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Inventory Model with Different Deterioration Rates with Stock and Price Dependent Demand under Time Varying Holding Cost

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Abstract:- An inventory model for deteriorating items with stock and price dependent demand is developed. Holding cost is considered as function of time. Shortages are not allowed. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.

Keywords:- Inventory model, Deterioration, Price dependent demand, Stock dependent demand, Time varying holding cost

I. INTRODUCTION

Deteriorating items inventory model have been studies by many authors in past. Certain products such as medicine, volatile liquids, food stuff decrease under deterioration during their normal storage period. Ghare and Schrader [1963] first developed an EOQ model with constant rate of deterioration. The model was extended by Covert and Philip [1973] by considering variable rate of deterioration. Shah and Jaiswal [1977] further extended the model by considering shortages. The related work are found in (Nahmias [1982], Raffat [1991], Goyal and Giri [2001], Ouyang et al. [2006], Wu et al. [2010]).

Aggarwal and Goel [1984] discussed an inventory model with weibull rate of decay with selling price dependent demand. Patra et al. [2010] developed a deterministic inventory model when deterioration rate was time proportional. Demand rate was taken as a nonlinear function of selling price, deterioration rate, inventory holding cost and ordering cost were all functions of time. Tripathy and Mishra [2010] dealt with development of an inventory model when the deterioration rate follows Weibull two parameter distribution, demand rate is a function of selling price and holding cost is time dependent. Patel and Parekh [2014] developed an inventory model with stock dependent demand under shortages and variable selling price.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with stock and price dependent demand with different deterioration rates for the cycle time under time varying holding cost. Shortages are not allowed. Numerical example is provided to illustrate the model. Sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

D(t) : Demand rate is a linear function of price and inventory level ($a + bI(t) - \rho p$, $a>0$, $0<b<1$, $\rho>0$)
A : Replenishment cost per order
c : Purchasing cost per unit
p : Selling price per unit
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta_1<1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta_2<1$
π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a function of price and inventory level.
- Replenishment rate is infinite and instantaneous.

- Lead time is zero.
- Shortages are not allowed.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

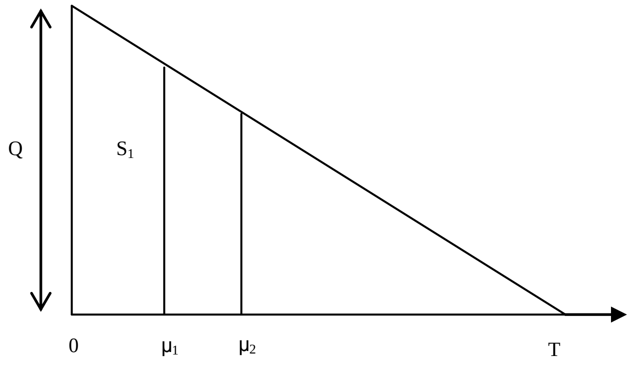


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bI(t) - \rho p), \qquad 0 \le t \le \mu_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bI(t) - \rho p), \qquad \mu_1 \le t \le \mu_2 \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bI(t) - \rho p), \qquad \mu_2 \le t \le T \qquad (3)$$

with initial conditions I(0) = Q, $I(\mu_1) = S_1$ and I(T) = 0.

Solutions of these equations are given by

$$I(t) = Q(1 - bt) - (at + \frac{1}{2}bt^2 - \rho pt - \frac{1}{2}\rho bpt^2 - abt^2 + \rho bpt^2), \qquad (4)$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a(\theta+b)(\mu_1^2 - t^2) - \frac{1}{2}\rho p(\theta+b)(\mu_1^2 - t^2) \\ - a(\theta+b)t(\mu_1 - t) + \rho p(\theta+b)t(\mu_1 - t) - ab\theta t(\mu_1^2 - t^2) + \rho pb\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1[1 + (\theta+b)(\mu_1 - t)] \qquad (5)$$

$$I(t) = \begin{bmatrix} a(T - t) - \rho p(T - t) + \frac{1}{2}ab(T^2 - t^2) - \frac{1}{2}\rho pb(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) - \frac{1}{6}\rho p\theta(T^3 - t^3) \\ - abt(T - t) + b\rho pt(T - t) - \frac{1}{6}ab\theta t(T^3 - t^3) + \frac{1}{6}\rho bp\theta t(T^3 - t^3) - \frac{1}{2}a\theta t^2(T - t) \\ + \frac{1}{2}\rho p\theta t^2(T - t) + \frac{1}{4}\rho bp\theta t^2(T^2 - t^2) - \frac{1}{4}ab\theta t^2(T^2 - t^2) \end{bmatrix}. \qquad (6)$$

(by neglecting higher powers of θ)

From equation (4), putting $t = \mu_1$, we have

$$Q = \frac{S_1}{(1 - b\mu_1)} + \frac{1}{(1 - b\mu_1)}\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 - \frac{1}{2}b\rho p\mu_1^2 - ab\mu_1^2 + \rho bp\mu_1^2\right). \qquad (7)$$

