiment is 13. According to the description of the experimental unit and method, $\text{COP}_{\text{SI}} \equiv \text{LMT}$. The range of light speed variations S_c is $0.408 \cdot 10^8$ m/s. Even if we assume that absolute uncertainty Δ_c of the light speed measurement is on the level of 2.7% of the result, i.e. $\Delta_c = 3.09096 \cdot 10^8 \cdot 0.027 = 0.0834559 \cdot 10^8$ m/s, the achieved comparative error is far from the recommended (12):

$$\varepsilon_c = \Delta_c/S_c = 0.0834559 \cdot 10^8 / (0.408 \cdot 10^8) = 0.2045 \gg 0.0014 \quad (17)$$

That is why, it would require taking extremely precise distance and displacement measurements or the microscope measurements that would be replaced by a photo detector that was used to measure the intensity of the light as a function of the distance across the glass plate and determine the position of the maximum intensity [11].

4.3 Newtonian constant of gravitation

The importance of a high precision of the Newtonian gravitational constant G is caused not only by a pure metrological interest but it has a key role in different theories including gravitation, cosmology, particle physics and astrophysics.

4.3.1 A research team has measured the gravitational constant using the original equipment [12]. The method is based on measuring the amount of torque applied to a thin ribbon set between heavy balls and applying voltage to a wire using a servo to counteract twisting due to Newtonian constant of gravitation G . Researchers discovered measurements revealed a value G_1 of $6.67545(18) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, with 27 ppm standard relative uncertainty, i.e. $r_{G_1} = 27 \cdot 10^{-6}$. It means that absolute uncertainty equals

$$\Delta_{G_1} = G_1 \cdot r_{G_1} = 1.802732 \cdot 10^{-15} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (18)$$

The observed and declared range of G_1 variations is (see Fig. 3, [12])

$$S_{G1} = 3 \cdot 10^{-15} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (19)$$

So, the comparative error achieved by researchers is

$$\varepsilon_G = \Delta_{G1} / S_{G1} = 0.6008 \quad (20)$$

In the discussed research there were mentioned about thirty (30) parameters taken into account during the published test results. According to dimensions of recorded variables, $\text{COP}_{\text{SI}} \equiv \text{LMT}\Theta$, where Θ – thermodynamic temperature. In this case, a number of DS variables ($z'' - \beta''$) causing a minimum comparative error $\varepsilon_{G^*} = \Delta_{G^*} / S_{G^*}$ is:

$$(z' - \beta') = (7 \cdot 3 \cdot 9 \cdot 9 - 1) / 2 = 850 \quad (21)$$

$$(z'' - \beta'') = (z' - \beta')^2 / (\Psi - \xi) = 850^2 / 38,265 = 19 \quad (22)$$

Let's calculate a minimum achievable comparative error ε_{G1^*}

$$\varepsilon_{G^*} = \Delta_{G^*} / S_{G^*} \leq 850 / 38,265 + 19 / 850 = 0.0222 + 0.0224 = 0.0446 \quad (23)$$

Therefore, $\varepsilon_{G^*} < \varepsilon_{G1}$. That is why, the research into ways to the better measure of G needs to be continued.

4.3.2 Using the same methodology and substituting the index of '1' to '2', we analyze one of the last works by measuring the gravitational constant [13]. The researchers measured the attraction between a cloud of cold rubidium atoms and tungsten weights. They came up with a value for G_2 of $6.67191(99) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ with a relative uncertainty of $r_{G2} = 150 \cdot 10^{-6}$ and absolute uncertainty $\Delta_{G2} = 1.000788 \cdot 10^{-14} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. There is described their technique in great detail. According to (Fig. 1 [13]), a range of G variations is $S_{G1} = 2 \cdot 10^{-14} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. So, the achieved comparative error is only

$$\varepsilon_{G2} = \Delta_{G2} / S_{G2} = 0.5 \quad (24)$$

In the frame of the suggested approach, $\varepsilon_{G2} < \varepsilon_{G1}$. It allows declaring the new apparatus and techniques helped to reach higher accuracy of G measurement.

V. CONCLUSION

5.1 We used information theory of thermodynamic processes for the formulation of general principles and derived effects, which are amenable to rigorous experimental verification.

5.2 A measure of evaluation of the achievable accuracy of measurement of fundamental physical constants is suggested and there is formulated the method of calculating the comparative error realized during the experiment.

5.3 The present analysis of published studies on the measurement of the fundamental physical constants allows us to hope that the above mentioned approach will be used to compare the accuracy achieved in various experimental settings and by applying methods that are differing each from other.

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