

## Transient Natural Convection Heat Transfer of Al<sub>2</sub>O<sub>3</sub>-Water Nanofluid in Enclosure: A Numerical Study

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**Abstract:-** Transient natural convection heat transfer and fluid flow of Al<sub>2</sub>O<sub>3</sub>-water nanofluid in an enclosure has been studied by a numerical method. Isothermal boundary conditions are applied to the horizontal walls of the enclosure, while the two vertical ones are kept adiabatic. The governing equations were discretized using the finite volume method and SIMPLER algorithm. The results are presented over a range of Rayleigh numbers from  $7 \times 10^3$  to  $5 \times 10^4$ , volume fraction,  $\phi = 0.02$  and aspect ratio ranging from 0.6 to 1. It was observed from the results that different behaviors (enhancement or deterioration) are predicted in the heat transfer and fluid flow as the time, Rayleigh number and aspect ratio increases. The enhancement is more pronounced at high Rayleigh number and the heat transfer decreases by increasing aspect ratio.

**Keywords:-** Enclosure, Nanofluid, Natural convection, Numerical study, Transient Nomenclature

A	Aspect ratio (W/H)	$\phi$	Nanoparticle volume fraction
Cp	Specific heat at constant pressure (J kg <sup>-1</sup> K <sup>-1</sup> )	$\rho$	Density (kg m <sup>-3</sup> )
g	Gravitational acceleration (m s <sup>-2</sup> )	$\mu$	Dynamic viscosity (kg m <sup>-1</sup> s <sup>-1</sup> )
H	Height of the enclosure (m)		Subscripts
K	Thermal conductivity (W m <sup>-1</sup> K <sup>-1</sup> )	nf	nanofluid
P	Pressure (N m <sup>-2</sup> )	f	fluid
Pr	Prandtl number	h	hot
Ra	Rayleigh number	c	cold
t	Time (s)	s	solid
T	Temperature (K)		
u, v	Velocity components (m s <sup>-1</sup> )		along x,y direction respectively
W	Width of the enclosure (m)		
x, y	coordinates		
Greek symbols			
$\alpha$	Fluid thermal diffusivity (m <sup>2</sup> s <sup>-1</sup> )		
$\beta$	Thermal expansion coefficient (K <sup>-1</sup> )		

### I. INTRODUCTION

Natural convection in fluid-filled rectangular enclosures has received considerable attention over the past several years due to its wide applications in engineering design of advanced technology. These applications span such diverse fields as electronic industry, cooling systems for nuclear reactors, heat exchangers, solar energy collectors, etc. A large cross section of fundamental research on this topic has been reviewed by Catton [1] and Bejan [2]. Most of the previous work has addressed natural convection in a cavity filled with ordinary fluids, having in general relatively low thermal conductivities. Thus, the heat transfer capacity through such systems is relatively limited and it is of importance, from the industrial and energy point of view, to improve it. One way to overcome this drawback is to disperse nano-scale particles in the base flow. The heat transfer characteristics of the resulting mixtures are considerably enhanced due to the presence of the nanoparticles in the fluids that increase significantly the effective thermal conductivity of the latter. This technique was developed by Choi [3] who was the first to introduce the term nanofluid to refer to a fluid in which nanoparticles are suspended. A review of the literature on this subject shows that rather little work has been carried out on natural convection in such nanofluids confined in enclosures. Some researchers analyzed the convective heat transfer of nanofluids by considering different models of nanofluid properties in an enclosure. Nasrin et al. [4] analyzes heat transfer and fluid flow of natural convection in a vertical closed chamber filled with Al<sub>2</sub>O<sub>3</sub>/water nanofluid that operates under differentially heated walls. The results highlight the range where the heat transfer uncertainties can be affected by the volume fraction of the nanoparticles. In addition, decreasing the Prandtl number results in amplifying the effects of nanoparticles due to increased effective thermal diffusivity. Alloui et al. [5] reports an analytical and numerical study of natural convection in a shallow rectangular cavity filled with nanofluids. The critical Rayleigh number for the onset of supercritical convection of nanofluids is predicted