From equations (5) and (6), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) - \rho p(\mu_1 - \mu_2) + \frac{1}{2}a(\theta+b)(\mu_1^2 - \mu_2^2) - \frac{1}{2}\rho p(\theta+b)(\mu_1^2 - \mu_2^2) \\ - a(\theta+b)\mu_2(\mu_1 - \mu_2) + \rho p(\theta+b)\mu_2(\mu_1 - \mu_2) - ab\theta\mu_2(\mu_1^2 - \mu_2^2) + b\rho\theta p\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} \qquad (8)$$
$$+ S_1[1 + (\theta+b)(\mu_1 - \mu_2)]$$

$$I(\mu_2) = \left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) \\ - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) \end{array}\right]. \qquad (9)$$

So from equations (8) and (9), we get

$$S_1 = \frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]}$$

$$\left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) \\ - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right]. \qquad (10)$$

Putting value of S_1 from equation (10) into equation (7), we have

$$Q = \frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}$$

$$\left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) - ab\mu_2(T-\mu_2) \\ + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) \\ + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right]$$

$$+ \frac{\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}b\rho p\mu_1^2 - ab\mu_1^2\right)}{(1-b\mu_1)}. \qquad (11)$$

Using (11) in (4), we have

$$I(t) = \frac{(1-bt)}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}$$

$$\left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) - ab\mu_2(T-\mu_2) \\ + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) \\ + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right]$$

$$+ \frac{(1-bt)\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}b\rho p\mu_1^2 - ab\mu_1^2\right)}{(1-b\mu_1)} - \left(at + \frac{1}{2}bt^2 - \rho pt + \frac{1}{2}\rho bpt^2 + abt^2\right) \qquad (12)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit include the following elements:

(i) Ordering cost (OC) = A (13)

(ii) $HC = \int_0^T (x+yt)I(t)dt = \int_0^{\mu_1} (x+yt)I(t)dt + \int_{\mu_1}^{\mu_2} (x+yt)I(t)dt + \int_{\mu_2}^{T} (x+yt)I(t)dt$

$$= x\left(aT - \rho pT + \frac{1}{2}abT^2 - \frac{1}{2}\rho bpT^2 + \frac{1}{6}a\theta T^3 - \frac{1}{6}\rho\theta pT^3\right)T + \frac{1}{6}y\left(\frac{5}{12}ab\theta - \frac{5}{12}\rho bp\right)T^6$$

$$+ x\left(\begin{array}{l} a\mu_1 - \rho p\mu_1 \\ + \left(\begin{array}{l} \dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) \\ - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right] \\ \left(1+(\theta+b)\mu_1\right) + \frac{1}{2}a(\theta+b)\mu_1^2 - \frac{1}{2}\rho p(\theta+b)\mu_1^2 \end{array}\right)\mu_2 \end{array}\right)$$

$$- x\left(\begin{array}{l} a\mu_1 - \rho p\mu_1 \\ + \dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3)\right] \\ \left(1+(\theta+b)\mu_1 + \frac{1}{2}a(\theta+b)\mu_1^2 - \frac{1}{2}\rho p(\theta+b)\mu_1^2\right)\mu_1 \end{array}\right)$$

$$- x\left(\begin{array}{l} a\mu_1 - \rho p\mu_1 \\ + \dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} b\rho p\mu_2(T-\mu_2) - ab\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) \\ + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) \\ + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array}\right] \\ \left(1+(\theta+b)\mu_1 + \frac{1}{2}a(\theta+b)\mu_1^2 - \frac{1}{2}\rho p(\theta+b)\mu_1^2\right)\mu_1 \end{array}\right)$$

$$\left(\begin{array}{l}
-\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pT+\frac{1}{4}\rho b\theta pT^{2}+\frac{1}{2}ab-\frac{1}{2}a\theta T-\frac{1}{4}ab\theta T^{2}\right)\\ +y\left(-a+\rho p+\frac{1}{6}\rho b\theta pT^{3}-abT+\rho bpT-\frac{1}{6}ab\theta T^{3}\right)\end{array}\right)\mu_2^3\\
-\frac{1}{4}\left(x\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)+y\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pT+\frac{1}{4}\rho b\theta pT^{2}+\frac{1}{2}ab-\frac{1}{2}a\theta T-\frac{1}{4}ab\theta T^{2}\right)\right)\mu_2^4\\
-\frac{1}{5}\left(x\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)+y\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)\right)\mu_2^5\\
+\frac{1}{2}\left(\begin{array}{l} x\left(-a+\rho p+\frac{1}{6}\rho b\theta pT^{3}-abT+\rho bpT-\frac{1}{6}ab\theta T^{3}\right)\\ +y\left(aT-\rho pT+\frac{1}{2}abT^{2}-\frac{1}{2}\rho bpT^{2}+\frac{1}{6}a\theta T^{3}-\frac{1}{6}\rho\theta T^{3}\right)\end{array}\right)T^{2}\\
-\frac{1}{4}\left(x\left(ab\theta-\rho b\theta p\right)+y\left(\frac{1}{2}a\left(\theta+b\right)-\frac{1}{2}\rho p\left(\theta+b\right)\right)\right)\mu_1^4
\end{array}\right)$$

