International Refereed Journal of Engineering and Science (IRJES)
ISSN (Online) 2319-183X, (Print) 2319-1821
Volume 4, Issue 4 (April 2015), PP.06-18

Quantum-inspired artificial fish swarm algorithm based on the Bloch sphere searching

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Abstract:- To enhance the performance of the intelligent optimization algorithm, a new model of performing search on the Bloch sphere is proposed. Then, by integrating the model into the artificial fish swarm optimization, we present a quantum-inspired artificial fish swarm optimization algorithm. In proposed method, the fishes are encoded with the qubits described on the Bloch sphere. The vector product theory is adopted to establish the axis of rotation, and the Pauli matrices are used to construct the rotation matrices. The four fish behaviors, such as moving, tracking, capturing, aggregating, are achieved by rotating the current qubit about the rotation axis to the target qubit on the Bloch sphere. The Bloch coordinates of qubit can be obtained by measuring with the Pauli matrices, and the optimization solutions can be presented through the solution space transformation. The highlight advantages of this method are the ability to simultaneously adjust two parameters of a qubit and automatically achieve the best match between two adjustment quantities, which may accelerate the optimization process. The experimental results show that the proposed method obviously outperforms the classical one in convergence speed and achieves better levels for some benchmark functions.

Keywords:- quantum computing; fish swarm optimizing; Bloch sphere searching; algorithm designin

I. INTRODUCTION

In 2002,Dr. Li Xiaolei proposed about Artificial Fish Swarm,AFS[1] for the first time,The algorithm simulates such several typical behaviors as aggregation, tracking, capture about feeding fish.Artificial Fish Swarm is a typical optimization algorithm,it has the characteristics of simplicity and parallel.After the algorithm proposed, caused wide attention of scholars at home and abroad,In this aspect of the improved algorithm,has put forward the fuse with cultural algorithm early or late ,AFA[2], based on the chaos search ,AFA[3-5],using Taboo Strategy,AFA[6],based on K-means clustering ,AFA[7,8]and other new improved model.In the aspect of application, AFA has been successfully used in Image compression[9], Task scheduling[10],Vehicle routing optimization[11]],Intrusion detection[12], Image segmentation[13] and so on.

Quantum computing is a new computing model,with its unique polymorphisms, superposition, parallelism has attracted extensive attention of scholars at home and abroad.In the real quantum system , quantum bit is based on Bloch spherical description, and contains two adjustable parameters.However, In the current quantum intelligent optimization algorithm, individual coding using Quantum bit encoding described by unit circle, with only one adjustable parameter.And the evolutionary mechanism utilize the quantum rotation gates and quantum not-gate, In essence,Quantum bits are revolving around the center of rotation on the unit circle ,and only change a parameter of quantum bits at the same time, therefore, Quantum properties has not been fully reflected.Although the literature[14] put forward a kind of Quantum genetic algorithm based on Quantum bit coordinate coding, and in the algorithm of quantum bit based on Bloch spherical description,also the evolutionary operator also can adjust the two parameters of quantum bits at the same time, however, the algorithm does not take into account two parameters adjustment quantity matching problem, which current bits to the target bit approximation process is not along the shortest path, so, optimize performance is affected.This paper proposes a new individual coding method and evolutionary.It is different from the literature[14], This paper directly uses the Quantum bit coding based on Bloch spherical description(Not the coordinates of quantum bits), using Quantum bit in Bloch sphere pivoting method to realize individual evolution quantity.With the artificial fish swarm optimization as the breakthrough point, describes that the design and implement the scheme of algorithm in detail.Using the standard-function-extremal optimization as an example,to how that the method is very promising by contrast.

2 Basic operation of AFS

Fish do not have complex power of reasoning, However, through some simple individual behavior can make the group highlights the strong intelligent .Artificial fish swarm algorithm including foraging, cluster, collision, random such four basic operations,Its basic principle is as follows.

2.1 Foraging behavior

This behavior is a kind of basic behavior of artificial fish, The meaning is artificial fish in the current position within the range of perception select a new location randomly. If the i -th artificial fish current position is $\boldsymbol{X}_i$,so the foraging behavior in the neighborhood of $\boldsymbol{X}_i$ (or sensing range) select location $\boldsymbol{X}_j$ randomly can be described as

$$\boldsymbol{X}_j = \boldsymbol{X}_i + \text{Visual} \cdot rand(0,1) \quad (1)$$

In the formula Visual is the scope of vision, $rand(0,1)$ is $(0,1)$ within the random number.