explicitly. Furthermore, a linear stability analysis of the parallel flow solution is studied and the threshold for Hopf bifurcation is determined. a numerical study of natural convection in a square enclosure with non-uniform temperature distribution maintained at the bottom wall and filled with nanofluids is carried out using different types of nanoparticles by Ben-Cheikh et al. [6]. It is observed that the heat transfer enhancement strongly depends on the type of nanofluids. For  $Ra = 10^6$ , the pure water flow becomes unsteady. Also, the increase of the volume fraction of nanoparticles makes the flow return to steady state. Natural convection fluid flow and heat transfer inside I-shaped enclosures filled with Cu-water nanofluid has been investigated numerically using finite difference method by Mansour et al. [7]. the obtained results showed that the mean Nusselt number increased with increase in Rayleigh number and volume fraction of Cu nanoparticles regardless aspect ratio of the enclosure. Moreover the obtained results showed that the rate of heat transfer increased with decreasing aspect ratio of the cavity and nanoparticles volume fraction. Sivasankaran et al. [8] present numerical study is to investigate the convective flow and heat transfer behaviour of nanofluids with different nano-particles in a square cavity. It is found that the heat transfer rate increases on increasing the volume fraction of the nanofluid for all types of nanoparticles considered. The increment in average Nusselt number is strongly dependent on the nanoparticle chosen. natural convection heat transfer characteristics in a differentially heated enclosure filled with a CuO-EG-Water nanofluid for different published variable thermal conductivity and variable viscosity models was studied by Abu-Nada and Chamkha [9]. Different behaviors (enhancement or deterioration) are predicted in the average Nusselt number as the volume fraction of nanoparticles increases depending on the combination of CuO-EG-Water variable thermal conductivity and viscosity models employed. Abu-Nada [10] investigates Heat transfer enhancement in horizontal annuli using variable properties of Al<sub>2</sub>O<sub>3</sub>-water nanofluid. It was observed that for  $Ra \geq 10^4$ , the average Nusselt number was reduced by increasing the volume fraction of nanoparticles. However, for  $Ra = 10^3$ , the average Nusselt number increased by increasing the volume fraction of nanoparticles. For  $Ra \geq 10^4$ , the Nusselt number was deteriorated everywhere around the cylinder surface especially at high expansion ratio. Haddad et al. [11] studied Natural convection heat transfer and fluid flow of CuO-Water nanofluids using the Rayleigh Bénard problem. By considering the role of thermophoresis and Brownian motion, an enhancement in heat transfer is observed at any volume fraction of nanoparticles. However, the enhancement is more pronounced at low volume fraction of nanoparticles and the heat transfer decreases by increasing nanoparticle volume fraction.

Laminar conjugate heat transfer by natural convection and conduction in a vertical annulus formed between an inner heat generating solid circular cylinder and an outer isothermal cylindrical boundary has been studied numerically by Shahi et al. [12]. Results are presented for the flow and temperature distributions and Nusselt numbers on different cross sectional planes and longitudinal sections for Rayleigh number ranging from  $10^5$  to  $10^8$ , solid volume fraction of  $0 < \phi < 0.05$  with copper-water nanofluid as the working medium. Considering that the driven flow in the annular tube is strongly influenced by orientation of tube. Aminossadati and Ghasemi [13] present a numerical study of natural convection cooling of a heat source embedded on the bottom wall of an enclosure filled with nanofluids. The results indicate that adding nanoparticles into pure water improves its cooling performance especially at low Rayleigh numbers. The type of nanoparticles and the length and location of the heat source proved to significantly affect the heat source maximum temperature. Lin and Violi [14] analyze the heat transfer and fluid flow of natural convection in a cavity filled with Al<sub>2</sub>O<sub>3</sub>/water nanofluid that operates under differentially heated walls. They determine the impact of fluid temperature on the heat transfer of nanofluids. Decreasing the Prandtl number results in amplifying the effects of nanoparticles due to increased effective thermal diffusivity. Hwang et al. [15] have carried out a theoretical investigation of the thermal characteristics of natural convection of an alumina-based nanofluid in a rectangular cavity heated from below using Jang and Choi's model [16] for predicting the effective thermal conductivity of nanofluids (and various models for predicting the effective viscosity). The used viscosity and thermal conductivity relationships have been reported by Nguyen et al. [17] and Chon et al. [18] respectively. They compared obtained results of the mentioned model with results of Brinkman model [19] for dynamic viscosity and Maxwell model [20] for thermal conductivity. Their results showed that at high Rayleigh numbers, the heat transfer rate was sensitive to viscosity model while thermal conductivity model did not influence on heat transfer rate. Buongiorno [21] proposed a model for convective transport in nanofluids incorporating the effects of Brownian diffusion and thermophoresis. This model was applied to analysing the onset of convection in a horizontal layer uniformly heated from below (the Rayleigh\_Bénard problem) by Tzou [22,23]. In [22] the case of stress-free boundaries was analysed. In [23] the combination of one free and one rigid boundary was investigated, and the case of two rigid boundaries was briefly mentioned. On the basis of his calculations, Tzou [22,23] concluded that for nanofluids the critical Rayleigh number (the value of the Rayleigh number at which instability appears) was lower by one or two orders of magnitude than for regular fluids.

The present study conducts a numerical study of transient natural convection heat transfer in a rectangular enclosure filled with Al<sub>2</sub>O<sub>3</sub>-water nanofluid. The top and bottom boundaries are subjected to constant temperature difference, while the other boundaries are assumed to be adiabatic. An accurate finite

volume scheme technique is devised for the purpose of solution of the governing equations. The effects of the Rayleigh number and aspect ratio of enclosure on flow pattern, temperature field were investigated. Based on author's knowledge above literature survey, these effects are not taken into account for a mentioned problem.

