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Time Truncated Chain Sampling Plans for Generalized Rayleigh Distribution

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Abstract:- In this paper, a Chain sampling plan is developed based on truncated lifetimes when the lifetime of an item follows a generalized Rayleigh distribution with known value of shape parameter. The design parameters such as the minimum sample size and the acceptance number are obtained by satisfying the producer's and consumer's risks at the specified quality levels, under the assumption that the termination time and the number of items are pre-fixed. The results are illustrated by an example.

Keywords:- Consumer's risk, Chain sampling plan, Generalized Rayleigh distribution, Operating characteristics, Producer's risk ,Truncated life test.

I. INTRODUCTION

Acceptance sampling plans are the practical tools for quality assurance applications involving product quality control. Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality. Whenever a sampling inspection is considered, the lot is either accepted or rejected along with associated producer and consumer's risk. In a time truncated sampling plan suppose n units are placed in a life test and the experiment is stopped at a predetermined time T, where the number of failures is recorded until the pre specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Sampling inspection in which the criteria for acceptance and nonacceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan and so the use of chain sampling plan is often recommended when an extremely high quality is essential. More recently, Aslam and Jun (2009) proposed the group acceptance sampling plan based on the truncated life test when the lifetime of an item follows the inverse Rayleigh and Log-logistic distribution. These truncated lifetests were discussed by many authors, Kantam R.R. L., Rosaiah K. and Srinivasa Rao G. (2001) discussed acceptance sampling based on life tests with Log-logistic models. Rosaiah K. and Kantam R.R.L. (2005) discussed acceptance sampling based on the inverse Rayleigh distribution. Gupta and Groll(1961), Baklizi and EI Masri(2004), and Tsai, Tzong and Shou(2006), Balakrishnan, Victor Leiva & Lopez (2007).All these authors developed the sampling plans for life tests using single acceptance sampling.The purpose of this study is to find the probability of Acceptance for chain sampling plan, assuming the experiment is truncated at preassigned time and lifetime follows a generalized Rayleigh distribution.

II. GLOSSARY OF SYMBOLS

n	-	Sample size
λ	-	Shape parameter
T	-	Prefixed time
β	-	Consumer's risk
p*	-	Minimum probability
d	-	Number of defectives
i	-	Acceptance criteria
Pa(p)	-	Probability of Acceptance

III. GENERALISED RAYLEIGH DISTRIBUTION :

The Rayleigh distribution was derived by Rayleigh (1880) to handle problems in the field of acoustics. Tsai and Wu (2006) developed an acceptance sampling plan assuming that the lifetime of a product has a generalised Rayleigh distribution.The cumulative distribution (c.d.f) of Rayleigh distribution is given by

$$F(t, b) = 1 - \exp(-t^2/(2b^2)), \; t > 0, \qquad \rightarrow \quad \textbf{(1)}$$

Where b > 0 is the scale parameter. If some other parameters are involved, then they are assumed to be known, for example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t / λ_0.

IV. CHAIN SAMPLING PLAN

Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) making use of cumulative results of several samples help to overcome the shortcomings of the Single Sampling Plan.The distinguish feature is that the current lot under inspection can also be accepted if one defective unit is observed in the sample provided that no other defective units were found in the samples from the immediately preceeding i lots, i.e. the chain. It avoids rejection of a lot on the basis of a single nonconfirming unit and improves the poor discrimination between good and bad quality. The conditions for application and operating procedure of chsp-1 are as follows

4.1 Conditions for application of ChSP -1:

The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.

1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected to be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

4.2 Operating Procedure

The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceeding i samples of size n.

The Chain sampling Plan is characterized by the parameters n and i. We are interested in determining the sample size required for in the case of generalized Rayleigh distribution and various values of acceptance number i.The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β. The probability of acceptance in the case of chain sampling plan is given by

$$P_a(p) = (1-p)^{n} + np(1-p)^{n-1}(1-p)^{ni}$$

Where $p = F_{GE}(t, \lambda) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\lambda}\right)^2}$

In Table 1 we present the minimum values of n, satisfying equation for p* = 0.75, 0.90, 0.95, 0.99 and for t / λ_0 = 0.628,0.942,1.257,1.571,2.356,3.141,4.712, keeping α_0 fixed. These choices are consistent with Gupta and Groll (1961), Gupta (1962),Kantam et al (2001),Baklizi and EI Masri (2004),Balakrishnan et Al (2007). The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function sampling plan. Once the minimum sample size is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough. For a fixed T, p is a decreasing function of $\lambda \geq \lambda_0$ .For a fixed i the operating chartacteristic function values as a function of λ / λ_0 are presented in Table 2 for different values of p* and for the chsp-1.