As the minimum optimization as an example, make $f(\boldsymbol{X})$ as individual $\boldsymbol{X}$'s objective function value, if $f(\boldsymbol{X}_i) > f(\boldsymbol{X}_j)$,According to the following formula, take a step in this direction

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \frac{\boldsymbol{X}_j - \boldsymbol{X}_i^t}{\|\boldsymbol{X}_j - \boldsymbol{X}_i^t\|} \text{Step} \cdot rand(0,1) \quad (2)$$

In the formula, Step is forward step.In the opposite,re-select location $\boldsymbol{X}_j$ randomly,to determine whether satisfied the forward conditions ,After Try_number's attempts ,If it still does not satisfied forward conditions, then execute the random behavior.

2.2 Cluster behavior

If the artificial fish current position is $\boldsymbol{X}_i$, the number of partners in the Visual neighborhood is n_f ,and Visual neighborhood heart is $\boldsymbol{X}_c$ .if $f(\boldsymbol{X}_c) \cdot n_f < \delta \cdot f(\boldsymbol{X}_i)$, as it has shown that the neighborhood center has many food and not crowded, in this time according to the following formula ,go for a step forward $\boldsymbol{X}_c$,

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \frac{\boldsymbol{X}_c - \boldsymbol{X}_i^t}{\|\boldsymbol{X}_c - \boldsymbol{X}_i^t\|} \text{Step} \cdot rand(0,1) \quad (3)$$

Otherwise execute foraging behavior.

In the formula δ is crowding factor.

2.3 The following behavior

Suppose the artificial fish current position is $\boldsymbol{X}_i$,in its Visual neighborhood,the minimum of n_f partners in the target value is $\boldsymbol{X}_j$ .if $f(\boldsymbol{X}_j) \cdot n_f < \delta \cdot f(\boldsymbol{X}_i)$, according to the following formula go for a step forward $\boldsymbol{X}_j$, Otherwise execute Foraging behavior.

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \frac{\boldsymbol{X}_j - \boldsymbol{X}_i^t}{\|\boldsymbol{X}_j - \boldsymbol{X}_i^t\|} \text{Step} \cdot rand(0,1) \quad (4)$$

In the formula δ is crowding factor.

2.4 Random behavior

The description of the Rondom behavior is relatively simple,just to pick a random state in the field, and then according to the following formula go for a step forward, it is a default behavior of Foraging behavior.

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \text{Visual} \cdot rand(0,1) \quad (5)$$

3 Quantum-inspired artificial fish swarm algorithm

This paper presents a new Quantum-inspired search mode, And integrate with the artificial fish swarm algorithm, which we call the Quantum-inspired artificial fish swarm algorith, called QIAFS.

3.1Bloch spherical quantum bit description

In quantum computing,a quantum bit is in a two-level quantum system which can be described as two-dimensional complex in Hilbert space . Any state of quantum bits can be written as

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \quad (6)$$

In the formula , $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$.

So,quantum bit belongs to description of the continuous variable θ and ϕ in the vector space, And a quantum bit can be described as many different state.Quantum bit can be described by a little Bloch sphere embedded in three-dimensional Descartes coordinate system. As shown in Fig1 .In the Fig1 $x = \cos\phi\sin\theta$, $x = \sin\phi\sin\theta$, $z = \cos\theta$ .Such like this, Quantum state $|\varphi\rangle$ can be written as

$$|\varphi\rangle = \left[\sqrt{\frac{1+z}{2}}, \frac{x+iy}{\sqrt{2(1+z)}}\right]^{T} \quad (7)$$

So, any point on the Bloch spherical surface $P(x,y,z)$ corresponds to a quantum bit $|\varphi\rangle$ one by one.

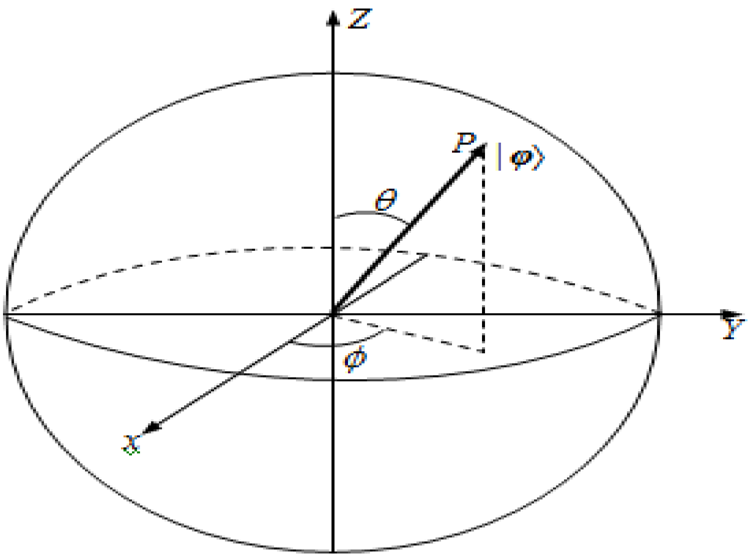

Fig.1 A quantum bit description on the Bloch sphere

3.2 The rotation of a quantum bit on the Bloch sphere

In this paper, we will build a search term in Bloch sphere,Even if quantum bits rotating around a fixed axis toward the target bit on the Bloch sphere. We can change two parameters θ and ϕ of quantum bit by rotation at the same time.And it also can achieve best matching of two adjustment automatically. So that we can simulate quantum behavior better and improve the optimization ability obviously.the key point of rotation is the design of rotating axis, The design method we presented can be stated as the following theorem.