## II. PHYSICAL CONFIGURATION

Fig. 1 shows a schematic diagram of the differentially heated enclosure. The top and bottom horizontal walls have constant but different temperatures  $T_h$  and  $T_c$ , respectively where  $T_h > T_c$ . The left and right vertical surfaces are kept adiabatic. The fluid in the enclosure is Al<sub>2</sub>O<sub>3</sub>/water nanofluid.  $W$  and  $H$  are the length and height of the enclosure. The gravitational force acts in the vertically downward direction.

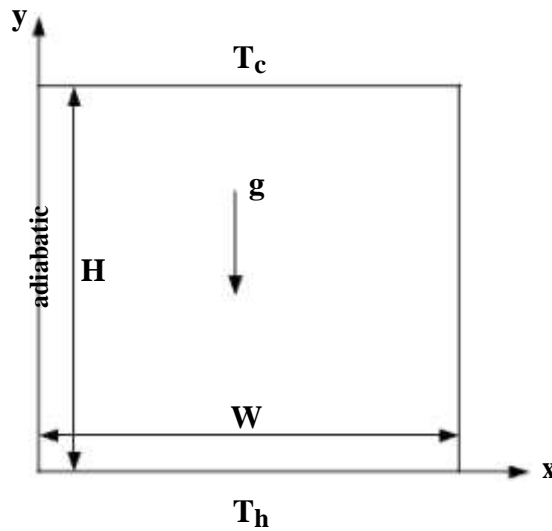


Fig.(1): Physical configuration and coordinate system of the enclosure

### 2.1. Assumptions

The mathematical equations describing the physical model are based on the following assumptions:

- (i) The thermo physical properties given in Table 1 are constant except for the density in the buoyancy force (Boussinesq's hypothesis).
- (ii) The fluid phase and nanoparticles are in a thermal equilibrium state.
- (iii) Nanoparticles are spherical.
- (iv) The nanofluid in the cavity is Newtonian, incompressible, and laminar and
- (v) Radiation heat transfer between the horizontal walls of the cavity is negligible when compared with the other mode of heat transfer.

### 2.2. Governing equations

The system of equations governing the two dimensional motion of a nanofluid in the enclosure can be written in dimensional form as

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

x-momentum equation:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

y-momentum equation:

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho \beta (T - T_c) g \quad (3)$$

Energy equation:

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

The effective density of the nanofluid is given as:

$$\rho_{eff} = \rho_f (1 - \phi) + \rho_p \phi \quad (5)$$

and  $\phi$  is the solid volume fraction of nanoparticles.

The thermal diffusivity of the nanofluid is:

$$\alpha_{eff} = \frac{k_{eff}}{\rho_{eff} c_{p,eff}} \quad (6)$$

Where, the heat capacitance of the nanofluid is given by:

$$c_{p,eff} = (1 - \phi) c_{p,f} + \phi c_{p,p} \quad (7)$$

The thermal expansion coefficient of the nanofluid can be determined by (8)

The effective dynamic viscosity of the nanofluid is given by: (9)

the effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnetts model (10)

The use of this equation is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles.

The appropriate initial and boundary conditions applied on the walls of the enclosure

are: At  $t = 0$ :  $u = v = 0$ ,  $T = T_c$  everywhere,

At  $t > 0$ :

1- on the left and right vertical walls:  $u = v = 0$

2- on the bottom wall:  $T = T_c$

3- on the top wall:  $T = T_h$

4- on all solid boundaries:  $u = v = 0$

**Table 1: Thermo-physical properties of water-Al<sub>2</sub>O<sub>3</sub> nanofluid at Pr = 6.2**

Physical properties	Water	Al <sub>2</sub> O <sub>3</sub>
Cp (J/kg.°C)	4179	850
ρ (kg/m <sup>3</sup> )	997.1	3900
k (W/m.°C)	0.61	46
β (1/k)	21 x 10 <sup>-5</sup>	1.67 x 10 <sup>-5</sup>

### III. NUMERICAL TECHNIQUE

The governing equations, together with boundary conditions, are discretised by implicit finite volume method, which is explained by Patankar [24]. The QUICK scheme is used for the convection terms and the central difference scheme is used for diffusion terms. The velocity and pressure are coupled by SIMPLE algorithm [24]. The solution domain is discretised with uniform mesh in both x and y directions. The effect of grid size is tested to select the appropriate grid density among a range from 41x41 to 161x161. It is observed from the grid independence test that an 121x121 grid is enough to investigate the problem. the nonlinear governing partial differential equations are transferred into a system of linear equations by applying Newton's method; the resulting algebraic equations are solved by Biconjugate Gradient Stabilized (BiCGStab) iterative method. the Backward Differences Formulae (BDF) of second order was used as a time step method. The time step is chosen to be uniform. To check the convergence of the sequential iterative solution, the sum of the absolute differences of the solution variables between two successive iterations has been calculated. When this summation falls below the convergence criterion, convergence is obtained, which the convergence criterion has been chosen as 10<sup>-5</sup> in this study.