$$-\frac{1}{3}\left(\begin{array}{l}
x\left(\frac{1}{2}a\left(\theta+b\right)-\frac{1}{2}\rho p\left(\theta+b\right)\right)\\
+y\left(\begin{array}{l}
-a+\frac{1}{\left[1+\left(\theta+b\right)\left(\mu_1-\mu_2\right)\right]}\\
\left[\begin{array}{l}
a\left(T-\mu_2\right)-\rho p\left(T-\mu_2\right)+\frac{1}{2}ab\left(T^{2}-\mu_2^{2}\right)-\frac{1}{2}\rho bp\left(T^{2}-\mu_2^{2}\right)+\frac{1}{6}a\theta\left(T^{3}-\mu_2^{3}\right)-\frac{1}{6}\rho\theta p\left(T^{3}-\mu_2^{3}\right)\\
-ab\mu_2\left(T-\mu_2\right)+b\rho p\mu_2\left(T-\mu_2\right)-\frac{1}{6}ab\theta\mu_2\left(T^{3}-\mu_2^{3}\right)+\frac{1}{6}\rho b\theta p\mu_2\left(T^{3}-\mu_2^{3}\right)-\frac{1}{2}a\theta\mu_2^{2}\left(T-\mu_2\right)\\
+\frac{1}{2}\rho\theta p\mu_2^{2}\left(T-\mu_2\right)+\frac{1}{4}\rho b\theta p\mu_2^{2}\left(T^{2}-\mu_2^{2}\right)-\frac{1}{4}ab\theta\mu_2^{2}\left(T^{2}-\mu_2^{2}\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)\\
-\frac{1}{2}a\left(\theta+b\right)\left(\mu_1^{2}-\mu_2^{2}\right)+\frac{1}{2}\rho p\left(\theta+b\right)\left(\mu_1^{2}-\mu_2^{2}\right)+a\left(\theta+b\right)\mu_2\left(\mu_1-\mu_2\right)-\rho p\left(\theta+b\right)\mu_2\left(\mu_1-\mu_2\right)\\
+ab\theta\mu_2\left(\mu_1^{2}-\mu_2^{2}\right)-b\rho\theta p\mu_2\left(\mu_1^{2}-\mu_2^{2}\right)
\end{array}\right]\\
\left(-\theta-b\right)-a\left(\theta+b\right)\mu_1+\rho p\left(\theta+b\right)\mu_1-ab\theta\mu_1^{2}+\rho b\theta p\mu_1^{2}+\rho p
\end{array}\right)
\end{array}\right)\mu_1^3$$

$$-\frac{1}{2}\left(x\left(\begin{array}{l}
-a+\frac{1}{\left[1+\left(\theta+b\right)\left(\mu_1-\mu_2\right)\right]}\\
\left[\begin{array}{l}
a\left(T-\mu_2\right)-\rho p\left(T-\mu_2\right)+\frac{1}{2}ab\left(T^{2}-\mu_2^{2}\right)-\frac{1}{2}\rho bp\left(T^{2}-\mu_2^{2}\right)+\frac{1}{6}a\theta\left(T^{3}-\mu_2^{3}\right)-\frac{1}{6}\rho\theta p\left(T^{3}-\mu_2^{3}\right)\\
-ab\mu_2\left(T-\mu_2\right)+b\rho p\mu_2\left(T-\mu_2\right)-\frac{1}{6}ab\theta\mu_2\left(T^{3}-\mu_2^{3}\right)+\frac{1}{6}\rho b\theta p\mu_2\left(T^{3}-\mu_2^{3}\right)-\frac{1}{2}a\theta\mu_2^{2}\left(T-\mu_2\right)\\
+\frac{1}{2}\rho\theta p\mu_2^{2}\left(T-\mu_2\right)+\frac{1}{4}\rho b\theta p\mu_2^{2}\left(T^{2}-\mu_2^{2}\right)-\frac{1}{4}ab\theta\mu_2^{2}\left(T^{2}-\mu_2^{2}\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)\\
-\frac{1}{2}a\left(\theta+b\right)\left(\mu_1^{2}-\mu_2^{2}\right)+\frac{1}{2}\rho p\left(\theta+b\right)\left(\mu_1^{2}-\mu_2^{2}\right)+a\left(\theta+b\right)\mu_2\left(\mu_1-\mu_2\right)-\rho p\left(\theta+b\right)\mu_2\left(\mu_1-\mu_2\right)\\
+ab\theta\mu_2\left(\mu_1^{2}-\mu_2^{2}\right)-b\rho\theta p\mu_2\left(\mu_1^{2}-\mu_2^{2}\right)
\end{array}\right]\\
\left(-\theta-b\right)-a\left(\theta+b\right)\mu_1+\rho p\left(\theta+b\right)\mu_1-ab\theta\mu_1^{2}+\rho b\theta p\mu_1^{2}+\rho p
\end{array}\right)\right)$$

$$-\frac{1}{2}\left(y\left(\begin{array}{l} a\mu_1 - \rho p\mu_1 + \dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) \\ - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array} \right] \\ \left(1+(\theta+b)\mu_1 + \frac{1}{2}a(\theta+b)\mu_1^2 - \frac{1}{2}\rho p(\theta+b)\mu_1^2\right) \end{array} \right) \right)\mu_1^2$$