Theorem1 Remember points P and Q are respectively corresponding vector P = [px, py, pz] and Q = [qx, qy, qz] on the Bloch sphere , and the axis where quantum bit turn from point P to Q on Bloch sphere is the vector product of P and Q, Raxis=P×Q,as shown in Fig2 .

Using the definition of vector product definition, the theorem is easy to prove.

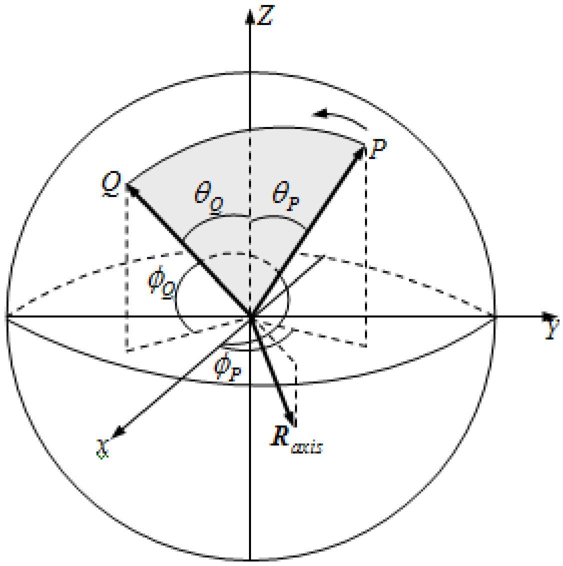

Fig.2 The rotation axis of a quantum bit on the Bloch sphere

Make $|\varphi_{ij}(t)\rangle$ and $|\varphi_{bj}(t)\rangle$ are respectively corresponding vector $P_{ij}=[p_{ijx},\ p_{ijy},\ p_{ijz}]$ and $P_{bj}=[p_{bjx},\ p_{bjy},\ p_{bjz}]$ on the Bloch sphere,According to the above theorem,the axis of $|\varphi_{ij}(t)\rangle$ turn toward $|\varphi_{bj}(t)\rangle$ can be described as $\boldsymbol{R}_{ij}=\dfrac{\boldsymbol{P}_{ij}\times\boldsymbol{P}_{bj}}{\|\boldsymbol{P}_{ij}\times\boldsymbol{P}_{bj}\|}$ (8)

According to the principles of quantum computing, on the Bloch sphere,the spin matrix that around the axis of a vector quantity $\boldsymbol{n}=[n_x,n_y,n_z]$ and arc δ can be described as

$$\boldsymbol{R}_{\boldsymbol{n}}(\delta)=\cos\frac{\delta}{2}\boldsymbol{I}-\mathrm{i}\sin\frac{\delta}{2}(\boldsymbol{n}\times\boldsymbol{\sigma}) \quad (9)$$

In the formula , $\boldsymbol{I}$ is unit matrix, $\boldsymbol{\sigma}=[\boldsymbol{\sigma}_x,\boldsymbol{\sigma}_y,\boldsymbol{\sigma}_z]$ .so, the current quantum bit $|\varphi_{ij}(t)\rangle$ is on the Bloch sphere now, the spin matrix of the around axis $\boldsymbol{R}_{ij}$ turn to target bit $|\varphi_{bj}(t)\rangle$ is

$$\boldsymbol{M}(\delta_{ij})=\cos\frac{\delta_{ij}}{2}I-\mathrm{i}\sin\frac{\delta_{ij}}{2}(\boldsymbol{R}_{ij}\times\boldsymbol{\sigma}) \quad (10)$$

The operating of $|\varphi_{ij}(t)\rangle$ turn to $|\varphi_{bj}(t)\rangle$ is

$$|\varphi_{ij}(t)\rangle=\boldsymbol{M}(\delta_{ij})\,|\varphi_{ij}(t)\rangle \quad (11)$$

In the formula t is iteration steps.

