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Functional Simulation of the Integrated Onboard System For a Commercial Launch Vehicle

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Abstract:- In the article has been chosen and simulated the integrated onboard guidance system of a commercial launch vehicle with application of GPS technologies. In this study we shall consider a concept of integrated onboard systems for launch vehicles in the context of the current task, and provide mathematical models of all its elements for different variants of their structure and composition. Was set the conceptual design of an integrated navigation system for the space launch vehicle qualified to inject small artificial Earth satellites into low and medium circular orbits. The conceptual design of the integrated navigation system based on GPS technology involves determination of its structure, models and algorithms, providing the required accuracy and reliability in inject payloads with due regard to restrictions on weight and dimensions of the system.

Keywords:- Gimbaled inertial navigation system (GINS), global positioning system (GPS), inertial navigation system (INS), mathematical model (MM), navigation, pseudorange, pseudovelocity, launch vehicle, self-guided system (SGS), Kalman filter, control loop, control system (CS), trajectory, boost phase (BPH), distributor, coordinate, orientation.

I. INTRODUCTION

A key tendency in the development of affordable modern navigation systems is displayed by the use of integrated GPS/INS navigation systems consisting of a gimbaled inertial navigation system (GINS) and a multichannel GPS receiver [1]. The investigations show [2, 3], that such systems of navigation sensors with their relatively low cost are able to provide the required accuracy of navigation for a wide class of highly maneuverable objects, such as airplanes, helicopters, airborne precision-guided weapons, spacecraft, launch vehicles and recoverable orbital carriers.

Problem setting.

The study of applications of GPS navigation technologies for highly dynamic objects ultimately comes to solving the following problems [4]:

1. Creation of quality standards (optimality criteria) for solving the navigation task depending from the type of an object, its trajectory characteristics and restrictions on the weights, dimensions, costs, and reliability of the navigation system.
2. Choosing and justification of the system interconnecting the GPS-receiver and GINS: uncoupled, loosely coupled, tightly coupled (ultra-tightly coupled).
3. Making mathematical models (MM) of an object's motion, including models of external factors beyond control influencing object (disturbances). This requires to make two types of object models: the most detailed and complete one, which will be later included in the model of the environment when simulating the operation of an integrated system, and a so-called on-board model, which is much simpler and more compact than the former one, and which will be used in the future to solve the navigation problem being a part of the on-board software.
4. Making MM for GINS in consideration of use of gyroscopes and accelerometers (i.e. it is required to make a model for navigation measurements supplied by GINS, taking into account systematic (drift) and random measurement errors).
5. Making a model of the navigation field of GPS, including system architecture, a method of calculating ephemeris of navigation satellites in consideration of possible errors, clock drifts on board the navigation satellites, and taking into account conditions of geometric visibility of a navigation satellite on different parts of the trajectory of a highly dynamic object.
6. Making a model of a multichannel GPS receiver, including models of code measurements (pseudorange and pseudovelocity) and, if necessary, phase measurements, including the whole range of chance and indeterminate factors beyond control, existing when such measurements are conducted (such as multipath effect).

7. Choosing an algorithm to process measured data in an integrated system in agreement with the speed-of-response requirement (the possibility to process data in real time) and demanded accuracy in solving a navigation task.
8. Creating an object-oriented computer complex for the implementation of the above models and algorithms with the objective to model the process of functioning of the integrated navigation system of a highly dynamic object.

Let's consider the above objectives, having regard to peculiarities of the subject of inquiry, namely a commercial launch vehicle, designed to launch payloads into low Earth orbit (LEO) or geostationary orbit (GSO), in more details.

Within the framework of this study we shall consider a light launch vehicle which has been jointly developed by the European Space Agency (ESA) and the Italian Space Agency (ASI) since 1998 [5]. It is qualified to launch satellites ranging from 300 kg to 2000 kg into low circular polar orbits. As a rule, these are low cost projects conducted by research organizations and universities monitoring the Earth in scientific missions as well as spy satellites, scientific and amateur satellites. The main characteristics of the launch vehicle are given in [19]. The launch vehicle Vega [6] is the prototype of the vehicle under development.

