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Multi-Objective Optimization of Carbon Fibre Reinforced Plastic (CFRP) Circular Hollow Section Using Genetic Algorithm for Engineering Structures

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Abstract:- The multi-objective optimization of carbon fibre reinforced plastics (CFRP) circular hollow sections using genetic algorithm for engineering structures is discussed. A MATLAB program incorporating Genetic Algorithm was used to obtain an optimal section. The result show that the optimal mass of CFRP required is 320kg while the thickness, inner radius and external radius were obtained as 44.2mm, 165mm and 127mm respectively to adequately sustain a load of 1000kN without failure, which represent loads from offshore structures, bridges, high raise buildings etc.

Keywords:- Genetic Algorithm, Multi-objective optimization, CFRP, Hollow Section

I. INTRODUCTION

In the last 60 years since the Second World War, Fibre Reinforced Polymer (FRP) has been used in many structural applications due to their excellent strength and weight characteristics as well as the ability for their properties to be tailored to the requirements of several complex applications (Iyer and Sen, 1991). FRP is increasingly used in the last decades in civil engineering constructions. The fibre composite members have been used in many countries to construct large-scale fibre composite structures such as traffic bridges and pedestrian bridges. Pedestrian bridges in rural areas are the most famous application of the fibre composites, but there are limited design guidelines for such applications (Abbro et al, 2007). This paper considers the optimization of Carbon Fibre Reinforced Polymer (CFRP) hollow sections under load as a contribution to the use of CFRP materials in civil engineering infrastructure.

II. BACKGROUND OF STUDY

Optimization of composite materials is an active research area with many open questions (Browne, 2013). These optimization problems typically have reasonably small dimension (fewer than 20 variables) but are subject to many manufacturing constraints. For example, Starnes and Haftka (1979) looked at composite panels and optimized them for maximum buckling load subject to strength and displacement constraints. Tenek and Hagiwara (1994) used homogenisation techniques to maximise the fundamental eigen frequency of both isotropic and composite plates. To perform the optimization they used sequential linear programming (SLP) methods. Setoodeh et al (2009) and Lindgaard and Lund (2010) optimized the layout of fibre angles in a composite material in order to maximize the buckling load of the material. Karakaya and Soykasap (2011) used a genetic algorithm and simulated annealing to optimize composite plates

a) Multi-Objective Optimization

Definition of a multi-objective optimization problem: The general multi-objective optimization problem is posed as follows (Marler and Arora, 2004):

Minimize $$F(x) = [F_1(x), F_2(x), \dots, F_k(x)]^T \qquad 1$$

Subject to $$g_j(x) \le 0, \qquad j = 1, 2, \dots, m, \qquad 2$$

$$h_l(x) = 0, \qquad l = 1, 2, \dots, e,$$

Where k is the number of objective functions, m is the number of inequality constraints, and e is the number of equality constraints. $x \in E^n$ is a vector of design variables (also called decision variables), where n is the number of independent variables x_i. $F(x) \in E^k$ is a vector of objective functions $F_i(x) : E^n \to E^1$. $F_i(x)$ are also called objectives, criteria, payoff functions, cost functions, or value functions. The gradient of $F_i(x)$ with respect to x is written as $\nabla_x F_i(x) \in E^n$. X_i^* is the point that minimizes the objective function $F_i(x)$. Any comparison (≤, ≥, etc.) between vectors applies to corresponding vector components.

III. METHODOLOGY

In this paper, MATLAB package is employed to assess the optimization of the hollow CFRP section. Specifically, the GA program will be written using MATLAB as a substitute for steel hollow sections, to obtain the optimal values of CFRP sections under the load.

a) Genetic Algorithms

The procedure used for this optimization is genetic algorithm and it is generally referred to as unconstrained nonlinear optimization.