#### 3.1 Code validation

The model validation is an important part of a numerical investigation. Hence, the validation of present computational code is verified against the numerical result of Lin and Violi [14] which was reported for natural convection heat transfer of nanofluids in a square cavity. Simulations are carried out for natural convection while employing the dimensionless parameters  $Gr = 10^4$ ,  $dp = 5$  nm and  $\phi = 0.05$  for both the streamlines and isotherms. This validation boosts the confidence in our numerical code to carry on with the above stated objective of the current investigation. As shown in Fig. 2, the new model is able to reproduce the published result of Lin and Violi [14].

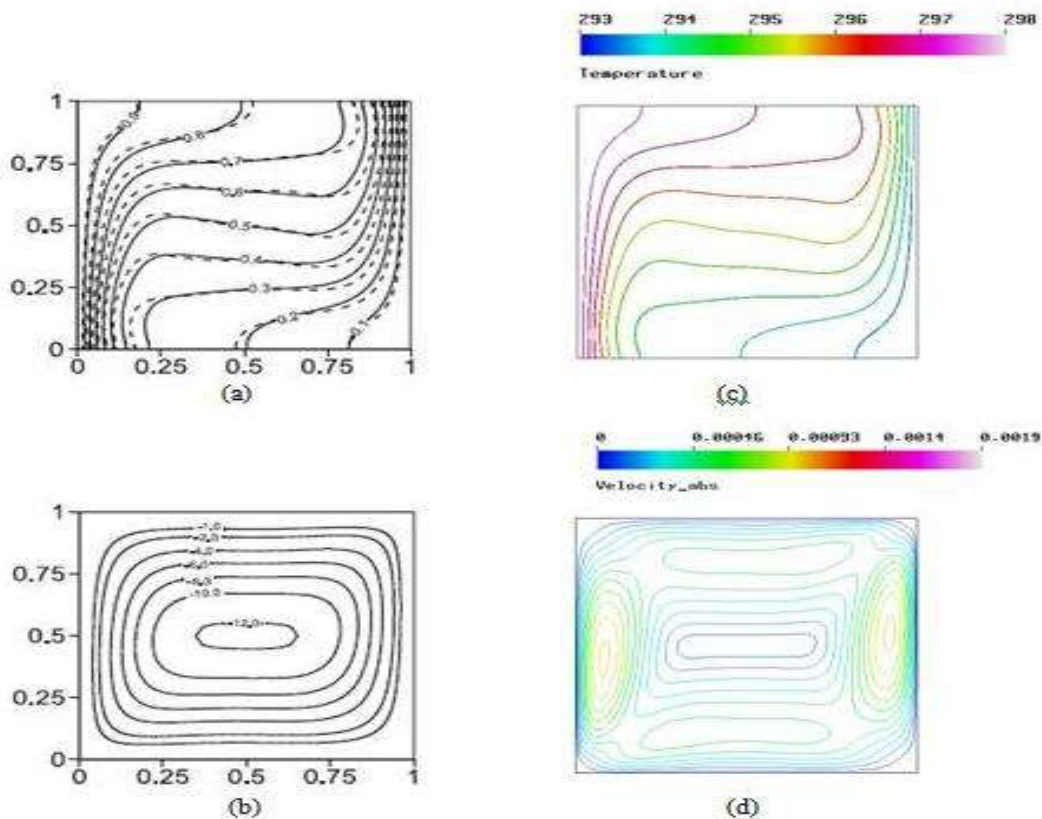


Fig.(2): comparison between present work isotherms (c) and streamlines (d) with that of Lin and Violi [14] (a,b) using  $Gr = 10^4$ ,  $d_p = 5$  nm,  $\phi = 0.05$

#### IV. RESULTS AND DISCUSSION

Numerical analysis of buoyancy induced flow in differentially heated enclosure filled with Al<sub>2</sub>O<sub>3</sub>-water based nanofluid has been performed. The effect of Rayleigh number, Ra, ranging from  $7 \times 10^3$  to  $5 \times 10^4$ , volume fraction of nanofluid,  $\phi = 0.02$  and aspect ratio of the cavity, A (width/height) ranging from 0.6 to 1 at different time steps are analyzed. Figs. (3,5,7) represent the time history of the temperature contours at different Rayleigh numbers while Figs. (4,6,8) display the time varying velocity contours. Figs. (9,11) indicate the effect of varying the aspect ratio at constant Rayleigh number on the time history of the temperature and velocity contours. From Figs.(3,5,7) one can observe that at early time steps, The isotherms are distributed almost equally throughout the enclosure, parallel to the horizontal wall with a symmetry behavior indicating that the conduction is the dominant mode of heat transfer. With the time increasing, the isotherms show vertical temperature stratification in the core region of the cavity. The isotherms cluster along the thermally active walls and form thermal boundary layers where convection dominates the flow regime. By increasing the value of Rayleigh number, time to reach the convection mode is less, flow strength increases and the boundary layers become more distinguished. Isotherms show that temperature gradients near the heater and cold wall become more severe. Figs. (4,6,8) show that convection is weakened and conduction is the dominant mode of heat transfer at the first time steps, after that, Due to the temperature distribution imposed at the bottom wall and to the boundary conditions on vertical walls, the buoyant forces generated due to the fluid temperature differences, The flow pattern becomes a single egg shaped cell occupying the whole cavity. The core region of the cell is at the middle of the enclosure. As the Rayleigh number increases, symmetrical flow patterns are observed in the enclosure. The streamlines show two counter rotating circulating cells. In fact, the buoyant forces generated due to the fluid temperature differences cause the fluid to rise in the middle and to descend on the sides of the enclosure. This movement of the fluid forms counter rotating circulating cells within the enclosure. The streamlines and isotherms for various aspect ratio are displayed in Figs.(9,10,11,12). Figs.(9,11) declare that for lower aspect ratio, time to reach the steady state temperature distribution is more less. Unsymmetrical isotherms can also see. For the flow patterns, Figs.(10,12) clearly show that the two counter rotating circulating cells pattern is changes to the single egg shaped cell occupying the whole cavity as the aspect ratio decreases.