3.3 QIAFS Coding scheme

In the QIAFS, individual uses quantum bit coding based on the Bloch spherical description.Set the population size is N,the space dimension is D,remember the t-th generation of the population is $\boldsymbol{P}(t)=[\boldsymbol{p}_1(t),\boldsymbol{p}_2(t),\cdots,\boldsymbol{p}_N(t)]^{\mathrm{T}}$, the i -th individuals can be encoding (initialization) as

$$\boldsymbol{p}_i(0)=[|\varphi_{i1}(0)\rangle,|\varphi_{i2}(0)\rangle,\cdots,|\varphi_{iD}(0)\rangle] \quad (12)$$

In the formula,

$$|\boldsymbol{\varphi}_{ij}(0)\rangle = \cos(\frac{\theta_{ij}}{2})|0\rangle + e^{\mathrm{i}\phi_{ij}}\sin(\frac{\theta_{ij}}{2})|1\rangle$$, $\theta_{ij} = rand(0,1)\pi$, $\phi_{ij} = rand(0,1)2\pi$.

3.4 The projection measurement of quantum bits

According to the principles of quantum computing, the Bloch coordinate of a quantum bit $|\boldsymbol{\varphi}\rangle$ can use the calculation of the Pauli matrices basis vectors that obtained by projection measure, the definition of Pauli matrices is

$$\boldsymbol{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \boldsymbol{\sigma}_y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \boldsymbol{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (13)$$

For the j -th quantum bit $|\boldsymbol{\varphi}_{ij}\rangle$ of i -th individuals, its coordinates of projection measurement formula is,

$$\begin{cases} x_{ij} = \langle \boldsymbol{\varphi}_{ij} | \boldsymbol{\sigma}_x | \boldsymbol{\varphi}_{ij} \rangle \\ y_{ij} = \langle \boldsymbol{\varphi}_{ij} | \boldsymbol{\sigma}_y | \boldsymbol{\varphi}_{ij} \rangle \\ z_{ij} = \langle \boldsymbol{\varphi}_{ij} | \boldsymbol{\sigma}_z | \boldsymbol{\varphi}_{ij} \rangle \end{cases} \quad (14)$$

In the formula, $i = 1, 2, \cdots, N, \; j = 1, 2, \cdots, D$.

In the QIAFS, The three coordinates of quantum bit can be seen as three parallel gene, each individual contains three parallel gene chains, each gene represents a chain optimization solution. Thus, each individual represent as three optimization solution on of the search space.

3.5 The transformation in Solution space

In the QIAFS algorithm, each individual group contains three Bloch coordinates, each axis represents an optimal solution. Because Bloch coordinates in the interval $[-1, 1]$, so we need to solute the space transformation.Remember optimize issues for the j -th dimension range of variables is $[\mathrm{Min}_j, \mathrm{Max}_j]$, thus the solution space transformation can be described as the three following type.

$$\begin{cases} X_{ij} = [\mathrm{Min}_j(1 - x_{ij}) + \mathrm{Max}_j(1 + x_{ij})]/2 \\ Y_{ij} = [\mathrm{Min}_j(1 - y_{ij}) + \mathrm{Max}_j(1 + y_{ij})]/2 \\ Z_{ij} = [\mathrm{Min}_j(1 - z_{ij}) + \mathrm{Max}_j(1 + z_{ij})]/2 \end{cases} \quad (15)$$

In the formula, $i = 1, 2, \cdots, N, \; j = 1, 2, \cdots, D$.

3.6 The Optimal update

The three groups of individuals corresponding solution (X_{ij}, X_{ij}, X_{ij}) are substituted into the objective function to calculate the individual target.The minimum target so far obtained is gf_{best} , $\boldsymbol{gp}_{best}$ is corresponding individual.so $f(\boldsymbol{p}_i) = \min(f(X_i), f(Y_i), f(Z_i))$, $f_{best} = \min_{1 \le i \le N}(f(\boldsymbol{p}_i))$,if $gf_{best} > f_{best}$,thus $gf_{best} = f_{best}$, $\boldsymbol{gp}_{best} = \boldsymbol{p}_{best}$.

3.7 Four kinds of search behavior of

QIAFS

1) Foraging behavior

2) For the i-th artificial fish $\boldsymbol{p}_i^t$, use the global optimum for the target individual quantum bit establish the rotation axis, with rotate angle $\theta_0 \cdot rand(0,1)$ (where θ_0 is initial value for the angle) around the axis of rotation on $\boldsymbol{p}_i^t$,and after rotation denoted by $\hat{\boldsymbol{p}}_i^t$.

If $f(\boldsymbol{p}_i^t) > f(\hat{\boldsymbol{p}}_i^t)$, then stop; Otherwise, repeat the rotation process.After Try_number times try, if it still not satisfied the formula $f(\boldsymbol{p}_i^t) > f(\hat{\boldsymbol{p}}_i^t)$, Then execute Random behavior.