$$+\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}\rho bp + \frac{1}{2}\rho\theta pT + \frac{1}{4}\rho b\theta pT^2 + \frac{1}{2}ab - \frac{1}{2}a\theta T - \frac{1}{4}ab\theta T^2\right) \\ + y\left(-a + \rho p + \frac{1}{6}\rho b\theta pT^3 - abT + \rho bpT - \frac{1}{6}ab\theta T^3\right) \end{array} \right)T^3 + \frac{1}{5}\left(x\left(\frac{5}{12}ab\theta - \frac{5}{12}\rho b\theta p\right) + y\left(\frac{1}{3}a\theta - \frac{1}{3}\rho\theta p\right)\right)T^5$$

$$+\frac{1}{4}\left(x\left(\frac{1}{3}a\theta - \frac{1}{3}\rho\theta p\right) + y\left(-\frac{1}{2}\rho bp + \frac{1}{2}\rho\theta pT + \frac{1}{4}\rho b\theta pT^2 + \frac{1}{2}ab - \frac{1}{2}a\theta T - \frac{1}{4}ab\theta T^2\right)\right)T^4$$

$$+x\left(\begin{array}{l} \dfrac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) \\ - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) \\ + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array} \right] \\ + \dfrac{a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}\rho bp\mu_1^2 - ab\mu_1^2}{(1-b\mu_1)} \end{array} \right)\mu_1$$

$$\left(\begin{array}{l} + \frac{1}{4}y\left(-\frac{1}{2}b - \frac{1}{2}\rho bp + ab\right)\mu_1^4 + \frac{1}{5}y(ab\theta - \rho b\theta p)\mu_2^5 + \frac{1}{4}\left(x(ab\theta - \rho b\theta p) + y\left(\frac{1}{2}a(\theta+b) - \frac{1}{2}\rho p(\theta+b)\right)\right)\mu_2^4 + \frac{1}{3}\left(x\left(\frac{1}{2}a(\theta+b) - \frac{1}{2}\rho p(\theta+b)\right)\right) \\ + y\left(\begin{array}{l} -a + \dfrac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]} \\ \left[\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) - ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) \\ - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) + \frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) \\ - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) - \frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) + ab\theta\mu_2(\mu_1^2-\mu_2^2) \\ - b\rho\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array} \right] \\ (-\theta-b) - a(\theta+b)\mu_1 + \rho p(\theta+b)\mu_1 - ab\theta\mu_1^2 + \rho b\theta p\mu_1^2 + \rho p \end{array} \right)\mu_2^3 \end{array} \right)$$

$$
\begin{aligned}
&+\frac{1}{2}\Bigg(x\Bigg(-a+\frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\
&\Big[a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab(T^2-\mu_2^2)-\frac{1}{2}\rho bp(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)-\frac{1}{6}\rho\theta p(T^3-\mu_2^3)-ab\mu_2(T-\mu_2)+b\rho p\mu_2(T-\mu_2)\\
&-\frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2)\\
&-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)\\
&+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)\Big]\\
&(-\theta-b)-a(\theta+b)\mu_1+\rho p(\theta+b)\mu_1-ab\theta\mu_1^2+\rho b\theta p\mu_1^2+\rho p\Bigg)\Bigg)\mu_2^2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\Bigg(y\Bigg(a\mu_1-\rho p\mu_1+\frac{1}{\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\
&\Big[a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab(T^2-\mu_2^2)-\frac{1}{2}\rho bp(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)-\frac{1}{6}\rho\theta p(T^3-\mu_2^3)-ab\mu_2(T-\mu_2)+b\rho p\mu_2(T-\mu_2)\\
&-\frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2)\\
&-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)\\
&+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)\Big]\\
&\left(1+a(\theta+b)\mu_1\right)+\frac{1}{2}a(\theta+b)\mu_1^2-\frac{1}{2}\rho p(\theta+b)\mu_1^2\Bigg)\Bigg)\mu_2^2
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{3}\Bigg(x\left(-\frac{1}{2}b-\frac{1}{2}\rho bp+ab\right)\\
&+y\Bigg(-\frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\
&\Bigg(b\Big[a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab(T^2-\mu_2^2)-\frac{1}{2}\rho bp(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)-\frac{1}{6}\rho\theta p(T^3-\mu_2^3)-ab\mu_2(T-\mu_2)+b\rho p\mu_2(T-\mu_2)\\
&-\frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2)-a(\mu_1-\mu_2)\\
&+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)\Big]\Bigg)\\
&-\frac{b\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2\right)}{(1-b\mu_1)}-a+\rho p\Bigg)\Bigg)\mu_1^3
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\Bigg(x\Bigg(-\frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\
&\Bigg(b\Big[a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab(T^2-\mu_2^2)-\frac{1}{2}\rho bp(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)-\frac{1}{6}\rho\theta p(T^3-\mu_2^3)-ab\mu_2(T-\mu_2)+b\rho p\mu_2(T-\mu_2)\\
&-\frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2)-a(\mu_1-\mu_2)\\
&+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)\Big]\Bigg)\\
&-\frac{b\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2\right)}{(1-b\mu_1)}-a+\rho p\Bigg)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}y\Bigg(\frac{1}{(1-b\mu_1)\left[1+(\theta+b)(\mu_1-\mu_2)\right]}\\
&\Bigg(a\Big[a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab(T^2-\mu_2^2)-\frac{1}{2}\rho bp(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)-\frac{1}{6}\rho\theta p(T^3-\mu_2^3)-ab\mu_2(T-\mu_2)+b\rho p\mu_2(T-\mu_2)\\
&-\frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3)+\frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)+\frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2)-\frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2)-a(\mu_1-\mu_2)\\
&+\rho p(\mu_1-\mu_2)-\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2)+a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2(\mu_1^2-\mu_2^2)-b\rho\theta p\mu_2(\mu_1^2-\mu_2^2)\Big]\\
&+\frac{a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}\rho bp\mu_1^2-ab\mu_1^2}{(1-b\mu_1)}\Bigg)\Bigg)\mu_1^2
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{6}y\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)\mu_2^6-x\left(aT-\rho pT+\frac{1}{2}abT^2-\frac{1}{2}\rho bpT^2+\frac{1}{6}a\theta T^3-\frac{1}{6}\rho\theta T^3\right)\mu_2 \\
&-\frac{1}{2}\left(x\left(-a+\rho p+\frac{1}{6}\rho b\theta pT^3-abT+\rho bpT-\frac{1}{6}ab\theta T^3\right)+y\left(aT-\rho pT+\frac{1}{2}abT^2-\frac{1}{2}\rho bpT^2+\frac{1}{6}a\theta T^3-\frac{1}{6}\rho\theta T^3\right)\right)\mu_2^2-\frac{1}{5}y\left(ab\theta-\rho b\theta p\right)\mu_1^5
\end{aligned}
\tag{14}
$$

