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The (R, Q) Control of A Mixture Inventory Model with Backorders and Lost Sales with Controllable Set Up Cost And Variable Lead Time with Service Level Constraint

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Abstract:- This paper modifies Chuang et al's[6] inventory model by adding service level constraint. In this study, I investigate the periodic review inventory model with a mixture of back orders and lost sales. The lead time varies with the ordered quantity. The set-up cost is not constant, it is controllable. Also, instead of having a stock out cost in the objective function, a service level constraint is employed. Furthermore, with the help of a numerical example the problem is described.

Keywords:- Mixture inventory, Periodic review, Service level, Lead time,JIT

I. INTRODUCTION

In traditional EOQ and EPQ models, set up cost is treated as constant. However, in practice, set-up cost can be controlled and reduced through various efforts such as worker training, procedural changes and specialized equipment acquisition. Through the Japanese experience of using Just-in-Time (JIT) production, the advantages and benefits associated with efforts to reduce set up cost can clearly be perceived. The ultimate goal of JIT from an Inventory stand point is to produce small lot-size with good quality, lower investment in inventory, less scrap and reduced storage space requirement. In order to achieve this goal, investing capital in reducing set-up cost is regard as one of the effective ways.

According to Silver et al [29] the implementation of electronic data interchange (EDI) may reduce the fixed set-up cost and result in new replenishment policy and the corresponding lower cost. Nasri et al[19]studied JIT manufacturing system and pointed out that the impact of investing in reduced set-up cost has been observed in many manufacturing setting including job shops, batch shops and flow shops. This type of investment differs from the traditional approach of investment aimed at increasing capacity because, in most production systems, production scheduling is affected directly by set-up cost. In addition, set-up control has been a topic of interest for many researchers in the field of inventory/production management. Porteus [23] studied the impact of capital investment in reducing set-up cost in the standard undiscounted EOQ model. Porteus [24] later extended this research to consider the discounted model. The effects of set-up cost reduction on the EOQ model with stochastic lead time is investigated by Nasri et al [19], Kim et al [13] focused on several classes of set-up cost reduction functions and described a general solution procedure on the EPQ model. Paknejad et al[22] presented a quality adjusted lot-sizing model with stochastic demand and constant lead time, and studied the benefits of lower set up cost in the model. Sarkar and Coates[27]extended EPQ model with set up cost reduction under stochastic lead time and finite number of investment possibilities to reduce set up cost. Ouyang et al [21] developed a mixture inventory model involving set up cost reduction with a service level constraint. Chuang et al [6] present a note on periodic review inventory model with controllable set up cost and lead time.Hariga[10] modified Kim and Benton's [12] model by rectifying the annual backorder cost and proposing another relation for the revised lot-size.Hariga's [10] model is more consistent with JIT's objectives in the sense that it results in smaller lot-sizes.

In most of the literature dealing with inventory problems, either deterministic models, lead time is treated as constant which therefore is not subject to control (Montgomery et al [15], Silver and Peterson[29]). However it can be controlled and reduced through various efforts. Bendaya et al [2] considered lead time as decision variable in his inventory model. in his an inventory model In many practical situations lead time can be reduced at an added crashing cost; in other words, it is controllable. By shortening lead time we can lower the safety stock, reduce the stock out loss, save product costs and improve the customer service level so as to gain competitive edges in business. The Japanese experiences of using Just-in-time (JIT) production have evidenced the advantages and benefits associated with the efforts to reduce lead time. The spirits of JIT emphasizes small lot production and provide high quality products.The goal of JIT inventory philosophies is the focus that keeps the inventory level and lead time to a practical minimum. Morton [17] analyzed Toyota production system and clearly addressed that lead time reduction is a crux of elevating productivity. In real world, in order to avoid the shortage,we have to maintain a buffer stock and determine the ordering time.Eppen et al [7] and Fotopouls et al

[8] determined the safety stock.Again reorder level is determined by Ray et al[25] and Urban et al [31] when demands are correlated.