2) Cluster behavior

For the i-th artificial fish $\boldsymbol{p}_i^t$,suppose the number of partners located in the neighborhood is n_f ,and the fish in the neighborhood centers seem as the virtual artificial fish $\hat{\boldsymbol{p}}_c$ .if $f(\hat{\boldsymbol{p}}_c)\cdot n_f < \delta f(\boldsymbol{p}_i^t)$ (δ is crowding factor), it represent that neighborhood center with more food and less crowded,at this time use each quantum bit of point $\hat{\boldsymbol{p}}_c$ as the target to establish rotation axis,and use the $\theta_0 \cdot rand(0,1)$ as the rotation angle,therefore execute Foraging behavior.

3) Collision behavior

For the i-th artificial fish $\boldsymbol{p}_i^t$,suppose its min target within n_f partners in Visual neighborhood is $\boldsymbol{p}_j^t$.Use the each quantum bit establish the rotation axis, with rotate angle $\theta_0 \cdot rand(0,1)$ (where θ_0 is initial value for the angle) around the axis of rotation on $\boldsymbol{p}_i^t$,therefore execute Foraging behavior.

4) Random behavior

For the i-th artificial fish $\boldsymbol{p}_i^t$,choose a partner randomly in its Visual neighborhood ,Then establish the rotation axis based on each quantum bit, use the $\theta_0 \cdot rand(0,1)$ as the rotating angle to implement rotate around the axis on $\boldsymbol{p}_i^t$.in the QIAFS,it also is the default of Foraging behavior.

3.8 QIAFS Embodiments

(1) Initialization,include: Population sizeN, Initial individual $\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_N$, Initial angle θ_0, Field of view Visual, Retries times Try number, Crowding factor δ.

(2) Calculate the individual value of the objective function, save the global best individual.

(3) Individuals in the foraging, clusters, collision, random four behaviors, by comparison, and then select a best practices to implemen, updating individual location information.

(4) If the termination condition is satisfied, then output the reason, otherwise turn(2).

4 The convergence of QIAFS

Suppose $\boldsymbol{P}_t = \{\boldsymbol{p}_1^t, \boldsymbol{p}_2^t, \cdots, \boldsymbol{p}_N^t\}$ is the t-th artificial and population of QIAFS,wherein the definition for the i-th individual is $\boldsymbol{p}_i^t$,

$$\boldsymbol{p}_i^t = \begin{bmatrix} \cos(\theta_{i1}^t/2) \\ e^{\mathrm{i}\phi_{i1}^t}\sin(\theta_{i1}^t/2) \end{bmatrix} \cdots \begin{bmatrix} \cos(\theta_{iD}^t/2) \\ e^{\mathrm{i}\phi_{iD}^t}\sin(\theta_{iD}^t/2) \end{bmatrix} \quad (16)$$

About the convergence of QIAFS , we have the following conclusions.

Theorem 2 QIAFS is a convergent with probability 1.

Prove Set digits of θ and ϕ respectively is υ_1 and υ_2, the state space of population is $(\upsilon_1\upsilon_2)^{D\times N}$,as known by population update strategy, $\boldsymbol{P}_{t+1}$ is relevant with $\boldsymbol{P}_t$ only, so iterative sequence $\{\boldsymbol{P}_t, t\ge 1\}$ is finite homogeneous Markov chains.

remerber $\boldsymbol{bp}^t = \{\boldsymbol{p}^t \mid f(\boldsymbol{p}^t) = \min_{1\le i\le N} f(\boldsymbol{p}_i^t)\}$ is the best individual of t -th population $\boldsymbol{P}_t$,

$s^* = \{\boldsymbol{bp}^k \mid \min_{1\le k\le (\upsilon_1\upsilon_2)^{D\times N}} f(\boldsymbol{bp}^k) = f^*\}$ is a global optimal solution set, f^* is the minimum objective function value for the overall situation.

Suppose $T = \{t \mid \boldsymbol{bp}^t \notin s^*\}$, $\boldsymbol{P}_t^k$ represent the k -th stats of artificial fish after t times iterations in the state space $k = 1, 2, \cdots, (\upsilon_1\upsilon_2)^{D\times N}$.

The following calculate the step transition transition probability $P_t(i\to j) = P(\boldsymbol{P}_t^i \to \boldsymbol{P}_{t+1}^j)$ of stochastic process $\{\boldsymbol{P}_t, t\ge 1\}$.

Because QIAFS use the elitist strategy, so $f(\boldsymbol{P}_{t+1}^j) \le f(\boldsymbol{P}_t^i)$ .When $i\notin T, j\in T$, $P_t(i\to j) = 0$,when $i\in T, j\notin T$, $P_t(i\to j)\ge 0$.