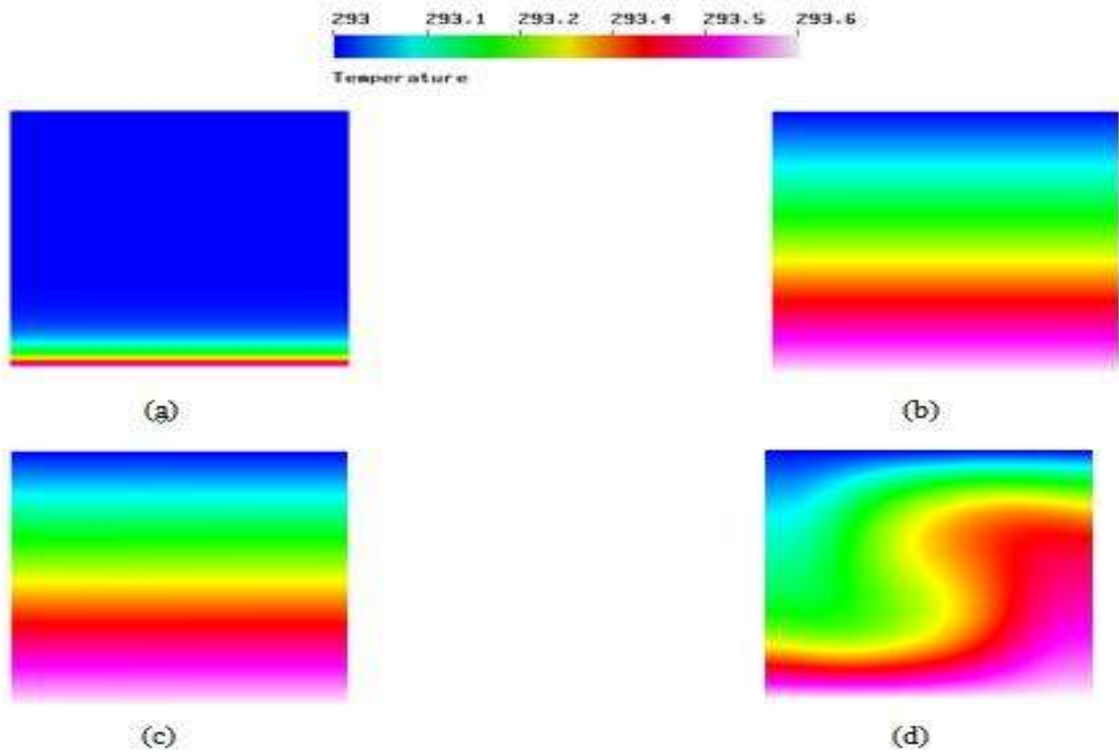


Fig.(3): temperature contours at  $Ra=7000, AR=1$  for different time periods, (a)  $t=2$  sec, (b)  $t=150$  sec, (c)  $t=300$  sec, (d)  $t=450$  sec