Recently,several continuous review inventory models have been developed [Liao and Shyu[14],Baganha et al[1] Beyer et al[3], Cakanyildirim [4], Charnes et al[5] , Moon and Choi [16] Salameh et al[26]]. But in the periodic review inventory model, literature discussing mixture inventory with service level is few. The cases of lost sales is considered in most of the inventory models i.e.Nahimas[18] where a demand occurs when the system is out of stock, is lost for ever.Jonasen et al [11] developed an inventory model of (r,Q) control with lost sales. In this paper we propose a mixture inventory model i.e.mixture of back order and lost sales. When stock out occurs, only a fraction γ $(0<\gamma<1)$ of the unsatisfied demand is backordered and the remaining fraction is lost. Ouyang[20]modified Parknejad et al's[22]model by considering a mixture of backorders and lost sales when stock out occurs. Besides instead of having a stock out cost in the objective function,a service level constraint is included. I solve this inventory model by using the minimax distribution free approach, which was originally proposed by Scarf [28]and has been disseminated by Gallego and Moon[9].Silver et al [30]studied distribution free approach on some production or inventory models. A numerical example is solved in support of the model and sensitivity analysis is performed.

II. NOTATIONS

D = Average demand per year

Q = Replenishment order quantity.

L = Length of the lead time

h = Stock holding cost /unit/review interval.

A_0= Original set-up cost.

A = Fixed cost of placing a replenishment order.

A^* = Optimal Set-up cost.

EAC= Expected annual cost.

ELS(r) = Expected lost sales incurred during an order cycle when the reorder level is r.

γ= The fraction of the demand during the stock out period that will be backordered, $0\leq\gamma\leq1$.

μ = Expected demand per unit time.

μ_L = Expected demand per unit time during lead time where $\mu_L = \mu L$

σ= Standard deviation of the demand per unit time.

σ_L= Standard deviation of the demand per unit time during lead time where $\sigma_L = \sigma L$

III. ASSUMPTIONS

The following assumptions are made:

(i) We consider lead time L depends on the ordered quantity i.e. $L=\sqrt{(pQ+b)}$ where p,b are constants.

(ii) The reorder point r =expected demand during lead time +safety stock (SS) and SS =k (standard deviation of lead time demand) i.e. $r=\mu L+k\sigma\sqrt{L}$ where k is the safety factor.

(iii) Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r.

IV. MODEL FORMULATION

Ouyang and Chuang [ ] considered an inventory system for a periodic review model with controllable lead time and asserted the following function of total expected annual cost which is the sum of set-up cost, holding cost, stock-out cost and lead time crashing cost.

In contrast to the Ouyang and Chung's [ ] model,we consider the set-up cost A as a decision variable and seek to minimize the sum of the capital investment cost of reducing set-up cost A and the inventory related costs[as expected in (1)] by optimizing over T,a and constrained on $0<A\leq A_0$, where A_0 is the original set-up cost. Therefore, the objective of our problem is to minimize the following total expected annual cost

$$EAC(A,Q) = \eta\, M(A)+EAC(Q) \qquad (1)$$

over $A\in(0, A_0]$ where η is the fractional opportunity cost of capital per year, M(A) follows a logarithmic investment function given by

$$M(A)=\frac{1}{\delta}\ln(\frac{A_0}{A}) \quad \text{for } A\in(0,A_0] \qquad (2)$$

where δ is the percentage decrease in A per dollar increase in investment. This logarithmic investment function has been utilized by Nasri et al [19],Porteus et al [23].

Therefore from the function (2) we note that the set-up cost level $A\in(0,A_0]$; It implies that if the optimal set-up cost obtained does not satisfy the restriction on A,then no set-up cost reduction investment is made.for this special case,the optimal set-up cost is the original set-up cost.