Suppose $P_t(i)$ is probability of in the state $\boldsymbol{P}_t$, remember $P_t = \sum_{i\in T} P_t(i)$, By the properties of Markov chain,the probability $\boldsymbol{P}_{t+1}$ of state $j\in T$ is

$$P_{t+1} = \sum_{i\in T}\sum_{j\in T} P_t(i)P_t(i\to j) + \sum_{i\notin T}\sum_{j\in T} P_t(i)P_t(i\to j)$$

Owing to $P_t = \sum_{i\in T}\sum_{j\in T} P_t(i)P_t(i\to j) + \sum_{i\in T}\sum_{j\notin T} P_t(i)P_t(i\to j)$

So $P_{t+1} = P_t - \sum_{i\in T}\sum_{j\notin T} P_t(i)P_t(i\to j) \le P_t$

So $\lim_{t\to\infty} P_t = 0$

$$\lim_{t\to\infty} P(f(bp^t) = f^*)$$
$$= 1 - \lim_{t\to\infty}\sum_{i\in T} P_t(i) = 1 - \lim_{t\to\infty} P_t = 1$$

Scilicet QIAFS is Convergent with probability 1.

5The experimental results and analysis

In this section, take the optimization of function extreme value for example, and to verify the superiority of QIAFS by the comparison among the common artificial fish algorithm, the Bloch quantum genetic algorithm in literature 14 and the elite genetic algorithm with reserve strategy.

5.1 Test Function

(1) $f_1(\boldsymbol{X}) = \sum_{i=1}^{D}(\lfloor x_i + 0.5\rfloor)^2$;

$\Omega = [-100,100]^D$; $x_i^* = 0$; $f(\boldsymbol{X}^*) = 0$

(2) $f_2(\boldsymbol{X}) = 418.98 - \frac{1}{D}\sum_{i=1}^{D} x_i \sin(\sqrt{|x_i|})$

$\Omega = [-500,500]^D$; $x_i^* = 420.97$; $f(\boldsymbol{X}^*) = 0$

(3) $f_3(\boldsymbol{X}) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2})$

$$-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$$

$\Omega = [-32, 32]^D$; $x_i^* = 0$; $f(X^*) = 0$

(4) $f_4(\boldsymbol{X}) = 78.33 + \frac{1}{D}\sum_{i=1}^{D}(x_i^4 - 16x_i^2 + 5x_i)$

$\Omega = [-100,100]^D$; $x_i^* = -2.9$; $f(\boldsymbol{X}^*) = 0$

$f_5(\boldsymbol{X}) = \frac{\pi}{D}\{10\sin^2(\pi y_1) +$

(5) $\sum_{i=1}^{D-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_D - 1)^2\} +$

$\sum_{i=1}^{D} u(x_i, 10, 100, 4)$

$y_i = 1 + \frac{1}{4}(x_i + 1)$;

$$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \le x_i \le a \\ k(-x_i - a)^m, & x_i < -a \end{cases} ;$$

$\Omega = [-50, 50]^D$; $x_i^* = -1$; $f(\boldsymbol{X}^*) = 0$

(6) $f_6(\boldsymbol{X}) = \frac{1}{10}\{\sin^2(3\pi x_1) +$

$\sum_{i=1}^{D-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] +$

$(x_D - 1)^2[1 + \sin^2(2\pi x_D)]\} +$

$\sum_{i=1}^{D} u(x_i, 5, 100, 4)$

$$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a \le x_i \le a \\ k(-x_i - a)^m, & x_i < -a \end{cases} ;$$

$\Omega = [-50, 50]^D$; $x_i^* = 1$;

(7) $f_7(\boldsymbol{X}) = \sum_{i=1}^{D-1}(x_i^2 + 2x_{i+1}^2 - 0.3\cos(3\pi x_i)\cos(4\pi x_{i+1}) + 0.3)$

$\Omega = [-100, 100]^D$; $x_i^* = 0$; $f(\boldsymbol{X}^*) = 0$

(8) $f_8(\boldsymbol{X}) = \sum_{k=1}^{D}\sum_{j=1}^{D}(\frac{y_{jk}^2}{4000} - \cos(y_{j,k}) + 1)$

$y_{jk} = 100(x_k - x_j^2)^2 + (1 - x_j)^2$

$\Omega = [-100,100]^D$; $x_i^* = 1$; $f(\boldsymbol{X}^*) = 0$

(9) $f_9(\boldsymbol{X}) = 3.86 - \sum_{i=1}^{4} c_i \exp(-\sum_{j=1}^{3} a_{ij}(x_j - b_{ij})^2)$

$x_i \in [0, 1]$; $\boldsymbol{X}^* = [0.115, 0.556, 0.853]$

$f(\boldsymbol{X}^*) = 0$

$$A = \begin{bmatrix} 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix} ; \quad C = [1.0\ 1.2\ 3.0\ 3.2]$$

$$B = \begin{bmatrix} 0.3689 & 0.1170 & 0.2673 \\ 0.4699 & 0.4387 & 0.7470 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{bmatrix}.$$