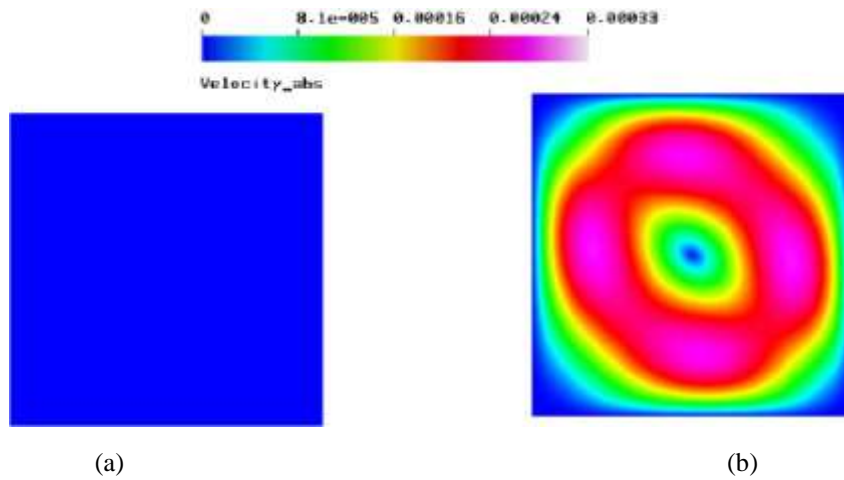
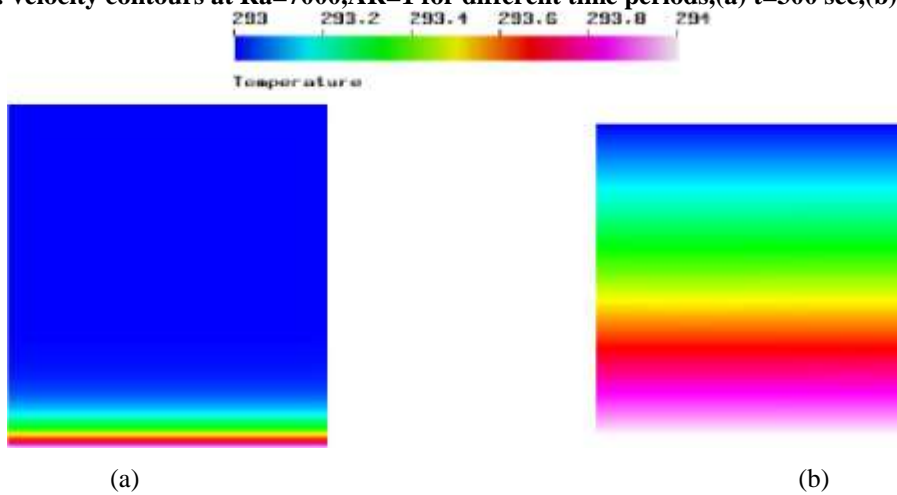


fig.(4): velocity contours at  $Ra=7000, AR=1$  for different time periods, (a)  $t=300$  sec, (b)  $t=450$  sec





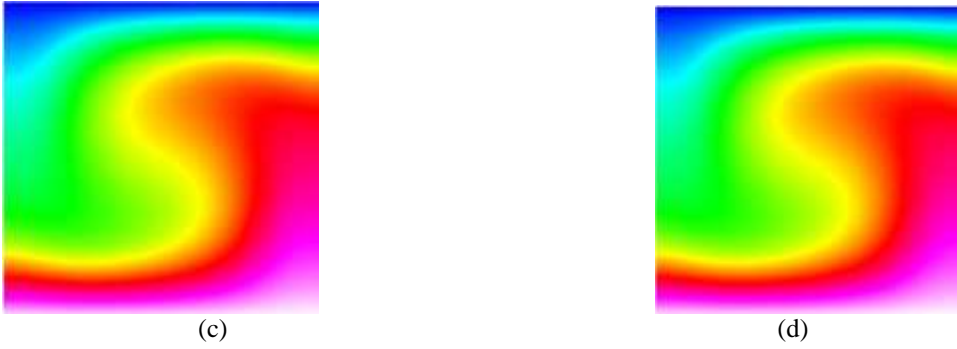


fig.(5): temperature contours at  $Ra=10000, AR=1$  for different time periods, (a)  $t=2$  sec, (b)  $t=150$  sec, (c)  $t=300$  sec, (d)  $t=450$  sec

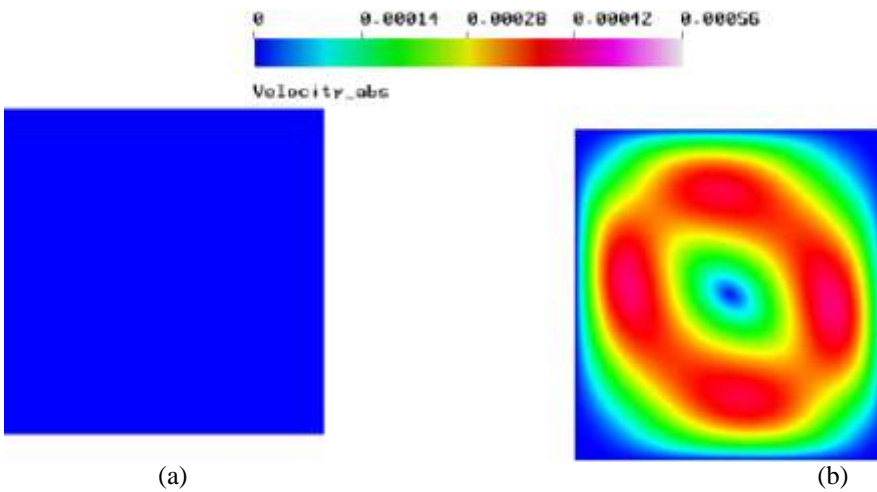


Fig.(6): velocity contours at  $Ra=10000, AR=1$  for different time periods, (a)  $t=150$  sec, (b)  $t=300$  sec