b) Objective function formulation

The objective is to design a minimum-mass *tubular CFRP* hollow section of length, *l* supporting a load, *P* without buckling. It is assumed that the hollow section is fixed at the base and pinned at the top.
The buckling load (also called the critical load) for a cantilever column is given as (Arora, 2004)

$$P_{cr} = \frac{\pi^2 EI}{4l^2} \qquad 3$$

Where, *I* is the moment of inertia for the cross section of the column
E is the the modulus of elasticity (Young's modulus).
The choice of the fixed-pined ends is to simulate a practical column end conditions in a structure.
Design variables are defined: *R* = mean radius of the column; *t* = wall thickness
Assuming that the column wall is thin (*R* >> *t*), the material cross-sectional area and moment of inertia are:

$$A = 2\pi Rt;\ I = \pi R^3 t \qquad 4$$

The total mass of the column to be minimized is given as

$$Mass = \rho(lA) = 2\rho l\pi Rt \qquad 5$$

c) Constraints formulation

The first constraint is that the stress (P/A) should not exceed the material allowable stress σ_a, to avoid crushing the material. This is expressed as the inequality $\sigma \ll \sigma_a$. Replacing σ by P/A and then substituting for *A*, we obtain

$$\frac{P}{2\pi Rt} \leq \sigma_a \qquad 6$$

While the limit state condition can be realized as;

$$\frac{P}{2\pi Rt} - \sigma_a \leq 0 \qquad 7$$

The column should not buckle under the applied load *P*, which implies that the applied load should not exceed the buckling load, that is, $P \leq P_{cr}$. Using the given expression for the buckling load and substituting for *I*, we obtain

$$P \leq \frac{\pi^3 ER^3 t}{4l^2} \qquad 8$$

While the limit state for structural safety can be expressed as;

$$P - \frac{\pi^3 ER^3 t}{4l^2} \leq 0 \qquad 9$$

Finally, the design variables *R* and *t* must be within the specified minimum and maximum values:

$$R_{min} \leq R \leq R_{max};\ t_{min} \leq t \leq t_{max} \qquad 10$$

The Redefinition of Equations 4 – 9 in MATLAB variables can be achieved as give below
Let R = x1, t = x2
Therefore, Equation 4 and 5 becomes

$$A = 2 * \pi * x1 * x2 \qquad 11$$

$$I = \pi \quad * x1^3 * x2 \qquad 12$$

Now, minimize

Mass, $f(x) = 2 * \pi * \rho * l * x1 * x2$ 13

Subject to

$$g_1(x) = [\frac{P}{2*pi*x1*x2}] - \sigma_a \leq 0 \qquad 14$$

$$g_2(x) = P - [\frac{pi^3*E*x1^3*x2}{4*L^2}] \leq 0 \qquad 15$$

$$g_3(x) = -x1 \leq 0; \qquad 16$$

$$g_4(x) = -x2 \leq 0; \qquad 17$$

$$0.001 \leq x1 \leq 1; 0.005 \leq x2 \leq 0.2 \qquad 18$$

Figure 3 and 4 shows the M-files for the objective and constraint functions, respectively

IV. RESULTS AND DISCUSSION

a) Input Parameters

The following are the assumed values of various parameters (from literature and experience) used in the program. These are:

Load, P = 10MN,
Length, L = 5m = 5000mm
Mass Density, $\rho = 1570 kg/m3,$
Allowable Stress $\sigma_a = 248 MPa,$
Young Modulus, $E = 230 GPa$
Area, A $= 2\pi Rt$
Moment of Inertia, I $= \pi R^3 t$

All constraints are normalized and rewritten using redefined design variables. Therefore the optimization problem is stated in the standard form as follows

minimize, $f(x) = 2 * pi * \rho * l * x1 * x2 \equiv 2 * \pi * 1570 * 5 * x1 * x2$ 19

Subject to

$g_1(x) = \left[\frac{P}{2*pi*x1*x2}\right] - \sigma_a \leq 0 \equiv \left[\frac{10(1.0\times10^6)}{2*pi*x1*x2}\right] - [248(1.0 \times 10^6)] \leq 0$ 20

$g_2(x) = P - [\frac{pi^3*E*x1^3*x2}{4*L^2}] \leq 0 \equiv 10(1.0 \times 10^6) - [\frac{pi^3*(230\times109)*x1^3*x2}{4*(5) \quad \times(5)}] \leq 0$ 21 $g_3(x) = -x1 \leq 0;$ 22

$g_4(x) = -x2 \leq 0;$ 23

$0.001 \leq x1 \leq 1; 0.005 \leq x2 \leq 0.2$ 24

The problem is solved using the *fminunc* function in the Optimization Toolbox.