(10) $$f_{10}(\boldsymbol{X}) = 100[(x_3 - 10\theta(x_1, x_2))^2 + (\sqrt{x_1^2 + x_2^2} - 1)^2] + x_3^2$$

$$\theta(x_1, x_2) = \begin{cases} \frac{1}{2\pi}\tan^{-1}\frac{x_2}{x_1}, & x_1 > 0 \\ \frac{1}{2\pi}\tan^{-1}\frac{x_2}{x_1} + 0.5, & x_1 < 0 \end{cases}$$

$$x_i \in [-10, 10]; \quad \boldsymbol{X}^* = [1,\ 0,\ 0]; \quad f(\boldsymbol{X}^*) = 0$$

5.2 (Parameter settings)

The ten functions above are all minimum optimization. The dimensions of the former eight high-dimensional function are D = 30, and the latter two low-dimensional function are 3.For comparison; we set the precision threshold for every function. As shown in table 1, when the optimization result is less than the corresponding threshold, the algorithm is convergent. If the algorithm is convergent, the optimization steps of the algorithm called the iteration steps; otherwise, the iteration steps are equal to the limited steps. Try number setting is 10; visual setting is 0.01(Max-Min). The Max and Min are respectively the test function of the independent variable of the maximum and the minimum. The congestion factor $\delta = 10$ .For QIAFS, the initial value of the corner $\theta_0 = 0.05\pi$; For AFS, the forward step is equal to 0.05(Max-Min); For BQGA, the corner step θ_0 is equal to 0.05π, the mutation probability is 10^{-3} ; For EGA, the crossover probability is 0.8; the mutation probability is 0.05. The population sizes for the four algorithm are 100, so is the limited steps.

Tab.1 The threshold value of 10 benchmark functions

function	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
Threshold	1.0	200	1.0	10	0.1	10^{-3}	2.0	700	10^{-9}	10^{-5}

5.3 Results contrast

To enhance the objectivity of the comparison results, the statistics of the results contrast was made after 50 times isolated running of every algorithm. The average time per iteration comparison is shown as Table 2. The convergence time (statistical difference convergence solution to X,Y solution, Z number of solution for both QIAFS and BQGA), the average number of steps, the average results comparison is shown as Table 3.

Tab.2 The average running time contrasts for each iteration (Unit: Sec)

function	QIAFS	AFS	BQGA	EGA
f_1	**15.4589**	**2.1083**	**0.0229**	**0.0009**
f_2	**18.2018**	**1.9192**	**0.0242**	**0.0012**
f_3	**19.2819**	**2.1627**	**0.0251**	**0.0014**

f_4	**8.41620**	**1.8256**	**0.0257**	**0.0018**
f_5	**20.2075**	**2.6023**	**0.0302**	**0.0044**
f_6	**19.6590**	**2.5276**	**0.0296**	**0.0043**
f_7	**19.7429**	**2.2785**	**0.0259**	**0.0023**
f_8	**31.2085**	**4.6551**	**0.1168**	**0.0308**
f_9	**4.61450**	**1.5158**	**0.0088**	**0.0023**
f_{10}	**3.96580**	**1.2965**	**0.0030**	**0.0013**

Tab.3 Four algorithms' optimization result contrasts

Function	Algorithm	convergence Frequency	X Solution	Y Solution	Z Solution	Average Steps	Average results
f_1	QIAFS	50	20	18	12	59.66	0
	AFS	3	—	—	—	98.96	5.200
	BQGA	0	0	0	0	100	2674
	EGA	0	—	—	—	100	16316
f_2	QIAFS	50	17	13	20	5.380	135.8
	AFS	30	—	—	—	72.54	191.5
	BQGA	8	6	2	0	88.28	224.3
	EGA	0	—	—	—	100	308.6
f_3	QIAFS	50	26	15	9	57.42	0.062
	AFS	0	—	—	—	100	6.008
	BQGA	0	0	0	0	100	10.71
	EGA	0	—	—	—	100	16.82
f_4	QIAFS	46	14	20	12	23.38	5.517
	AFS	14	—	—	—	90.30	11.33
	BQGA	1	0	0	1	98.42	15.97
	EGA	0	—	—	—	100	34.09
f_5	QIAFS	37	16	15	6	55.84	1.392
	AFS	7	—	—	—	92.48	2.109
	BQGA	0	0	0	0	100	2.8e+04
	EGA	0	—	—	—	100	1.7e+07
f_6	QIAFS	39	17	14	8	94.44	0.0007
	AFS	0	—	—	—	100	1.1682
	BQGA	0	0	0	0	100	1.8e+05
	EGA	0	—	—	—	100	6.3e+07
f_7	QIAFS	40	17	18	5	98.30	1.609
	AFS	1	—	—	—	99.36	7.053
	BQGA	0	0	0	0	100	8284
	EGA	0	—	—	—	100	46079