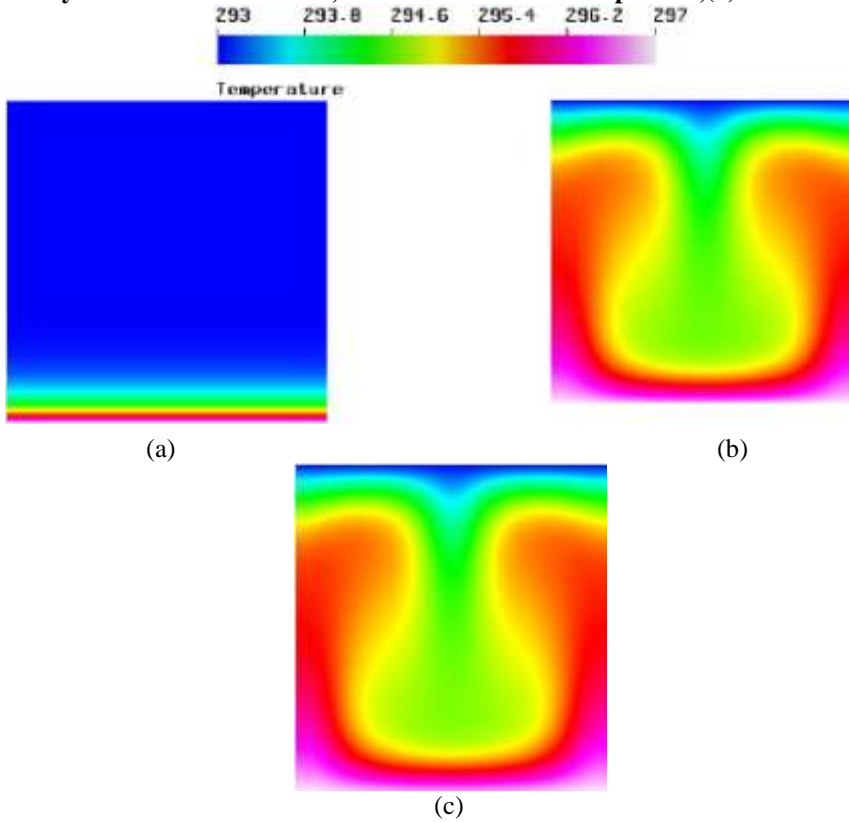


Fig.(7): temperature contours at  $Ra=50000, AR=1$  for different time periods, (a)  $t=2$  sec, (b)  $t=150$  sec, (c)  $t=300$  sec

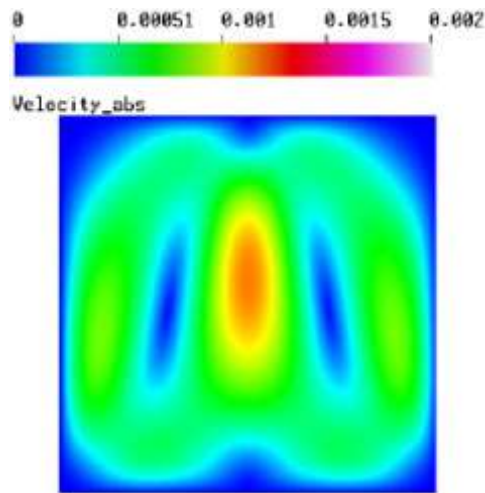


Fig.(8): velocity contour at  $Ra=50000, AR=1, t=150$  sec

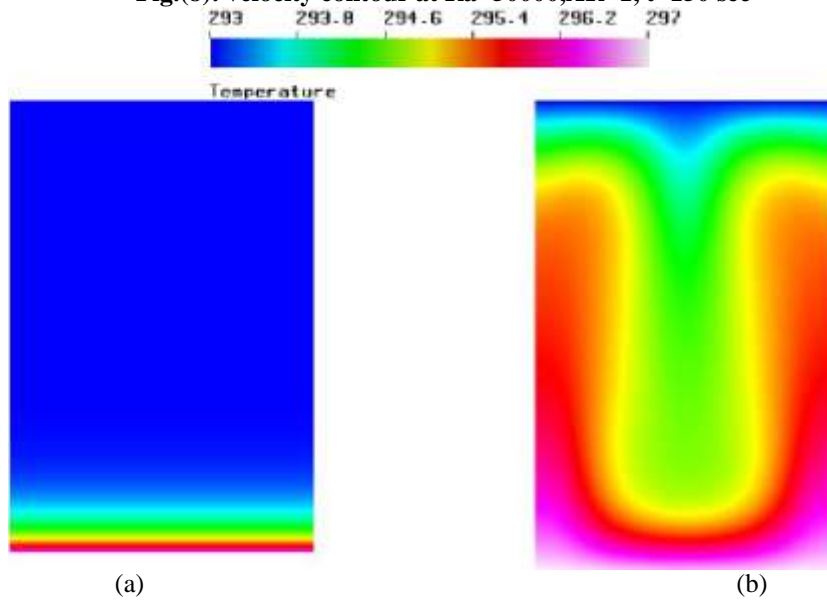


Fig.(9): temperature contours at  $Ra=50000, AR=0.8$  for different time periods, (a)  $t=2$  sec, (b)  $t=150$  sec

