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Optimization Model for a Distribution System based on Location-Routing with Distance and forbidden route

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Abstract :- In a distribution network it is important to decide the locations of facilities that impact not only the profitability of an organisation but the ability to serve customers. Generally the location-routing problem is to minimize the overall cost by simultaneously selecting a subset of candidate facilities and constructing a set of delivery routes that satisfy some restrictions. In this paper we impose forbidden route in the constraint. We use integer programming model to describe the problem. A feasible neighbourhood search is proposed to solve the result model

Keywords : Distribution system, Location routing, VRP, Forbidden route

I. INTRODUCTION

The design of a distribution system begins with the questions of where to locate the facilities and how to allocate customers to the selected facilities. These questions can be answered using location-allocation models, which are based on the assumption that customers are served individually on out-and-back routes. However, when customers have demands that are less-than-truckload and thus can receive service from routes making multiple stops, the assumption of individual routes will not accurately capture the transportation cost. Therefore, the integration of location-allocation and routing decisions may yield more accurate and cost-effective solutions.

Therefore determining the locations of facilities within a distribution network is an important decision that impacts not only the profitability of an organization but the ability to serve customers. Classical assumptions in location modeling are that deliveries are made on out-and-back routes visiting a single customer (or that customer's travel individually to the site). Under this assumption, the cost of delivery is independent of other deliveries made. In many contexts, however, deliveries are made along multiple stop routes visiting two or more customers; in this case, the cost of delivery depends on the other customers on the route and the sequence in which they are visited. In order to capture accurately the cost of multiple stop routes within a location model, the routing problem must be solved at the same time as the location problem.

In its most general form, the location-routing problem (LRP) seeks to minimize total cost by simultaneously selecting a subset of candidate facilities and constructing a set of delivery routes that should satisfy requirements, such as, customer demands, number of vehicles, and each route begins and end at the same facility. .

There are other things that should be considered such as:

1. *Perishable goods delivery* problems have temperature restrictions to prevent spoilage that often translate into route duration or route length constraints.
2. *Time critical delivery* problems, such as express package delivery, have time deadline restrictions that limit the duration or length of routes.

The Vehicle Routing Problem (VRP) is defined on a given graph $G = (V, A)$, where $V = \{v_1,v_2,\ldots,v_n\}$ is a set of vertices and $A \subseteq \{(v_i, v_j) : i \neq j, v_i, v_j \in V\}$ is the arc set. An optimal set of routes, composed of a cyclic linkage of arcs starting and ending at the depot, is selected to serve a given set of customers at vertices. The problem aims at minimizing the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used). This problem was first introduced by Dantzig and Ramser in 1959 to solve a real-world application concerning the delivery of gasoline to service stations. A comprehensive overview of the Vehicle Routing Problem can be found in Toth and Vigo (2002) which discusses problem formulations, solution techniques, important variants and applications.

Forbidden route involving pairs of edges occur frequently ("No left turn") and can occur dynamically due to rush hour constraints, lane closures, construction, etc. Longer forbidden subpaths are less common, but can arise, for example if heavy traffic makes it impossible to turn left soon after entering a multi-lane roadway from the right. If we are routing a single vehicle it is more natural to find a detour from the point of failure when a forbidden path is discovered.

Location-routing problems are clearly related to both the classical location problem and the vehicle routing problem. In fact, both of the latter problems can be viewed as special cases of the LRP. If we require all customers to be directly linked to a depot, the LRP becomes a standard location problem. If, on the other hand, we fix the depot locations, the LRP reduces to a VRP. From a practical viewpoint, location-routing forms part of *distribution management*, while from a mathematical point of view, it can usually be modeled as a *combinatorial optimization* problem. We note that this is an NP-hard problem, as it encompasses two NP-hard problems (facility location and vehicle routing). Since a number of problem versions exist, we cannot reproduce all the formulations here. In the first instance, the reader is referred to [2] for an excellent review of various formulations.

Most of the research to date has focused on heuristic methods since LRPs merge two NP-hard problems. The heuristics generally decompose the problem into its three components, facility location, customer allocation to facilities and vehicle routing, and solve a series of well-known problems such as *p*-median, location-allocation and vehicle routing. Exact methods have been developed for a small number of LRP models that are derived from two-index flow formulations for the vehicle routing problem (VRP). Laporte and Nobert [3] solve a single depot model by a constraint relaxation method.