We have assumed that when stock out occurs, only a fraction γ ($0 \prec \gamma \prec 1$) of the unsatisfied demand is backordered and the lead time demand x has a normal probability density function(p.d.f) f(x) with mean μL and standard deviation σ√L and the reorder point r= $\mu L + k\sigma\sqrt{L}$,where k be defined as safety factor. The expected number of back orders per cycle is γELS(r) and the expected lost sales per cycle is (1-γ) ELS(r).

The expected net inventory level just before the order arrives is r-μL+(1-γ)ELS(r) and the expected net inventory level at the beginning of the cycle is

Q+r-μL+(1-γ)ELS(r). Therefore, the expected annual holding cost is h[$\frac{Q}{2} + r - \mu L + (1-\gamma)ELS(r)]$ .An order of size Q is placed as soon as the inventory position reaches the reorder point r.

In this paper we modify some of the assumptions of Chuang et al's [6] as follows

(i) The lead time L =√(pQ+b) depends on the ordered quantity and it is a variable

(ii) Instead of having a stock-out cost term in the objective function, a service level constraint which implies that the stock-out level per cycle is bounded, is added to the model

Therefore our problem can be expressed as

$$\text{Min } EAC(Q, A) = \frac{\eta}{\delta}\ln\frac{A_0}{A} + \frac{AD}{Q} + h[\frac{Q}{2} + r - \mu_L + (1-\gamma)ELS(r)] \quad (3)$$

subject to the service level constraint is $\frac{ELS(r)}{Q} \le \alpha$ where $\alpha(0 \prec \alpha \prec 1)$ is the proportion of demands which are not met from the stock and hence 1-α is the service level.

Since the form of the probability distribution of lead time demand is unknown, the exact value of the expected demand at the end of the cycle ELS(r) can not be obtained, hence Minimax distribution free approach is applied for the problem.

Let F denote the class of p.d.f.s with finite mean μ_L and standard deviation σ_L, then the minimax principle for this problem is to find the most favourable p.d.f. f_x in F for each (Q, L) and then minimize over (Q,L) more exactly our problem is

$$\underset{(Q,L)}{\text{Min}}\ \underset{f_x \in F}{\text{Max}}\ EAC$$

This task can be achieved by utilizing the following proposition, which was asserted by Gallego and Moon[9].

Proposition

$$ELS(r) \le \frac{1}{2}\{\sqrt{\sigma_L{}^2 + (r-\mu_L)^2} - (r-\mu_L)$$
$$= \frac{1}{2}\{\sqrt{L\sigma^2 + k^2\sigma^2 L} - k\sigma\sqrt{L}\} \quad (4)$$

Moreover, the upper bound of (3) is tight.

Proof: The proof is similar to that of lemma 1 given by Gallego and Moon[9] and hence we omit it.

Since the reorder point r = expected demand during lead time + safety stock

i.e. $r = \mu L + k\sigma\sqrt{L}$; k=safety factor

V. THE BASIC ALGORITHM (GENETIC ALGORITHM)

Genetic Algorithm:

Genetic Algorithm is a class of adaptive search technique based on the principle of population genetics.The algorithm is an example of a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of parameter space.Genetic Algorithm work according to the principles of natural genetics on a population of string structures representing the problem variables.All these features make genetic algorithm search robust,allowing them to be applied to a wide variety of problems.

Implementing GA:

The following are adopted in the proposed GA to solve the problem :

(1) Parameters
(2) Chromosome represention
(3) Initial population production
(4) Evaluation
(5) Selection

(6) Crossover
(7) Mutation
(8) Termination

Parameters
Firstly,we set the different parameters on which this GA depends.all these are the number of generation (MAXGEN), population size (POPSIZE), probability of crossover(PCROS), probability of mutation (PMUTE).