f_8	QIAFS	47	16	30	1	88.70	660.05
	AFS	13	—	—	—	95.74	738.39
	BQGA	0	0	0	0	100	1.8e+13
	EGA	0	—	—	—	100	1.0e+16
f_9	QIAFS	49	14	20	15	58.80	1.3e-10
	AFS	10	—	—	—	90.00	4.9e-09
	BQGA	0	0	0	0	100	1.8e-03
	EGA	0	—	—	—	100	3.3e-02
f_{10}	QIAFS	50	17	30	3	33.42	5.2e-08
	AFS	28	—	—	—	77.10	1.7e-05
	BQGA	0	0	0	0	100	0.7259
	EGA	0	—	—	—	100	1.8623

Table 2 implied that on the running time, the four algorithms from long to short sort of QIAFS, AFS, BQGA, EGA. The analysis of the result as follows. In QIAFS and AFS, each iteration of each artificial fish have to perform four acts, but also perform foraging behavior Try number at most times, and then choose the best kind of behavior as the current search results. Therefore, the behavior of a complex algorithm to calculate the amount of fish is too large, leading to QIAFS and AFS running time significantly longer than BQGA and EGA. Whereas, QIAFS has adopted the three chains quantum bit encoding mechanism, each artificial fish simultaneously searching three new locations while iterating, and each search behavior must involve measurement and projection bit rotation matrix structure and so on, which significantly increased the amount of computation, leading to its running time is significantly longer than AFS. For BQGA, although which also adopted the three-chain quantum bit encoding mechanism, it does not involve measuring the quantum bit rotation matrix structure or operations, and the evolution process (only rotation and variation) is relatively simply. Therefore, the running time is less than the QIAFS and AFS, while it's longer than EGA because that every iteration needs to update three gene chains.

Table 3 shows that in terms of optimizing ability, QIAFS obviously better than that of AFS, and QIAFS and AFS are apparently better than BQGA and EGA. And four kinds of algorithm optimization ability of the order from high to low are QIFAS, AFS, BQGA, and EGA. This comparison results can by analyzed as follows. First, Quantum bit the introduction of the three chains encoding mechanism effectively improved the algorithm for the solution space ergodicity. According to the geometric features of the Bloch sphere, this kind of coding mechanism can increase the probability of access to the global optimal solution by expanse the number of the global optimal solution. This is the direct reason why the two kinds of quantum derivative algorithm are superior to the corresponding classical algorithm. Second, QIAFS and AFS effectively simulate foraging behavior of fish, is the root cause of its high-performance optimization capabilities. Four kinds of behavior each may find a good solution. Choose the best behavior as the search direction among four kinds of search direction, which reflected the diversity of search direction and the complementary advantages among four behaviors. This approach significantly increases the ability of searching optimization, which leads the two fish algorithms are superior to the two genetic algorithms. Third, besides QIAFS adopted the fish search mechanism, it also used a new method that is quantum bit pivoting. This is the reason why the optimization ability of QIAFS is apparently better than that of BQGA. Even though two algorithms both adopted the three chain quantum bit encoding, the algorithm adjustment quantum bit uses different methods. BQGA using direct method, which directly adjusted two parameters of the quantum bit. And also used the same adjustment amount ($\Delta\theta = \Delta\phi$). This method clearly cannot approach the target bit along the shortest path. Since if you want to approach the target bit along the shortest path, $\Delta\theta$ and $\Delta\phi$ must meet a certain matching relationship which is very difficult to express it clearly by a certain analytic formula. While in QIAFS, the adjustment of individual quantum bits adopted the indirect method. That is to make the quantum bits directly along the Bloch sphere axis of rotation on the way to the target bit of the great circle (at this time the shortest path exists). Although this method did not adjust the two parameters of the quantum bit directly, it automatically achieved the precise match of $\Delta\theta$ and $\Delta\phi$. So this method has higher ability of optimization.

Comprehensive two aspects of funning time and optimization ability, QIAFS precisely at the expense of the running time in exchange for improved optimization capabilities. This is consistent with no free lunch theorem. Besides, though the running time of QIAFS is longer, it still has a broad application prospect to a large number of offline optimization problems that not concerned about the running time.

VI. CONCLUSIONS

In this paper we propose a Quantum-inspired artificial fish swarm algorithm,using quantum bit inplement Individual coding based on Bloch spherical description,using quantum bit rotating around the axis based on Bloch sphere to implement four search behavior AFSA .The experimental results reveal the quantum bit coding function based on Bloch sphere, and individual evolutionary roteting around the axis which make the adjustment of quantum bit two parameters with best matches . it can effectively improve the ability of intelligent optimization algorithms effectively.

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