Laporte [2] develops an equivalent model and also extends the model to the case where the number of vehicles used is a variable in the model. Laporte et al. [5] solve a multi-depot problem in which at most *p* facilities are located by adapting REVERSE algorithm. The largest problems solved have seven candidate facilities and 40 customers. Laporte et al. [4] solve a multi-depot capacitated LRP using a constraint relaxation method. In their work, the largest problem solved to optimality has eight candidate facilities and 20 customers. Laporte et al. [4] use a branch and-bound procedure to solve asymmetric LRPs that include as many as three candidate facilities and 80 customers. . Guerra et al. [10] propose a heuristic algorithm for solving LRP in a logistic system. Toyoglu et al. [7] consider the LRP using a combination of facility location and vehicle routing problems. The main objective of their paper is to develop LRP with fewer constraints and variables. A nice survey of LRP can be found in Drexl and Schneider [1].

Success in developing exact methods for solving larger instances of LRPs is likely to come from leveraging the advances in exact methods for solving VRPs and other difficult combinatorial optimization problems. Motivated by the success of set partitioning formulations for a variety of transportation problems, such as the VRP with time windows (e.g. [8]), the pickup and delivery problem with time windows (e.g. [11]) and the crew scheduling problem (e.g. [9]), we propose a feasible neighborhood search in the context of developing exact algorithms for LRPs. The two main contributions of this paper is to present a new model for the LRP with distance constraints and forbidden route, then we improve the strategy of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps .

II. PROBLEM FORMULATION

In this section, we present a new set-partitioning-based formulation of the LRP with distance constraints. The objective of the LRP with distance constraints is to select a set of locations and to construct a set of associated delivery routes in such a way as to minimize facility costs plus routing costs. The set of routes must be such that each customer is visited exactly once by one route and that the length of each route does not exceed the maximum distance.

III. INITIAL MODEL

The model developed in this paper is based on Berger et al. (2007). Let I be the set of customer locations and J be the set of candidate facility locations. We define the graph $G=(N,A)$, where $N = I \cup J$ is the set of nodes and $A=N\times N$ is the set of arcs. We let d_{ij} for all $(i, j)\in A$ be the distance between nodes i and j. The distances satisfy the triangle inequality. The distances satisfy the triangle inequality. For applications in which the distance constraint applies to the length of the route to the last customer instead of the length of the return trip to the depot, we set d_{ij} to 0 for all *(i, j)* with $i \in I$ and $j \in J$. We define a feasible route *k* associated with facility *j* as a simple circuit that begins at facility *j*, visits one or more customer nodes and returns to facility *j* and that has a total distance of at most the maximum distance, denoted *M*. Then, we let P_j denote the set of all feasible routes associated with the facility *j* for all $j \in J$. The cost of a route $k \in P_j$ is the sum of the costs of the arcs in the route. The cost of an arc $(i, j) \in A$ is proportional to the distance d_{ij} to reflect distance related operating costs.

a. Parameters

$$a_{ijk} = \begin{cases} 1, \text{if route } k \text{ associated with facility } j \\ \text{visits customer } i, \forall\, i \in I, \forall\, j \in J, \forall\, k \in P_j \\ 0, \text{otherwise} \end{cases}$$

c_{jk} cost of route *k* associated with facility *j*
$, \forall\, j \in J, \forall\, k \in P_j\, f_j$ fixed cost associated with selecting facility
j, $\forall\, j \in J$
$\propto$ object w $\propto$ eighted factor

b. Decision Variables

$$x_j = \begin{cases} 1, \text{if facility } j \text{ is selected}, \forall\, j \in J \\ 0, \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1, \text{if route } k \text{ associated with facility } j \\ \text{is selected}, \forall\, j \in J, \forall\, k \in P_j \\ 0, \text{otherwise} \end{cases}$$

(LRP-DC) Minimize $\propto . \sum_{j \in J} f_i x_i + \sum_{j \in J} \sum_{k \in P_j} c_{jk} y_{jk}$ (1)

s.t. $\sum_{j \in J} \sum_{k \in P_j} c_{jk} y_{jk} = 1 \;\forall\, i \in I$ (2)

$x_j - y_{jk} \geq 0 \;\; \forall\, j \in J, \forall\, k \in P_j$ (3)

$x_j \in \{0, 1\} \;\forall\, j \in J$ (4)

$y_{jk} \in \{0, 1\} \;\forall\, j \in J, \forall\, k \in P_j$ (5)

The objective function (1) seeks to minimize the weighted sum of the facility costs and the routing costs. Constraints (2) are the set partitioning constraints that require each customer *i* be served by exactly one of the selected routes. Constraints (3) require that facility *j* be selected if a route *k* associated with facility *j* is selected. Constraints (4) and (5) are standard binary restrictions. The LRP with distance constraints is NP-hard. By placing very large costs on the arcs connecting two customer nodes, we obtain a special case of the model in which the selected routes contain exactly one customer.