(by neglecting higher powers of θ)

(iii) $DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$

$$
=c\theta\left(\begin{aligned}
&a\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)-\rho p\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}a\left(\theta+b\right)\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}\rho p\left(\theta+b\right)\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)\\
&-a\left(\theta+b\right)\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)+\rho p\left(\theta+b\right)\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)-ab\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)+\rho pb\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right)\\
&+\frac{1}{1+\left(\theta+b\right)(\mu_1-\mu_2)}\\
&\left(\begin{aligned}
&a\left(T-\mu_2\right)-\rho p\left(T-\mu_2\right)+\frac{1}{2}ab\left(T^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(T^3-\mu_2^3\right)-ab\mu_2(T-\mu_2)\\
&+b\rho p\mu_2(T-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(T^3-\mu_2^2\right)+\frac{1}{6}\rho b\theta p\mu_2\left(T^3-\mu_2^2\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)+\frac{1}{2}\rho\theta p\mu_2^2\left(T-\mu_2\right)\\
&+\frac{1}{4}\rho b\theta p\mu_2^2\left(T^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(T-\mu_2)+\rho p(T-\mu_2)-\frac{1}{2}a\left(\theta+b\right)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\left(\theta+b\right)\left(\mu_1^2-\mu_2^2\right)\\
&+a\left(\theta+b\right)\mu_2(\mu_1-\mu_2)-\rho p\left(\theta+b\right)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-\rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)
\end{aligned}\right)\\
&\left(\mu_2+\left(\theta+b\right)\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)\right)
\end{aligned}\right)
$$

$$
-c\theta\left(\begin{aligned}
&\frac{1}{2}a\mu_1^2-\frac{1}{2}\rho p\mu_1^2+\frac{1}{6}a\left(\theta+b\right)\mu_1^3-\frac{1}{6}\rho p\left(\theta+b\right)\mu_1^3-\frac{1}{4}ab\theta\mu_1^4+\frac{1}{4}\rho b\theta p\mu_1^4\\
&+\left(\begin{aligned}
&\frac{1}{1+\left(\theta+b\right)(\mu_1-\mu_2)}\\
&\left(\begin{aligned}
&a\left(T-\mu_2\right)-\rho p\left(T-\mu_2\right)+\frac{1}{2}ab\left(T^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(T^3-\mu_2^3\right)-ab\mu_2(\mu_1-\mu_2)\\
&+b\rho p\mu_2(\mu_1-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(T^3-\mu_2^3\right)+\frac{1}{6}\rho b\theta p\mu_2\left(T^3-\mu_2^3\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)+\frac{1}{2}\rho\theta p\mu_2^2\left(T-\mu_2\right)\\
&+\frac{1}{4}\rho b\theta p\mu_2^2\left(T^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a\left(\theta+b\right)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\left(\theta+b\right)\left(\mu_1^2-\mu_2^2\right)\\
&+a\left(\theta+b\right)\mu_2(\mu_1-\mu_2)-\rho p\left(\theta+b\right)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-\rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)
\end{aligned}\right)
\end{aligned}\right)\\
&\left(\mu_1+\frac{1}{2}\left(\theta+b\right)\mu_1^2\right)
\end{aligned}\right)
$$