Chromosome representation
An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators.Traditional binary vectors used to represent the chrosones are not effective in many non-linear problems.Since the proposed problem is highly non-linear,hence to overcome the difficulty,a real-number represention is used.In this representation,each chromosome Vi is a string of n numbers of genes G_{ij}, (j =1,2,…n) where these n numbers of genes respectively denote n number of decision variables(Xi, i=1,2,…n).

Initial population production
The population generation technique proposed in the present GA is illustrated by the following procedure: For each chromosome Vi,every gene G_{ij} is randomly generated between its boundary (LB_j,UB_j0 where LB_j and UB_j are the lower and upper bounds of the variables X_j,i= 1,2,….,n and i=1,2,…n, POPSIZE.

Evaluation
Evaluation function plays the some role in GA as that which the environment plays in natural evalution.Now, evaluation function (EVAL) for the chromosome V_i is equivalent to the objective function PF(X).These are following steps of evaluation.

Step 1: find EVAL (V_i) by EVAL (V_i) = f(X_1,X_2,…X_n)
where the genes G_{ij} represent the decision variable X_j, j =1,2,…n, POPSIZE and f is the objective function.

Step 2 : find total fitness of the population : F= $\sum_{i=1}^{POPSIZE} EVAL(V_i)$

Step 3 : calculate the probability pi of selection for each chromosome V_i as

$$Y_i = \sum_{j=1}^{i} p_j$$

Selection
The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as Stochastic random sampling, Roulette wheel selection have been proposed for various problems.In this paper we adopt roulette wheel selection process.
This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way:
(a) Generate a random (float) number r between 0 to 1.
(b) If r < Y_i then the first chromosome is V_i otherwise select the i^{th} chromosome
V_i (2 ≤ i ≤ POPSIZE) such that T_{i-1} ≤ r ≤ Y_i

Crossover
Crossover operator is mainly responsible for the search of new string. The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection chromosomes for new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and the resulting offspring's $V_m^{/}$ and are $V_n^{/}$ calculated as:

$$V_m^{/} = c.V_m + (1-c).V_n$$
$$V_n^{/} = c.V_n + (1-c).V_m$$

where c is a random number between 0 and 1.

Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly.It is applied to a single chromosome V_i. the selection of a chromosome for mutation is performed in the following way :

Step 1. Set i ← 1
Step 2. Generate a random number u from the range [0,1]
Step 3. If u < PMUTE,then go to step 2.
Step 4. Set i ← i + 1
Step 5. If i ≤ POPSIZE,then go to Step 2.

Then the particular gene G_{ij} of the chromosome V_i selected by the above mentioned steps is randomly selected.in this problem,the mutation is defined as

$$G_{ij}^{mut} = \text{random number from the range } (0,1)$$

Termination

If the number of iteration is less than or equal to MAXGEN then the process is going on,otherwise it terminates.

The GA's procedure is given below:

```
begin
  do {
       t ← 0

       while(all constraints are not satisfied)
        {
        initialize Population (t)
        }
        evaluate Population(t)
        while (not terminate)
        {
         t ← t +1
        select Population(t) from Population(t-1)
        crossover and mutate Population(t)
        evaluate Population(t)
        }
        Print Optimum Result
        }
        end.
```

VI. NUMERICAL ILLUSTRATION

In order to illustrate the above model, we consider an inventory system with the following datas used by Chuang et al [6],Ouyang et al [20] and Ouyang et al[21]. The problem is solved by Genetic algorithm.

h = \$15.0, A =50.0/per review ,D=600.0units per cycle,,σ =7.0, μ=5.0

TABLE: 1

α =0.02 SL=98%							
Changing parameter	**Change in parameter**	**Set-up cost reduction**			**Fixed set-up cost**		
		Optimal Order(Q)	**A***	**EAC**	**Optimal Order(Q)**	**EAC**	**Savings (%)**
γ	**0.0**	**229.2161**	**42.93**	**1238.33**	**332.7788**	**1309.77**	**5.77**
	0.5	**233.5628**	**48.49**	**1173.53**	**337.3219**	**1246.78**	**6.24**
	0.8	**243.8241**	**49.26**	**1152.34**	**361.3425**	**1224.93**	**6.30**
	1.0	**248.4321**	**54.17**	**1120.30**	**368.3217**	**1202.76**	**7.36**