V. DESCRIPTION OF TABLES AND AN EXAMPLE

In Table1, we provide the minimum sample size required for the proposed sampling plan for the generalised Rayleigh distribution. Assume that the life time distribution is an generalised Rayleigh distribution with and that the experimenter is interested in knowing that the true mean life is atleast 1000 hours with confidence 0.99. It is assumed that the maximum affordable time is 767 hoursand $t / \lambda_0 = 0.942$, from the table 1, we obtain n = 11.The lot will be accepted if during 767 hours not more than one failure is observed in a sample of 11 and if no defectives are found in the immediately preceeding i samples.For the sampling plan(n =

11, i= 2, t / λ_0 = 0.942) and confidence level p* = 0.99 under generalised Rayleigh distribution the values of the operating characteristic function from Table 2 as follows

λ / λ_0	**2**	**4**	**6**	**8**	**10**	**12**
L(p)	**0.328385**	**0.860968**	**0.964044**	**0.987452**	**0.994618**	**0.997338**

FIGURE 1:

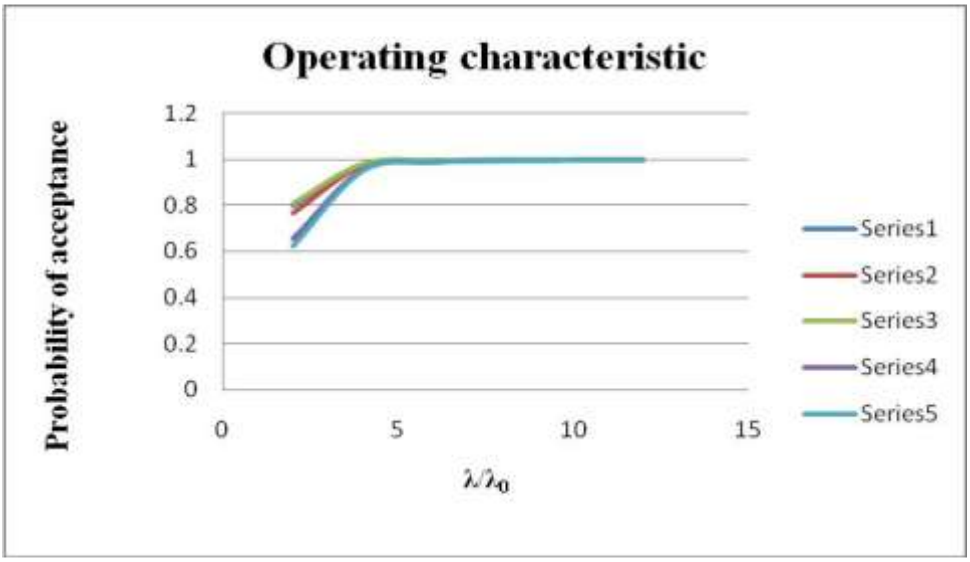

Table 1: OC curve for Probability of acceptance against λ/λ_0

Minimum sample size for the proposed plan in case of Generalised Rayleigh distribution with probability p* and the corresponding acceptance number i .

p*	**i**	**t/ λ 0**							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	**4.712**
0.75	1	9	4	3	2	1	1	1	**1**
	2	8	4	2	2	1	1	1	**1**
	3	8	4	2	2	1	1	1	**1**
	4	8	4	2	2	1	1	1	**1**
	5	8	4	2	2	1	1	1	**1**
	6	8	4	2	2	1	1	1	**1**
0.90	1	13	6	4	3	2	1	1	**1**
	2	12	6	3	2	1	1	1	**1**
	3	12	6	3	2	1	1	1	**1**
	4	12	6	3	2	1	1	1	**1**
	5	12	6	3	2	1	1	1	**1**
	6	12	6	3	2	1	1	1	**1**
0.95	1	16	8	5	3	2	1	1	**1**
	2	16	7	4	3	2	1	1	**1**
	3	16	7	4	3	2	1	1	**1**
	4	13	7	5	4	2	2	1	**1**
	5	16	7	4	3	2	1	1	**1**
	6	16	7	4	3	2	1	1	**1**
0.99	1	24	11	6	4	2	2	1	**1**
	2	24	11	6	4	2	1	1	**1**
	3	24	11	6	4	2	1	1	**1**
	4	24	11	6	4	2	1	1	**1**
	5	24	11	6	4	2	1	1	**1**
	6	**24**	**11**	**6**	**4**	**2**	**1**	**1**	**1**