The objective function for the optimization is the minimization of mass. The program is designed to read the required data and apply its necessary constraints. The program then searches for the minimum thickness and radius that can adequately carry the given load with the optimum mass. The result of the iteration process are shown in Plate 1. The results are used for numerical modeling.

a) Mass Reduction

The reduction of mass obtained after the optimization process showed that, CFRP had a mass of the structure at 320kg obtained from the optimization process when GA was employed. The mass of steel was at 1595.1kg when the optimization was performed using the same objective and constraint functions. This shows about 400% weight saving in using CFRP in place of steel.

V. CONCLUSION

From the optimization of the CFRP hollow section along with identification of the objective function, constraints and design variables, the study resulted in optimal values of 320.64kg, 44.2mm and 147mm for mass, thickness and mean radius respectively for a CFRP Hollow section will be able to adequately sustain a load of 1000kN without failure which represents loads from offshore structures, bridges, high raise buildings etc.

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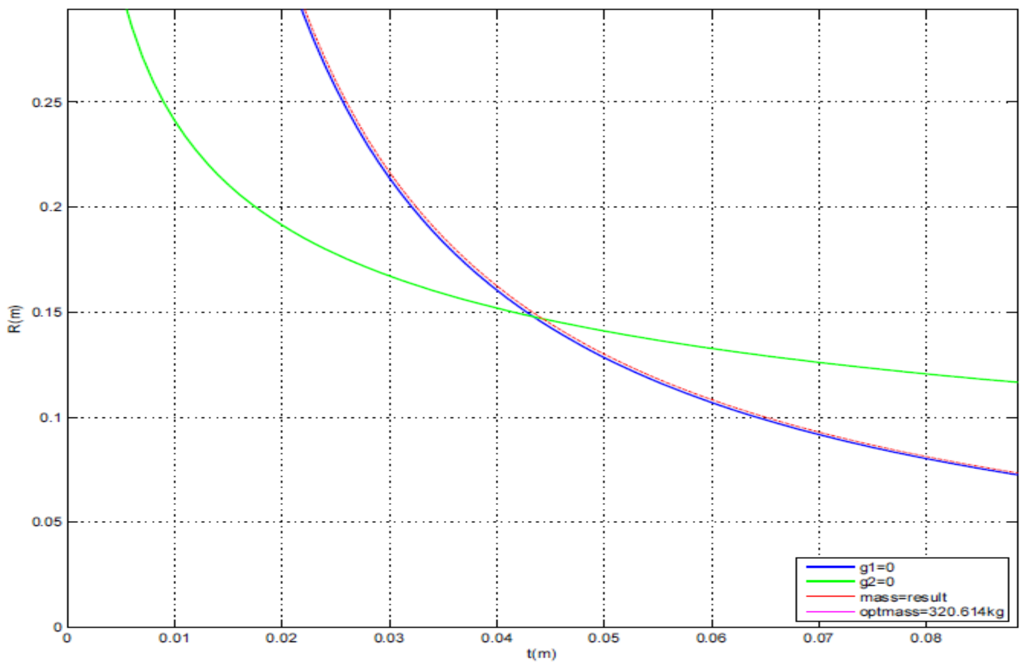