TABLE: 2

α =0.015 SL=98.5%							
Changing parameter	**Change in parameter**	**Set-up cost reduction**			**Fixed set-up cost**		
		Optimal Order(Q)	**A***	**EAC**	**Optimal Order(Q)**	**EAC**	**Savings (%)**
γ	**0.0**	**235.3206**	**43.73**	**1245.68**	**339.5959**	**1315.61**	**5.61**
	0.2	**255.8502**	**46.51**	**1212.13**	**378..7718**	**1282.19**	**5.77**
	0.3	**259.9286**	**50.16**	**1172.61**	**386.o445**	**1246.17**	**6.27**
	0.4	**266.8764**	**55.87**	**1144.58**	**397.0433**	**1227.30**	**7.23**

TABLE: 3

α =0.01 SL=99%							
Changing parameter	**Change in parameter**	**Set-up cost reduction**			**Fixed set-up cost**		
		Optimal Order(Q)	**A***	**EAC**	**Optimal Order(Q)**	**EAC**	**Savings (%)**
γ	**0.0**	**239.4417**	**43.89**	**1251.86**	**342.2090**	**1317.90**	**5.28**
	0.5	**256.4844**	**46.98**	**1201.67**	**381.0924**	**1284.31**	**5.99**
	0.8	**260.3799**	**50.94**	**1171.18**	**386.8227**	**1246..89**	**6.46**
	1.0	**267.6839**	**56.03**	**1145.30**	**396.8971**	**1227.17**	**7.14**

VII. RESULT DISCUSSION

From Table-1 to 3 we can infer that as γ increases optimal order quantity also increases.Total cost are decreased for both cases i.e. for set-up cost reduction and fixed set-up cost. Percentage of savings is increased due to set-up cost reduction for various service levels.

VIII. CONCLUSION

The purpose of this paper is to present a mixture of backorders and lost sales periodic review inventory control for minimizing the sum of the ordering cost, holding cost and backorder. A service level constraint is added instead of stockout cost. The optimal reorder level and lot-size for various service levels are determined.

From the above table we infer that as γ increases the reorder point also increases but the lot-size and the total cost decreases.

REFERENCES

[1]. Baganha. M. P, Pyke. D. F, Ferrer, (1996), "The undershoot of the reorder point, Tests of an approximation",International Journal Of Production Economics, 45, 311-320.

[2]. Ben-daya. M and raof. A (1994), 'Inventory models involvig lead time as decision variable",Journal of Operational Research Society,45,579-582.

[3]. Beyer. D, (1994), "An inventory model with Weiner demand process and positive lead time", Optimization, 29, 181-193

[4]. Cakanyildirim,M; Bookbinder,JH and Gerchak,Y.(2000), "Contious review inventory models where random lead time depends on lot size and reserved capacity",Int.J. Production Economics, 68,217-228.

[5]. Charnes J.M.,Marmorstein H and Zinn W(1995) Safely stock determination with serially correlated demand in a periodic review inventory system.J.Opl.Res Soc 46:1006-1013.

[6]. Chuang,B.R.,Ouyang,L.Y. and Chuang,K.W.(2004) "A note on periodic review inventory model with controllable set up cost and lead time". Computers and Operations Research, vol.31, pp.549-561.

[7]. Eppen,GD and Martin RK (1988) , "Determining safety stock in the presence of stochastic lead time and demand" ,Mgmt Science,34,1380-1390.

[8]. Foto pouls S. Wang MC and Rao SS, (1988), "Safety stock determination with correlated demands and arbitrary lead times". Eur J. Opl Res, 35, 172-181.