$$
\begin{aligned}
&+c\theta\left(\begin{aligned}
&\frac{1}{6}\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)T^6+\frac{1}{5}\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)T^5+\frac{1}{4}\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pT+\frac{1}{4}\rho b\theta pT^2+\frac{1}{2}ab-\frac{1}{2}a\theta T-\frac{1}{4}ab\theta T^2\right)T^4\\
&+\frac{1}{3}\left(-a+\rho p+\frac{1}{6}\rho b\theta pT^3-abT+\rho bpT-\frac{1}{6}ab\theta T^3\right)T^3+\frac{1}{2}\left(aT-\rho pT+\frac{1}{2}abT^2-\frac{1}{2}\rho bpT^2+\frac{1}{6}a\theta T^2-\frac{1}{6}\rho\theta pT^3\right)T^2
\end{aligned}\right)\\
&-c\theta\left(\begin{aligned}
&\frac{1}{6}\left(\frac{5}{12}ab\theta-\frac{5}{12}\rho b\theta p\right)\mu_2^6+\frac{1}{5}\left(\frac{1}{3}a\theta-\frac{1}{3}\rho\theta p\right)\mu_2^5+\frac{1}{4}\left(-\frac{1}{2}\rho bp+\frac{1}{2}\rho\theta pT+\frac{1}{4}\rho b\theta pT^2+\frac{1}{2}ab-\frac{1}{2}a\theta T-\frac{1}{4}ab\theta T^2\right)\mu_2^4\\
&+\frac{1}{3}\left(-a+\rho p+\frac{1}{6}\rho b\theta pT^3-abT+\rho bpT-\frac{1}{6}ab\theta T^3\right)\mu_2^3+\frac{1}{2}\left(aT-\rho pT+\frac{1}{2}abT^2-\frac{1}{2}\rho bpT^2+\frac{1}{6}a\theta T^2-\frac{1}{6}\rho\theta pT^3\right)\mu_2^2
\end{aligned}\right)
\end{aligned}
\tag{15}
$$

(iv) $SR = p\left(\int_0^T (a+bI(t) - p)dt\right)$

$$= p\left(\int_0^{\mu_1} (a+bI(t) - p)dt + \int_{\mu_1}^{\mu_2} (a+bI(t) - p)dt + \int_{\mu_2}^{T} (a+bI(t) - p)dt\right)$$

$$= p\left(a\mu_1 + b\left(\begin{array}{l} \dfrac{1}{(1-b\mu_1)\left(1+(\theta+b)(\mu_1-\mu_2)\right)} \\ \left(\left(\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ -ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ +\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(\mu_1-\mu_2) + \rho p(\mu_1-\mu_2) \\ -\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - \rho b\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array} \right) \right. \\ \left. + \dfrac{1}{(1-b\mu_1)}\left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2 + \frac{1}{2}\rho bp\mu_1^2 - ab\mu_1^2\right)\left(\mu_1 - \frac{1}{2}b\mu_1^2\right) \right. \\ \left. - \frac{1}{2}a\mu_1^2 - \frac{1}{6}b\mu_1^2 + \frac{1}{2}\rho p\mu_1^2 - \frac{1}{6}\rho bp\mu_1^3 + \frac{1}{3}ab\mu_1^3 \right) \end{array} \right) - \rho p\mu_1 \right)$$

$$+p\left(a\mu_2 + b\left(\begin{array}{l} a\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right) - \rho p\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right) + \frac{1}{2}a(\theta+b)\left(\mu_1^2\mu_2 - \frac{1}{3}\mu_2^3\right) - \frac{1}{2}\rho p(\theta+b)\left(\mu_1^2\mu_2 - \frac{1}{3}\mu_2^3\right) \\ -a(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2 - \frac{1}{3}\mu_2^3\right) + \rho p(\theta+b)\left(\frac{1}{2}\mu_1\mu_2^2 - \frac{1}{3}\mu_2^3\right) - ab\theta\left(\frac{1}{2}\mu_1^2\mu_2^2 - \frac{1}{4}\mu_2^4\right) + \rho pb\theta\left(\frac{1}{2}\mu_1^2\mu_2^2 - \frac{1}{4}\mu_2^4\right) \\ + \dfrac{1}{1+(\theta+b)(\mu_1-\mu_2)} \\ \left(\begin{array}{l} a(T-\mu_2) - \rho p(T-\mu_2) + \frac{1}{2}ab(T^2-\mu_2^2) - \frac{1}{2}\rho bp(T^2-\mu_2^2) + \frac{1}{6}a\theta(T^3-\mu_2^3) - \frac{1}{6}\rho\theta p(T^3-\mu_2^3) \\ -ab\mu_2(T-\mu_2) + b\rho p\mu_2(T-\mu_2) - \frac{1}{6}ab\theta\mu_2(T^3-\mu_2^3) + \frac{1}{6}\rho b\theta p\mu_2(T^3-\mu_2^3) - \frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ +\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2) + \frac{1}{4}\rho b\theta p\mu_2^2(T^2-\mu_2^2) - \frac{1}{4}ab\theta\mu_2^2(T^2-\mu_2^2) - a(T-\mu_2) + \rho p(T-\mu_2) \\ -\frac{1}{2}a(\theta+b)(\mu_1^2-\mu_2^2) + \frac{1}{2}\rho p(\theta+b)(\mu_1^2-\mu_2^2) + a(\theta+b)\mu_2(\mu_1-\mu_2) - \rho p(\theta+b)\mu_2(\mu_1-\mu_2) \\ + ab\theta\mu_2(\mu_1^2-\mu_2^2) - \rho b\theta p\mu_2(\mu_1^2-\mu_2^2) \end{array} \right) \\ \left(\mu_2 + (\theta+b)\left(\mu_1\mu_2 - \frac{1}{2}\mu_2^2\right)\right) \end{array} \right) - \rho p\mu_2 \right)$$