The planned payload to be delivered by the launch vehicle to a polar orbit at an altitude of ~700 km shall be 1500 kg. The launch vehicle is tailored for missions to low Earth and Sun-synchronous orbits. During the first mission the light class launch vehicle is to launch the main payload, a satellite weighing 400 kg, to an altitude of 1450 km with an inclination of the orbit 71.50^0.

The launch vehicle under consideration is the smallest one developed by ESA. We assume that the new launch vehicle will be able to meet the demands of the market for launching small research satellites and will enable universities to conduct research in space. The launcher will be primarily used for satellites that monitor the Earth surface. The injection is conducted according to the most popular and simplest (and the cheapest) scenario [7, 8], more specifically: the instrument unit and the navigation system ride atop the 3rd stage of the launch vehicle. Thus, launching until separation of the 4th stage carrying payload is conducted in accordance with the data provided by the navigation system which estimates 12 components of the launcher state vector, including position, velocity, orientation angles and angular velocities. Basically, launching may be done upon implementation of any of the possible algorithms, for example, a terminal one, that provides accuracy of the 3rd stage launching to the calculated point of separation of the 4th stage or the traditional algorithm which minimizes the deviation of the center of mass of the launcher from the preselected programmed trajectory [5].

The injection sequence which is being described here supposes conducting the following procedures at peak altitude reached by the 3rd stage, namely the computation of the required orientation of the 4th stage and the computation of the required impulse to transfer the payload carried by the 4th stage to an orbit of an artificial satellite of the Earth from the final point reached by the 3rd stage. Thus, the transfer of the 4th stage from the end-point of lifting the 3rd stage to an orbit of injecting the payload is performed by the software, i.e. without the use of navigation data, and thus the accuracy of injection of the payload into the required orbit is determined by two factors: the accuracy of lifting the 3rd stage in the predetermined terminal point and the accuracy of the program control in the 4th stage [5].

From the standpoint of the problem concerned, namely the synthesis of the navigational algorithm of the space launcher in the proposed injection sequence we are interested only in the first factor, i.e. accuracy of lifting of the 3rd stage to the point of separation 4th stage. This accuracy, other conditions being equal, is determined by the precision of solving a navigation task in lifting the 3rd stage in consideration of both components: the center of mass and the velocity of the stage. They predetermine the required impulse for the 4th stage [9].

Thus, we may determine the main criterion of the accuracy of the navigation task in relation to the integrated inertial navigation system of the space launch vehicle: we need to ensure maximum accuracy in determining the position and velocity vectors of the 3rd stage of the launch vehicle in the exo-atmospheric phase of the mission in the selected for navigation coordinate system. Clearly, this accuracy, in its turn, other things being equal, depends upon the accuracy of the initial conditions of travel of the 3rd stage, or in other words, the accuracy of navigation on the previous atmospheric phase of the mission [1].

Consequently, in the case of the proposed injection sequence the simplest and most obvious criterion for evaluation of the accuracy of the synthesized system should be adopted. It is required to ensure maximum accuracy in determining the vectors of position and the center -of- mass velocity of the launcher during the flight of the1st-3rd stages, i.e. in atmospheric and exo-atmospheric phases of the mission. This accuracy can be characterized by the value of the dispersions posteriori of the corresponding components of the mentioned vectors [10].

II. MATHERIALS AND METHODS

Now let's consider the possible integration schemes for GINS and GPS receiver with respect to this technical problem [11]. As it has been aforementioned, currently we can think of three possible integration schemes as follows [12-16]:

- uncoupled (separated subsystems);
- loosely coupled;
- Tightly coupled (ultra-tightly coupled).

Let's consider the peculiarities of these systems.

Uncoupled systems are the simplest option for simultaneous use of INS and GPS receiver [17]. Both systems operate independently. But, as INS errors constantly accumulate, it is necessary eventually to make correction of INS according to data provided by the GPS receiver. Creating such architecture requires minimal changes to the hardware and the software [11].