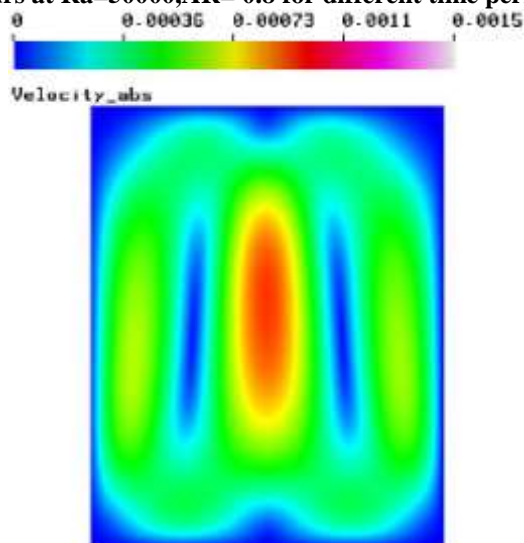


Fig.(10): velocity contour at  $Ra=50000, AR=0.8, t=150$  sec



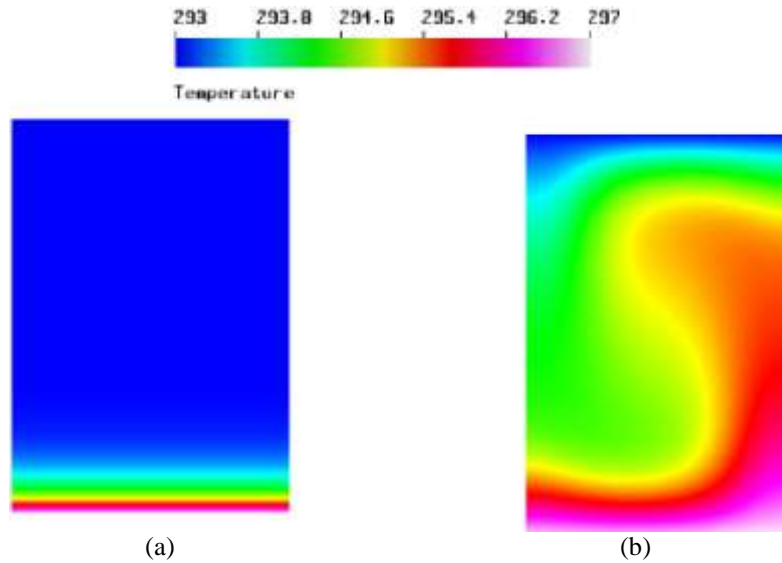


Fig.(11): temperature contours at  $Ra=50000, AR=0.6$  for different time periods, (a)  $t=2$  sec, (b)  $t=150$  sec

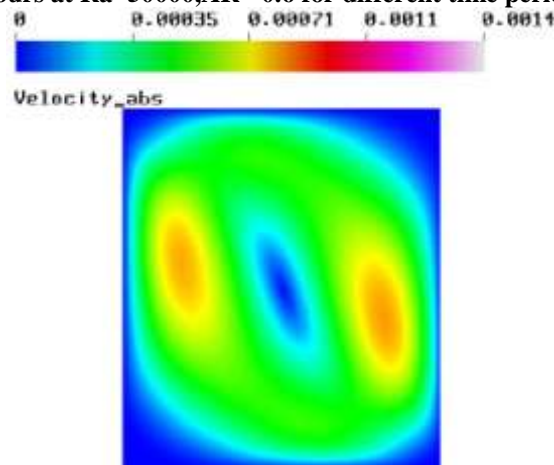


Fig.(12): velocity contour at  $Ra=50000, AR=0.6, t=150$  sec

## V. CONCLUSIONS

A numerical study has been performed to investigate the effect of Rayleigh number and the aspect ratio on the transient natural convection in an enclosure having isothermal horizontal walls and adiabatic vertical walls filled with  $Al_2O_3$ -water based nanofluid. The problem is solved using implicit finite volume method. Some important points can be drawn from the obtained results such as

- Increasing the value of Rayleigh number and decreasing the value of aspect ratio enhances the heat transfer and flow strength keeping other parameters fixed.
- Time to reach the convection mode is less when increasing the value of Rayleigh number and decreasing the value of aspect ratio.
- Increasing the time changes the isotherms from linear horizontal pattern to vertical curvilinear.
- Unsymmetrical isotherms convert to symmetrical as Rayleigh number increases and vice versa when aspect ratio decreases.
- Single cell streamlines becomes two counter rotating circulating cells as Rayleigh number increases and vice versa when aspect ratio decreases.

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