[9]. Gallego. G. and Moon. I, (1993) "The distributions free news boy problem review and extensions." Journal of Operational Research Society, 44, 825-834.

[10]. Hariga. G (1999), “A stoachstic inventory model with lead time and lot-size interaction”,Production Planning & Control,10,434 – 438.
[11]. Jonansen. S. G. and Hill. R. M, (2000), “The (r,Q) Control of a periodic- review inventory system with continuous demand and lost sales”, Int. J. Production Economics 68, 279-286.
[12]. Kim,J.S. and Benton,W.C.(1995), “Lot-size dependent lead times in a (Q,r) inventory system”, International Journal of Production Research,33,41-58.
[13]. Kim;K.L; Hayya J.C and Hong,J.D, (1992), “Set-up reduction in economic production quantity model”,Deciison Sciences,1992, 23,500-508.
[14]. Liao, CJ and Shyu,CH (1991), “An analytical determination of lead time with normal demand”, International Journal Of Production Economics,11,72-78.
[15]. Montgomery DC, Bazaraa MS and Keswani AK, (1973), “Inventory models with a mixture of back orders and lost sales”, Naval Research Logistics Quarterly, 20, 255-263.
[16]. Moon and Choi. S, (1994), “The distribution free continuous review inventory system with a service level constraint”, Comp Ind Eng, 27, 213-216.
[17]. Morton, TE, (1971), “The near-myopic nature of the hagged-proportional cost inventory problem with lost sales”. Operations Research 19, 1708-1716.
[18]. Nahmias. S, (1978), “Simple approximations for a variety of dynamic lead time lost-sales inventory models”. Operations Rescarch 27, 904-924.
[19]. Nasri.F,Affisco.J.F. and Paknejad,M.J.(1990), “Set-up cost reduction in a inventory model with finite range stochastic lead times”, International Journal of Production Research, Vol.28,pp.199-212.
[20]. Ouyang, LY; Yeh NC and Wu,KS, (1996), “Mixture inventory model with backorders and lost sales for variable lead time”, Opsearch, Vol. 37, No. 4 ,327-339.
[21]. Ouyang, LY and Chang HC, (2000), “Mixture inventory model involving setup cost reduction with a service level constraint”, Journal of Opl Res Soc, Vol. 47, 829-832.
[22]. Paknejad MJ, Nasri F and Affisco JF, (1995), “Defective units is a continuous review (s,Q) system, Int. J. prod. Res, 33, 2767-2777.
[23]. Porteus,E.L.(1985), “Investing in reduced set-ups in the EOQ model”, Management Sciences, Vol.31,pp.998-1010.
[24]. Porteus,E.L.(1986), “Investing in new parameter values in the discounted EOQ model”, Naval Research Logistics,Vol.33,pp. 39-48.
[25]. Ray, WD (1981), “Computation of reorder levels when the demands are correlated and the lead time random”.Journal of Opl Res Soc 32: 27-34.
[26]. Salameh,M.K;Abboud,N.E. El-Kassar,A.N. and Ghattas,RE,(2003),Continuous review inventory model with delay in payments,International Journal Of Production Economics,85, 91-95.
[27]. Sarkar,B.R and Coates, I.R (1990), “Manufacturing set-up cost reduction under variable lead times and finite opportunities for investment”, International Journal of Priduction Economics,49,237-247.
[28]. Scarf,H (1958), “Studies in the mathematical theory of inventory and production”,Stanford University Press,Stanford,California.
[29]. Silver, EA and Peterson,R.(1985), “Decision systems for inventory management and production planning”, New York:Wiley.
[30]. Silver,EA, Pyke DF and Peterson R, (1998) “Inventory Management and production planning and scheduling”, John Wiley, New York
[31]. Urban,T.L.(2000) “Reorder level determination with serially correlated demand”, Journal of Operational Research Society, 51,762-768.