In loosely coupled systems GINS and GPS also generate separate solutions, but there is a binding unit in which GPS-based measurements and GINS readouts make assessment of the status vector and make corrections of data provided by GINS [19].

A loosely coupled complex envisages an independent identification of navigation parameters both by GINS and Self-Guided System (SGS). Different navigation parameters (coordinates, velocities) are provided by GINS and SGS. They are then used in the Kalman filter to determine errors occurring in GINS with a purpose of their subsequent compensation [11, 18].

Such systems usually use two filters: the first one is a part of the satellite receiver, and the second one is used for co-operative processing of information. The advantage of this scheme is in high functional reliability of the navigation system. The drawback is in correlation of errors, arriving from SGS to the input of the second Kalman filter and the need of strict synchronization of measurements provided by INS and SGS [17]. In sources loosely coupled systems are divided into three following types [15]: the standard, "aggressive" and the so-called MAGR schemes. The difference between "aggressive" scheme and the standard one is that the former one uses the information on acceleration for extrapolation of navigation sighting executed by SGS provided by GINS in the period between measurements [11].

In tightly coupled systems the role of the INS is reduced only to the measurement of the primary parameters of translational and rotational motions. For this reason, in such systems INS are only inertial measurement units, and the GPS receiver is without own Kalman filter. In such a structure both INS and SGS provide a series of measurements for a common computing unit [19].

Tightly coupled systems are characterized by high accuracy compared with aforementioned systems, and the integrated filter makes it possible to use all available GPS satellites optimal way, but at cost of the functional redundancy of the system. Tightly coupled systems use the only "evaluator" (as a rule, the Kalman filter) that uses differences between pseudoranges and/or pseudo velocities, calculated (predicted) by INS and measured by Self-Guided System. Advantages of such a scheme are the following [6]:

- the problem of measurement correlation is absent;
- there is no need in synchronization of INS and Self-Guided System as just one clock generator is used;
- Search and selection of law quality measurements of pseudorenges.

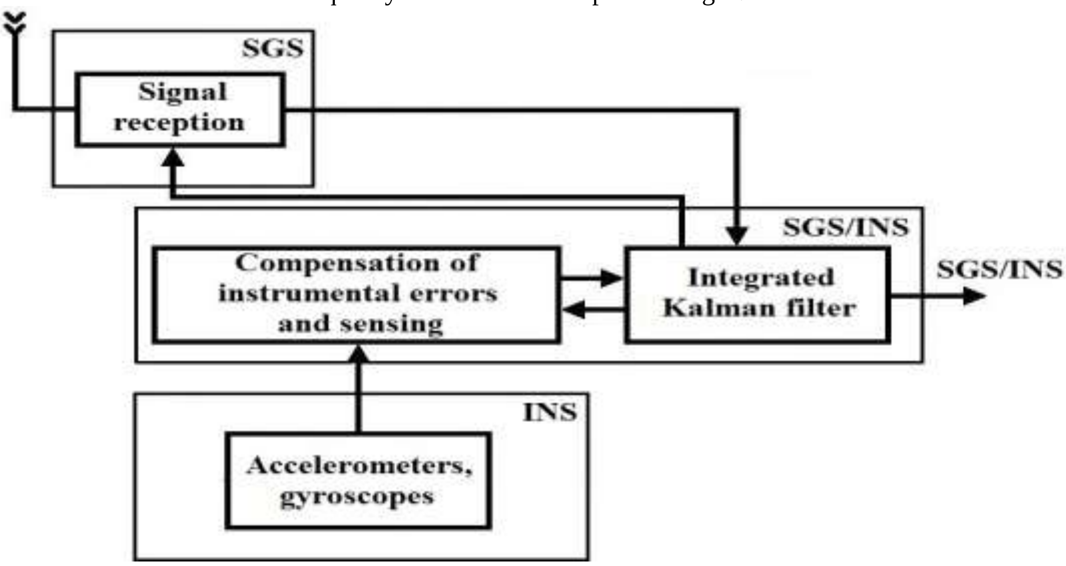

Fig. 1. Ultra-tightly coupled system [19]

The disadvantages of closely coupled systems are the following [6]:

- the need for special equipment for Self-Guided Systems;
- use of complex equations for measurements;
- Low reliability because INS failure may result if failure of the whole system.