As presented, the formulation LRP-DC potentially contains an exponential number of variables y_{jk} and an exponential number of constraints (3). Thus, for instances of practical size, enumerating all of the feasible routes and solving the resulting integer program is unlikely to be effective. Instead, we will use feasible neighborhood search for solving the model.

The model for vehicle routing with forbidden route can be written as:
(VRP-FR)

$$\min \sum_{(i,j)\in A;\,(i,j)\notin X} c_{ij}x_{ij} \quad (6)$$

s.t. $$\sum_{(s,j)\in\delta^+(i)} x_{ij} = 1 \qquad \forall i \in C, i \notin X \quad (7)$$

$$\sum_{(s,j)\in\delta^-(i)} x_{ij} = \sum_{(s,j)\in\delta^+(i)} x_{ij} \qquad i \notin X, \forall i \in V \quad (8)$$

$$\sum_{(s,j)\in\delta^-(i)} (T_{ji}^r + T_{ji}^r x_{ij}) \le \sum_{(s,j)\in\delta^+(i)} T_{ji}^r \qquad i \notin X, \forall r \in R, \forall i \in C \quad (9)$$

$$a_i^r x_{ij} \le T_{ji}^r \le b_i^r x_{ij} \qquad (i,j) \notin X, \forall r \in R, \forall (i,j) \in A \quad (10)$$

$$T_{ij}^r \ge 0 \qquad (i,j) \notin X, \forall r \in R, \forall (i,j) \in A \quad (11)$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \notin X, \forall (i,j) \in A \quad (12)$$

The combination of the two models we can get a model for LRP with Distance and Forbidden Route constraints.

IV. NEIGHBOURHOOD SYSTEM

It should be noted that, generally, in integer programming the reduced gradient vector, which is normally used to detect an optimality condition, is not available, even though the problems are convex. Thus we need to impose a certain condition for the local testing search procedure in order to assure that we have obtained the "best" suboptimal integer feasible solution.

Scarf (1986) has proposed a quantity test to replace the pricing test for optimality in the integer programming problem. The test is conducted by a search through the neighbours of a proposed feasible point to see whether a nearby point is also feasible and yields an improvement to the objective function.

Let $[\beta]_k$ be an integer point belongs to a finite set of neighbourhood $N([\beta]_k)$. We define a neighbourhood system associated with $[\beta]_k$, that is, if such an integer point satisfies the following two requirements

1. If $[\beta]_j \in N([\beta]_k)$ then $[\beta]_k \in [\beta]_j), j \neq k$.
2. $N([\beta]_k) = [\beta]_k + N(0)$

With respect to the neighbourhood system mentioned above, the proposed integerizing strategy can be described as follows.

Given a non-integer component, x_k, of an optimal vector , x_B. The adjacent points of x_k, being considered are $[x_k]$ and $[x_k] + 1$. If one of these points satisfies the constraints and yields a minimum deterioration of the optimal objective value we move to another component, if not we have integer-feasible solution.

Let $[x_k]$ be the integer feasible point which satisfies the above conditions. We could then say if $[x_k] + 1 \in N([x_k])$ implies that the point $[x_k] + 1$ is either infeasible or yields an inferior value to the objective function obtained with respect to $[x_k]$. In this case $[x_k]$ is said to be an "optimal" integer feasible solution to the integer programming problem. Obviously, in our case, a neighborhood search is conducted through proposed feasible points such that the integer feasible solution would be at the least distance from the optimal continuous solution.

V. THE BASIC APPROACH

Before we proceed to the case of MINLP problems, it is worthwhile to discuss the basic strategy of process for linear case, i.e., Mixed Integer Linear Programming (MILP) problems.

Consider a MILP problem with the following form

Minimize $P = c^T x$ (13)

Subject to $Ax \leq b$ (14)

$x \geq 0$ (15)

x_j integer for some $j \in J$ (16)

A component of the optimal basic feasible vector $(x_B)_k$, to MILP solved as continuous can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \cdots - \alpha_{kj}(x_N)_j - \cdots \alpha_{kn} - m(x_N)_N n \qquad (17)$$