$$-p\left(a\mu_1+b\left(\begin{array}{l}\frac{1}{2}a\mu_1^2-\frac{1}{2}\rho p\mu_1^2+\frac{1}{6}a(\theta+b)\mu_1^3-\frac{1}{6}\rho p(\theta+b)\mu_1^3-\frac{1}{4}ab\theta\mu_1^4+\frac{1}{4}\rho b\theta p\mu_1^4\\ +\frac{1}{\left(1+(\theta+b)(\mu_1-\mu_2)\right)}\\ \left(\begin{array}{l}a(T-\mu_2)-\rho p(T-\mu_2)+\frac{1}{2}ab\left(T^2-\mu_2^2\right)-\frac{1}{2}\rho bp\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)-\frac{1}{6}\rho\theta p\left(T^3-\mu_2^3\right)-ab\mu_2(T-\mu_2)\\ +b\rho p\mu_2(T-\mu_2)-\frac{1}{6}ab\theta\mu_2\left(T^3-\mu_2^2\right)+\frac{1}{6}\rho b\theta p\mu_2\left(T^3-\mu_2^2\right)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)+\frac{1}{2}\rho\theta p\mu_2^2(T-\mu_2)\\ +\frac{1}{4}\rho b\theta p\mu_2^2\left(T^2-\mu_2^2\right)-\frac{1}{4}ab\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(T-\mu_2)+\rho p(T-\mu_2)-\frac{1}{2}a(\theta+b)\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p(\theta+b)\left(\mu_1^2-\mu_2^2\right)\\ +a(\theta+b)\mu_2(\mu_1-\mu_2)-\rho p(\theta+b)\mu_2(\mu_1-\mu_2)+ab\theta\mu_2\left(\mu_1^2-\mu_2^2\right)-\rho b\theta p\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ \left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^2\right)\end{array}\right)-\rho p\mu_1\right)$$

$$+p\left(aT+b\left(\frac{1}{2}aT^2-\frac{1}{2}\rho pT^2+\frac{1}{6}abT^3-\frac{1}{6}\rho bpT^3+\frac{1}{12}a\theta T^4-\frac{1}{12}\rho\theta pT^4-\frac{1}{12}ab\theta T^5+\frac{1}{12}\rho b\theta pT^5\right)-\rho pT\right)$$

$$-p\left(a\mu_2+b\left(\begin{array}{l}a\left(T\mu_2-\frac{1}{2}\mu_2^2\right)-\rho p\left(T\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}ab\left(T^2\mu_2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}\rho bp\left(T^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{6}a\theta\left(T^3\mu_2-\frac{1}{4}\mu_2^4\right)\\ -\frac{1}{6}\rho\theta p\left(T^3\mu_2-\frac{1}{4}\mu_2^4\right)-ab\left(\frac{1}{2}T\mu_2^2-\frac{1}{3}\mu_2^3\right)+b\rho p\left(\frac{1}{2}T\mu_2^2-\frac{1}{3}\mu_2^3\right)-\frac{1}{6}ab\theta\left(\frac{1}{2}T^3\mu_2^2-\frac{1}{5}\mu_2^5\right)\\ +\frac{1}{6}\rho b\theta p\left(\frac{1}{2}T^3\mu_2^2-\frac{1}{5}\mu_2^5\right)-\frac{1}{2}a\theta\left(\frac{1}{3}T\mu_2^3-\frac{1}{4}\mu_2^4\right)+\frac{1}{2}\rho\theta p\left(\frac{1}{3}T\mu_2^3-\frac{1}{4}\mu_2^4\right)+\frac{1}{4}\rho b\theta p\left(\frac{1}{3}T^2\mu_2^3-\frac{1}{5}\mu_2^5\right)\\ -\frac{1}{4}ab\theta\left(\frac{1}{3}T^2\mu_2^3-\frac{1}{5}\mu_2^5\right)\end{array}\right)-\rho p\mu_2\right) \quad (16)$$

The total profit during a cycle, π consisted of the following:

$$\pi=\frac{1}{T}\left[SR-OC-HC-DC\right] \quad (17)$$

Substituting values from equations (13) to (16) in equation (17), we get total profit per unit. Putting $\mu_1= v_1T$ and $\mu_2=v_2T$ in equation (17), we get profit in terms of T and p. Differentiating equation (17) with respect to T and p and equate it to zero, we have

i.e. $$\frac{\partial\pi(T,p)}{\partial T}=0,\ \frac{\partial\pi(T,p)}{\partial p}=0 \quad (18)$$

provided it satisfies the condition

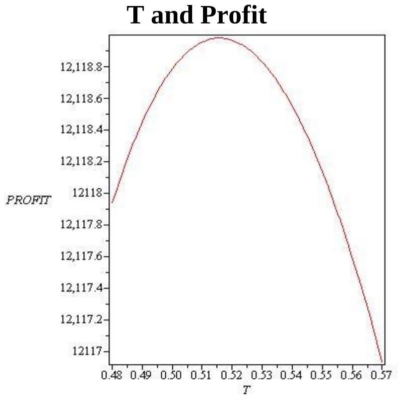
$$\frac{\partial^2\pi(T,p)}{\partial T^2}<0,\ \frac{\partial^2\pi(T,p)}{\partial p^2}<0 \text{ and } \left[\frac{\partial^2\pi(T,p)}{\partial T^2}\right]\left[\frac{\partial^2\pi(T,p)}{\partial p^2}\right]-\left[\frac{\partial^2\pi(T,p)}{\partial p\partial T}\right]^2>0. \quad (19)$$

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, ρ= 5, θ=0.05, x = Rs. 5, y=0.05, v_1 = 0.30, v_2 = 0.50, in appropriate units. The optimal values of T* =0.5156, p* = 50.6966, Profit*= Rs. 12118.9833 and optimum order quantity Q* = 128.9945.