The later drawback can be eliminated by introducing a parallel Kalman filter only for Self-Guided System [11]. Thus, the main differences between a tightly coupled system and a loosely coupled system are as follows:

- Use of the INN output information on acceleration in the code and carrier frequency tracking loop. This allows to narrow the loop bandwidth and improve performance and tuning accuracy;
- Use pseudoranges and pseudovelocities (insted of coordinates and velocities) to estimate errors in INS.

A separate embodiment of the tightly coupled systems is the so-called ultra-tightly coupled systems. In such systems (Fig. 1) estimations are undertaken in the integrated Kalman filter, and the GPS receiver is further simplified [20]. In this case, the Kalman filter is of order 40 and its implementation requires a computer with a very high speed [18].

Table 1 Key specification of ultra-tightly coupled MIGITS systems [20]

Specifications	C- MIGITS	P- MIGITS	M- MIGITS
Accuracy: **- coordinates** **- velocity**	 76 m 0,7 m/s	 19 m -	 16 m -
Sizes, mm	146x130x109	146x130x158	-
Number of receiver channels	5L1, C/A	5L1, C/A	10L1, L2, P/Y
Inertial unit	GIC-100	IMU-202	DQI (Digital Quartz IMU)
Operating time between failures, hours	2700	3600	10000
Weight, kg	2,0	3,2	2,8
Capacity, W	18	20	20
Power supply, V (direct current)	28	28	28

The experience of ultra-tight coupling of inertial and satellite systems is extremely interesting. In particular, we can meet in sources the so-called MIGITS (Miniature Integrated GPS/INS Tactical Systems) systems developed by Rockwell International. To date these are the most compact integrated systems [20]. Their key specifications are presented in Table 1.

III. RESULTS AND DISCUSSION

It is known that the algorithm of inertial navigation system is based upon integration of acceleration values of the launch vehicle sent by integrating accelerometers and reconstruction based upon calculation of the apparent way of its full position and the velocity in the coordinate system used to solve the navigation task by taking into account accelerations caused by the gravitational influence of the Earth [11]. Let's consider structures of mathematical methods and algorithms needed to reconstruct all the functions of INS.

The Mathematical Model of INS of the Launch Vehicle Using Gyrostabilized Platform.

In the INS under consideration the main components are the gyrostabilized platform and the block of integrating accelerometers.

The gyrostabilized platform (GSP) embodies the on-board navigation coordinate system (ONCS). However, in fact the coordinate system embodied by GSP differs from the ideal NCS due to the following reasons [21]:

1. Initial alignment of gyrostabilized platform on launch site is imperfect. In other words, orientation of the axes of the embodied coordinate system (hereinafter referred to as NCS1) differs from the orientation of an ideal navigation coordinate system (INCS), if latter is adopted as the main coordinate system solving navigation tasks by initial errors of GSP alignment. This difference is described by a sequence of three turns at angles corresponding to the initial rotation matrix errors calculated using the following method:

Let us assume that (x, y, z) are the coordinates of a point in the coordinate system XYZ. Then in the coordinate system $X'Y'Z'$ they will rearrange as follows (Fig. 2):

$$x' = x\cos\vartheta + y\sin\vartheta;$$
$$y' = -x\sin\vartheta + y\cos\vartheta;$$
$$z' = z.$$