Table 2: Operating Characteristic values for the time truncated chain sampling plan (n, i, t// λ 0) for a given p*, when i = 2 .

p*	n	t/λ_m 0	λ / λ_0 2	4	6	8	10	12
0.75	8	0.628	0.797927	0.979913	0.995654	0.998580	0.999409	**0.999713**
	4	0.942	0.765639	0.975646	0.994681	0.998256	0.999274	**0.999647**
	2	1.257	0.807193	0.981242	0.995960	0.998682	0.999452	**0.999734**
	2	1.571	0.653085	0.958007	0.990520	0.996854	0.998683	**0.999357**
	1	2.356	0.624566	0.953318	0.989398	0.996474	0.998522	**0.999278**
	1	3.141	0.351502	0.877895	0.969305	0.989402	0.995478	**0.997770**
	1	3.927	0.163574	0.763457	0.932822	0.975696	0.989395	**0.994705**
	1	4.712	0.065969	0.624566	0.877854	0.973318	0.979079	**0.989398**
0.90	12	0.628	0.656257	0.988009	0.990498	0.996844	0.998678	**0.999355**
	6	0.942	0.609586	0.949139	0.988322	0.996101	0.998362	**0.999199**
	3	1.257	0.663675	0.959686	0.990918	0.996989	0.998739	**0.999385**
	2	1.571	0.653085	0.958007	0.990520	0.996854	0.998683	**0.999357**
	1	2.356	0.624566	0.953318	0.989398	0.996474	0.998522	**0.999278**
	1	3.141	0.351502	0.877895	0.969305	0.989402	0.995478	**0.997770**
	1	3.927	0.163574	0.763457	0.932822	0.975696	0.989395	**0.994705**
	1	4.712	0.065969	0.624566	0.877854	0.973318	0.979079	**0.989398**
0.95	16	0.628	0.530266	0.930845	0.983653	0.994484	0.997671	**0.998858**
	7	0.942	0.539986	0.933513	0.984356	0.994730	0.997777	**0.998911**
	4	1.257	0.535478	0.932724	0.984165	0.994665	0.997750	**0.998897**
	3	1.571	0.46382	0.913588	0.979122	0.992898	0.996991	**0.998522**
	2	2.356	0.280816	0.840657	0.958039	0.985256	0.993656	**0.996857**
	1	3.141	0.351502	0.877895	0.969305	0.989402	0.995478	**0.997770**
	1	3.927	0.163574	0.763457	0.932822	0.975696	0.989395	**0.994705**
	1	4.712	0.065969	0.624566	0.877854	0.973318	0.979079	**0.989398**
0.99	24	0.628	0.341168	0.866486	0.965674	0.988047	0.994879	**0.997469**
	11	0.942	0.328385	0.860968	0.964044	0.987452	0.994618	**0.997338**
	6	1.257	0.343177	0.868461	0.966313	0.988287	0.994985	**0.997522**
	4	1.571	0.326785	0.861652	0.964320	0.987562	0.994668	**0.997364**
	2	2.356	0.280816	0.840657	0.958039	0.985256	0.993656	**0.996857**
	1	3.141	0.351502	0.877895	0.969305	0.989402	0.995478	**0.997770**
	1	3.927	0.163574	0.763457	0.932822	0.975696	0.989395	**0.994705**
	1	**4.712**	**0.065969**	**0.624566**	**0.877854**	**0.973318**	**0.979079**	**0.989398**

VI. CONCLUSIONS

In this paper, chain sampling plan for the truncated life test was proposed in the case of generalized Rayleigh distribution. The minimum sample size required and the acceptance number were calculated. From the figure 1 we can see that the probability of acceptance increases when λ / λ_0 increases and it reaches the maximum value 1 when λ / λ_0 is greater than 5.

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