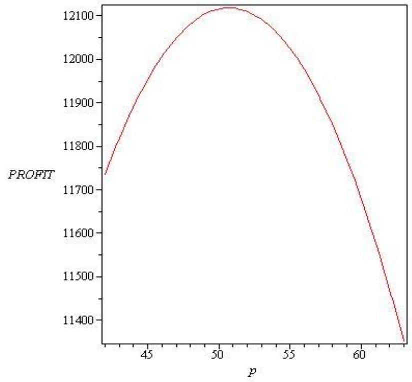
The second order conditions given in equation (19) are also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



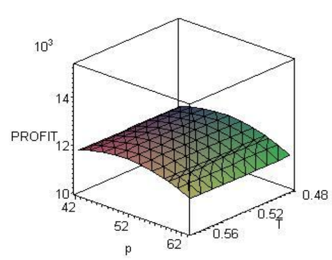
Graph 1

p and Profit



Graph 2

T, p and Profit



Graph 3

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis

Parameter	%	T	p	Profit	Q
a	+20%	0.5152	60.6956	17620.0982	155.0084
	+10%	0.5132	55.6932	14742.9178	141.4137
	-10%	0.5223	45.7060	9748.3399	117.4179
	-20%	0.5341	40.7223	7631.1214	106.4905
θ	+20%	0.5071	50.6956	12113.7200	126.9412
	+10%	0.5113	50.6960	12116.3381	127.9562
	-10%	0.5200	50.6972	12121.6565	130.0564
	-20%	0.5246	50.6978	12124.3588	131.1674
x	+20%	0.4488	50.7170	12059.3196	112.0074
	+10%	0.4788	50.7058	12088.1061	119.6306
	-10%	0.5620	50.6904	12152.4632	140.5975
	-20%	0.6229	50.6893	12189.2984	156.3844
A	+20%	0.5629	50.7619	12081.9071	140.8471
	+10%	0.5398	50.7300	12100.0385	135.0580
	-10%	0.4900	50.6614	12138.8670	122.5813
	-20%	0.4629	50.6241	12159.8509	115.7940
ρ	+20%	0.4847	42.3209	10009.8496	120.9312
	+10%	0.4980	46.1270	10968.3300	124.4147
	-10%	0.5399	56.2855	13526.1189	135.2842
	-20%	0.5778	63.2821	15286.3701	145.0437

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x and ρ, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and inventory dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

REFERENCES

[1]. Aggarwal, S.P. and Goel, V.P. (1984): Order level inventory system with demand pattern for deteriorating items; Eco. Comp. Econ. Cybernet, Stud. Res., Vol. 3, pp. 57-69.

[2]. Covert, R.P. and Philip, G.C. (1973): An EOQ model for items with Weibull distribution deterioration; American Institute of Industrial Engineering Transactions, Vol. 5, pp. 323-328.

[3]. Ghare, P.N. and Schrader, G.F. (1963): A model for exponentially decaying inventories; J. Indus. Engg., Vol. 15, pp. 238-243.

[4]. Goyal, S.K. and Giri, B. (2001): Recent trends in modeling of deteriorating inventory; Euro. J. Oper. Res., Vol. 134, pp. 1-16.

[5]. Nahmias, S. (1982): Perishable inventory theory: a review; Operations Research, Vol. 30, pp. 680-708.

[6]. Ouyang, L. Y., Wu, K.S. and Yang, C.T. (2006): A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments; Computers and Industrial Engineering, Vol. 51, pp. 637-651.

[7]. Patel, R. and Parekh, R. (2014): Deteriorating items inventory model with stock dependent demand under shortages and variable selling price, International J. Latest Technology in Engg. Mgt. Applied Sci., Vol. 3, No. 9, pp. 6-20.

[8]. Patra, S.K., Lenka, T.K. and Ratha, P.C. (2010): An order level EOQ model for deteriorating items in a single warehouse system with price depended demand in nonlinear (quadratic) form; International J. of Computational and Applied Mathematics, Vol. 5, pp. 277-288.

[9]. Raafat, F. (1991): Survey of literature on continuously deteriorating inventory model, Euro. J. of O.R. Soc., Vol. 42, pp. 27-37.

[10]. Shah, Y.K. and Jaiswal, M.C. (1977): An order level inventory model for a system with constant rate of deterioration; Opsearch; Vol. 14, pp. 174-184.

[11]. Tripathy, C.K. and Mishra, U. (2010): An inventory model for Weibull deteriorating items with price dependent demand and time varying holding cost; Applied Mathematical Sciences, Vol. 4, pp. 2171-2179.

[12]. Wu, K.S., Ouyang, L. Y. and Yang, C.T. (2006): An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging; International J. of Production Economics, Vol. 101, pp. 369-384.