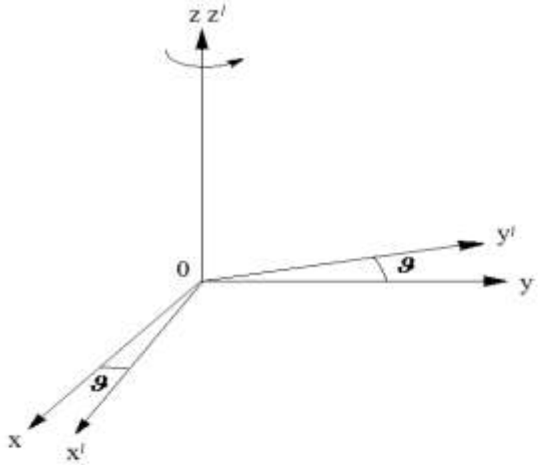

Fig. 2. Coordinate system NCS1

In matrix form, these equations will be as follows:

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} \cos\vartheta & \sin\vartheta & 0 \\ -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} x \\ y \\ z \end{vmatrix}.$$

Let us write down:

$R_z(a) = \begin{pmatrix} \cos\vartheta & \sin\vartheta & 0 \\ -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ – a standard rotation matrix about axis z through an angle ϑ.

Then we have the following relation:

$$X' = R_z(\vartheta)X.$$

Having similarly described coordinate transformation after rotations about the other two axes, and denoted respectively the matrices:

$$R_x(\vartheta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix}, \quad R_y(\vartheta) = \begin{pmatrix} \cos\vartheta & 0 & -\sin\vartheta \\ 0 & 1 & 0 \\ \sin\vartheta & 0 & \cos\vartheta \end{pmatrix}.$$

We will receive:

$$X_{NCS1} = R_{HB} X_{INCS},$$

where X_{INCS} are coordinates of an object in the inertial navigational coordinate system (INCS); X_{NCS1} are coordinates of an object in NCS1; R_{HB} is a matrix of transformation of INCS to NCS1:

$$R_{HB} = R_x(\delta H) \cdot R_y(\delta V) \cdot R_z(\delta A),$$

where δA is an angle of rotation of INCS corresponding to azimuth alignment error:

$$\delta A \in N\{0, \sigma_A\},$$

δH is an angle of rotation of INCS corresponding to horizon alignment error:

$$\delta H \in N\{0, \sigma_H\},$$

δV – angle of rotation of INCS corresponding to vertical alignment error:

$$\delta V \in N\{0, \sigma_V\}.$$

Another source of errors in the orientation of GSP besides imperfection of the initial alignment, is gyroscope precession depending upon the time of flight duration of the launch vehicle and g-force occurring during the flight, and it leads to the so-called drift of GSP. The drift of gyroplatform caused by angular drift can be described by the following matrix [22, 23]:

$$D_{GSP} = (E + M_{GSP}),$$

where $M_{GSP} = \begin{pmatrix} 0 & \varepsilon_3 & -\varepsilon_2 \\ -\varepsilon_3 & 0 & \varepsilon_1 \\ \varepsilon_2 & -\varepsilon_1 & 0 \end{pmatrix}$, $\bar{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is a gross error in the orientation of GSP axes in projection onto INCS axes.

Dynamics of drift shall be described as follows [24, 25]:

$$\bar{\varepsilon}_{i+1} = \bar{\varepsilon}_i + \bar{\omega}\Delta t,$$

where $\bar{\varepsilon}_{i+1}, \bar{\varepsilon}_i$ is a gross error corresponding to point in time t_{i+1}, t_i;

$\bar{\omega}\Delta t$ is a total "drift" of GSP axes within time interval $\Delta t.$

In its turn, $\omega\Delta t$ may be represented as a sum of "free" GSP drift which is a time function, i.e. independent of external factors but dependent merely on time, and deviation in the orientation of the axes caused by g-forces [23, 26]:

$$\omega\Delta t = \overline{\dot{\varepsilon}_{CB}}\Delta t + M_{\varepsilon}\frac{1}{g}\frac{1}{\Delta t}\Delta W,$$

where $\overline{\dot{\varepsilon}_{CB}}$ is a vector of "free" drift of GSP axes;