Figure 1: Graphical display of the iteration process

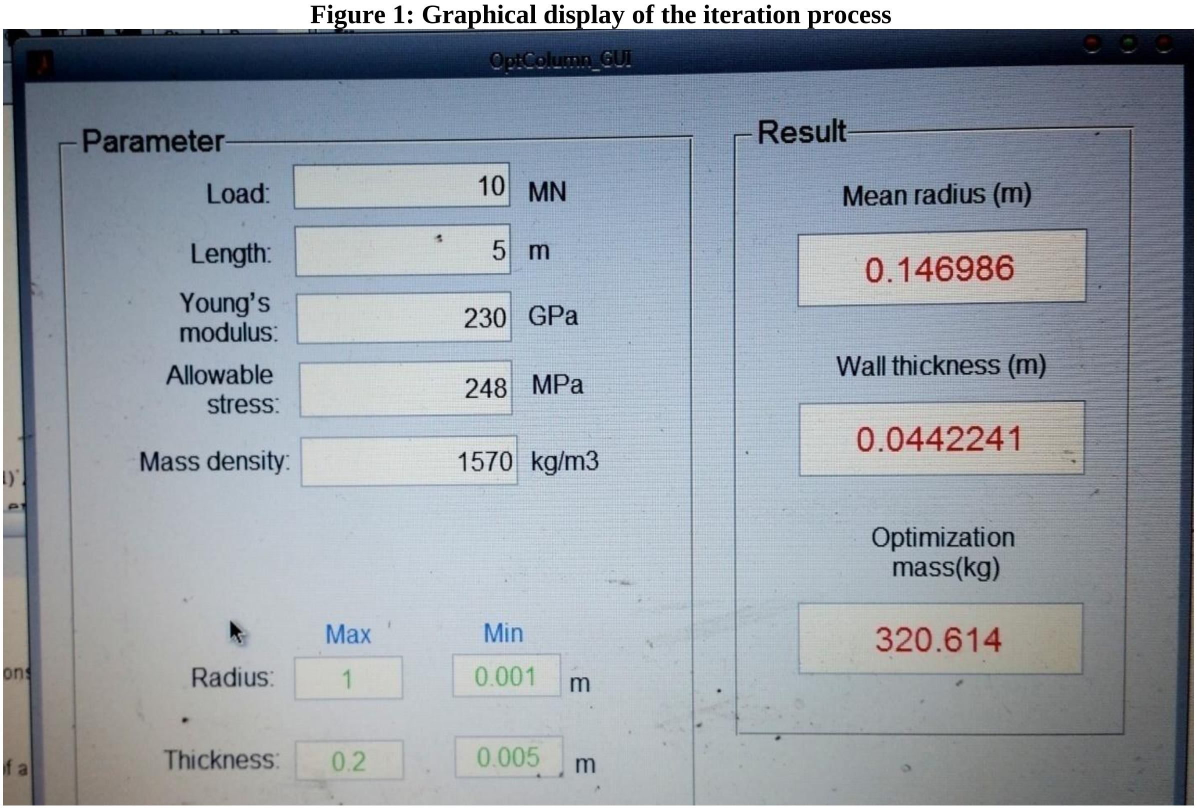

Figure 2: Output of optimization showing the optimum mass, mean radius, and thickness.

```
s.lt=handles.metricdata.length;     % length of the column
s.dn=handles.metricdata.density;    % mass of the density
s.ld=handles.metricdata.load; % load
s.ym=handles.metricdata.modulu; % Young's modulus
s.as=handles.metricdata.stress; % allowable stress
rm=handles.metricdata.Rmax; % Allowable maximum radius
rn=handles.metricdata.Rmin; % Allowable minimum radius
tm=handles.metricdata.Tmax; % Allowable maximum thickness
tn=handles.metricdata.Tmin; % Allowable minimum thickness

ObjectiveFunction = @(x) 2*pi*s.lt*s.dn*x(1)*x(2);
```

Figure 3: M-File for invoking the Objective Function

```
function [g, h] = con_f(x,s)
x1 = x(1); x2 = x(2);
% Set input parameters
P =s.ld*1e6; % loading (Kg)
E = s.ym*1e9; % Young's modulus (Pa)
L = s.lt; % length of the column (m)
Sy = s.as*1e6; % allowable stress (Pa)

% Inequality constraints

g(1) = P/(2*pi*x1*x2)-Sy;    % A=2*pi*R*t
g(2) = P - (pi^3*E*x1^3*x2)/(4*L^2); % I=pi*R^3*t
g(3) = -x1;
g(4) = -x2;

% Equality constraint (none)
h = [];
return
```

Figure 4: M-File for invoking Constraints functions