Note that, this expression can be found in the final tableau of Simplex procedure. If $(x_B)_k$ is an integer variable and we assume that β_k is not an integer, the partitioning of β_k into the integer and fractional components is that given

$$[\beta_k] + f_k\,,\; 0 \leq f_k \leq 1 \qquad (18)$$

suppose we wish to increase $(x_B)_k$ to its nearest integer, $([\beta] + 1)$. Based on the idea of suboptimal solutions we may elevate a particular nonbasic variable, say $(x_N)_{j^*}$, above its bound of zero, provided α_{kj^*}, as one of the element of the vector α_{j^*}, is negative. Let Δ_{j^*} be amount of movement of the non variable $(x_N)_{j^*}$, such that the numerical value of scalar $(x_B)_k$ is integer. Referring to Eqn. (17), Δ_{j^*} can then be expressed as

$$\Delta_{j^*} = \frac{1-f_k}{-\alpha_{kj^*}} \qquad (19)$$

while the remaining nonbasic stay at zero. It can be seen that after substituting (18) into (19) for $(x_N)_{j^*}$ and taking into account the partitioning of β_k given in (18), we obtain

$$(x_B)_k = [\beta] + 1$$

Thus, $(x_B)_k$ is now an integer.

It is now clear that a nonbasic variable plays an important role to integerized the corresponding basic variable. Therefore, the following result is necessary in order to confirm that must be a non-integer variable to work with in integerizing process.

Theorem 1. *Suppose the MILP problem* (13)-(16) *has an optimal solution, then some of the nonbasic variables.* $(x_N)_j, j = 1, \ldots, n$, *must be non-integer variables.*

Proof:

Solving problem as a continuous of slack variables (which are non-integer, except in the case of equality constraint). If we assume that the vector of basic variables consists of all the slack variables then all integer variables would be in the nonbasic vector x_N and therefore integer valued.

VI. DERIVATION OF THE METHOD

It is clear that the other components, $(x_B)_{i\neq k}$, of vector x_B will also be affected as the numerical value of the scalar $(x_N)_{j^*}$ increases to Δ_{j^*}. Consequently, if some element of vector α_{j^*}, i.e., α_{j^*} for $i \neq k$, are positive, then the corresponding element of x_B will decrease, and eventually may pass through zero. However, any component of vector *x* must not go below zero due to the non-negativity restriction. Therefore, a formula, called the minimum ratio test is needed in order to see what is the maximum movement of the nonbasic $(x_N)_{j^*}$ such that all components of *x* remain feasible. This ratio test would include two cases.

1. A basic variable $(x_B)_{i\neq k}$ decreases to zero (lower bound) first.
2. The basic variable, $(x_B)_k$ increases to an integer.

Specifically, corresponding to each of these two cases above, one would compute

$$\theta_1 = \min_{i\neq k | \alpha_{j^*}>0} \left\{\frac{\beta_i}{\alpha_{j^*}}\right\} \quad (20)$$

$$\theta_2 = \Delta_{j^*} \quad (21)$$

How far one can release the nonbasic $(x_N)_{j^*}$ from its bound of zero, such that vector x remains feasible, will depend on the ratio test θ^* given below

$$\theta^* = \min(\theta_1, \theta_2) \quad (22)$$

obviously, if $\theta^* = \theta_1$, one of the basic variable $(x_B)_{i\neq k}$ will hit the lower bound before $(x_B)_k$ becomes integer. If $\theta^* = \theta_2$, the numerical value of the basic variable $(x_B)_k$ will be integer and feasibility is still maintained. Analogously, we would be able to reduce the numerical value of the basic variable $(x_B)_k$ to its closest integer $[\beta_k]$. In this case the amount of movement of a particular nonbasic variable, $(x_N)_{j^*}$, corresponding to any positive element of vector α_{j}, is given by

$$\Delta_{f'} = \frac{f_k}{\alpha_{kj}} \quad (23)$$

in Linear Programming (LP) terminology the operation conducted in Eqns. (17) and (18) is called the pricing operation. The vector of reduced costs d_i is used to measure the deterioration of the objective function value caused by releasing a nonbasic variable from its bound. Consequently, in deciding which nonbasic should be released in the integerizing process, the vector d_i must be taken into account, such that deterioration is minimized. Recall that the minimum continuous solution provides a lower bound to any integer-feasible solution. Nevertheless, the amount of movement of particular nonbasic variable as given in Eqns. (11) or (15), depends in some way on the corresponding element of vector α_j. Therefore it can be observed that the deterioration of the objective function value due to releasing a nonbasic variable $(x_N)_{j^*}$ so as to integerize a basic variable $(x_B)_k$ may be measured by the ration

$$\left|\frac{d_k}{\alpha_{kj^*}}\right| \quad (24)$$

where $|a|$ means the absolute value of scalar a.

In order to minimize the detonation of the optimal continuous solution we then use the following strategy for deciding which nonbasic variable may be increased from its bound of zero, that is,

$$\min_j \left\{ \left| \frac{d_k}{\alpha_{kj^*}} \right| \right\}, \; j = 1, \ldots, n - m \qquad (25)$$

From the "active constraint" strategy and the partitioning of the constraints corresponding to basic (B), superbasic (S) and nonbasic (N) variables we can write

$$\begin{bmatrix} B & S & N \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x_b \\ x_N \\ x_S \end{bmatrix} = \begin{bmatrix} b \\ b_N \end{bmatrix} \qquad (26)$$

or

$$Bx_b + Sx_N + Nx_S = b \qquad (27)$$

$$x_N = b_N \qquad (28)$$

The basis matrix B is assumed to be square and nonsingular, we get

$$x_B = \beta - Wx_S - \alpha x_N \qquad (29)$$

Where

$$\beta = B^{-1} \qquad (30)$$

$$W = B^{-1}S \qquad (31)$$

$$\alpha = B^{-1} \qquad (32)$$

Expression (28) indicates that the nonbasic variables are being held equal to their bound. It is evident through the "nearly" basic expression of Eqn. (29), the integerizing strategy discussed in the previous section, designed for MILP problem can be implemented. Particularly, we would be able to release a nonbasic variable from its bound, Eqn.(28) and exchange it with a corresponding basic variable in the integerizing process, although the solution would be degenerate. Furthermore, the Theorem (1) above can also be extended for MINLP problem.

Theorem 2. *Suppose the MINLP problem has a bounded optimal continuous solution, then we can always get a non-integer y_i in the optimum basic variable vector.*

Proof.

1. If these variables are nonbasic, they will be at their bound. Therefore they have integer value.
2. If a y_j is superbasic, it is possible to make y_j basic and bring in a nonbasic at its bound to replace it in the superbasic.

However, the ratio test expressed in (14) cannot be used as a tool to guarantee that the integer solution optimal found gill remains in the feasible region. Instead, we use the feasibility test from Minos in order to check whether the integer solution is feasible or infeasible.

6.1 *Pivoting*.

Currently, we are in a position where particular basic variable, $(x_B)_k$ is being integerized, thereby a corresponding nonbasic variable, $(c_N)_{j^*}$, is being released from its bound of zero. Suppose the maximum movement of $(x_N)_{j^*}$ satisfies

$$\theta^* = \Delta_{j^*}$$

such that $(x_B)_k$ is integer valued to exploit the manner of changing the basis in linear programming, we would be able to move $(x_N)_{j^*}$ into B (to replace $(x_B)_k$) and integer-valued $(x_B)_k$ into *S* in order to maintain the integer solution. We now have a degenerate solution since a basic variable is at its bound. The integerizing process continues with a new set $[B, S]$. In this case, eventually we may end up with all of the integer variables being superbasic.

Theorem 3. *A suboptimal solution exists to the MILP and MINLP problem in which all of the integer variables are superbasic.*

Proof.

1. If all of the integer variables are in *N*, then they will be a bound.
2. If an integer variable is basic it is possible to either
   - Interchange it with a superbasic continuous variable, or
   - Make this integer variable superbasic and bring in a nonbasic at its bound to replace it in the basis which gives a degenerate solution.

The other case which can happen is that a different basic variables $(x_B)_{i \neq k}$ may hit its bound before $(x_B)_k$ becomes integer. Or in other words, we are in a situation where

$$\theta^* = \Delta_1$$

In this case we move the basic variable $(x_B)_j$ into N and its position in the basic variable vector would be replaced by nonbasic $(x_B)_{j^*}$. Note $(x_B)_k$ is still a non-integer basic variable with a new value.

VII. FEASIBLE NEIGHBOURHOOD HEURISTIC SEARCH

While a straightforward brand-and-bound approach could be adopted, for many classes of large-scale problems such a procedure would be prohibitively expensive in terms of total computing time. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

This may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as nonbasic.

The procedure may be summarized as follows:

Step 1 : Solve the problem ignoring integrality requirements.

Step 2 : Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution.

Step 3 : Divide the set I of integer variables into the set I_1 at their bounds that were nonbasic at the continuous solution and the set I_2, $I = I_1 + I_2$.

Step 4 : Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5 : At the solution obtained in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to set I_2 and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set I_2.

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.

In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasic, the integers in the set I_2 must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is included in the problem.

VIII. CONCLUSIONS

This paper presents a LRP model in which there are some forbidden route. The framework of the model stems from VRP with time windows with forbidden route. Then we exclude the forbidden route from the previous assigned route. We solve the model using a feasible neighbourhood search.

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