$$M_{\varepsilon} = \begin{pmatrix} \dot{\varepsilon}_{1y} + \dot{\varepsilon}_{1HP}\dfrac{\Delta W_Z}{g}\dfrac{1}{\Delta t} & 0 & \dot{\varepsilon}_{1k} \\ 0 & \dot{\varepsilon}_{2y} + \dot{\varepsilon}_{2HP}\dfrac{\Delta W_Z}{g}\dfrac{1}{\Delta t} & \dot{\varepsilon}_{2k} \\ 0 & \dot{\varepsilon}_{3y} + \dot{\varepsilon}_{3HP}\dfrac{\Delta W_Z}{g}\dfrac{1}{\Delta t} & \dot{\varepsilon}_{1y} \end{pmatrix},$$

where $\dot{\varepsilon}_{iy}$ is a vector of the drift speed of axes which is proportional to acceleration; $\dot{\varepsilon}_{ik}$ is a vector of the drift speed of GSP axes due to errors in disbalance of GSP gyroscopes during acceleration acting towards kinematic moments; $\dot{\varepsilon}_{iHP}$ is a vector of the drift speed of GSP axes due to lack of rigidity errors.

Increment of apparent velocity is measured using the block of integrating accelerometers. The block of accelerometers has alignment errors, as is the case with the gyro platform. Simulation of the errors shall be made according to a scheme similar to the simulation of initial alignment errors for GSP [25, 27], and the corresponding matrix R_a describing transformation from the coordinate system RCS2 materialized through actual alignment of accelerometers to the onboard navigation coordinate system (ONCS) is the following [28]:

$$R_a = R_x(\delta X) \cdot R_y(\delta Y) \cdot R_z(\delta Z),$$

where $\delta X, \delta Y, \delta Z$ are angles corresponding to errors in the orientation of the block of accelerometers.

Besides, the gain of each accelerometer is, strictly speaking, a random variable. In addition, measurements performed by each accelerometer are accompanied by additive measurement error interpreted herein as a random variable [29].

Against this background, the simulation algorithm for measurement of increment of the apparent velocity read from accelerometers can be represented as follows [30]:

1. Suppose that the vector of real apparent velocity increment is ΔW_{INCS}. Then, after transformation INCS into the onboard navigation coordinate system (ONCS), we obtain apparent velocity increment vector in ONCS:

$$\Delta W_{ONCS} = R_a \times D_{GSP} \times R_{HB} \times \Delta W_{INCS}.$$

2. The values of the components of the apparent velocity increment vector read from accelerometers shall be calculated according to the following equations

$$\begin{cases} \Delta W_{ACC_x} = (\Delta W_{ONCS_x} + \delta W_x)S_x; \\ \Delta W_{ACC_y} = (\Delta W_{ONCS_y} + \delta W_y)S_y; \\ \Delta W_{ACC_z} = (\Delta W_{ONCS_z} + \delta W_z)S_z, \end{cases}$$

where $\delta W = (\delta W_x, \delta W_y, \delta W_z)^T$ is a vector of random measurement errors;

S_x, S_y, S_z are values of the gain of accelerometers.

Additionally, accelerometers have deadband areas, so increment of the apparent velocity transferred by accelerometers is compared with a certain threshold value. If the value ΔW_{ACC} is less than the threshold value, the output value will be 0.

A functional diagram of the model of the onboard measuring system (OMS) is shown in Fig. 3.

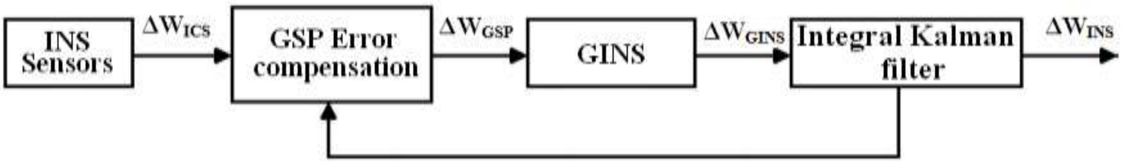

Fig. 3. Functional model of OMS

Algorithm for Solving a Navigation Task Using GSP. A functional circuit for INS using GSP is represented in Fig. 4.

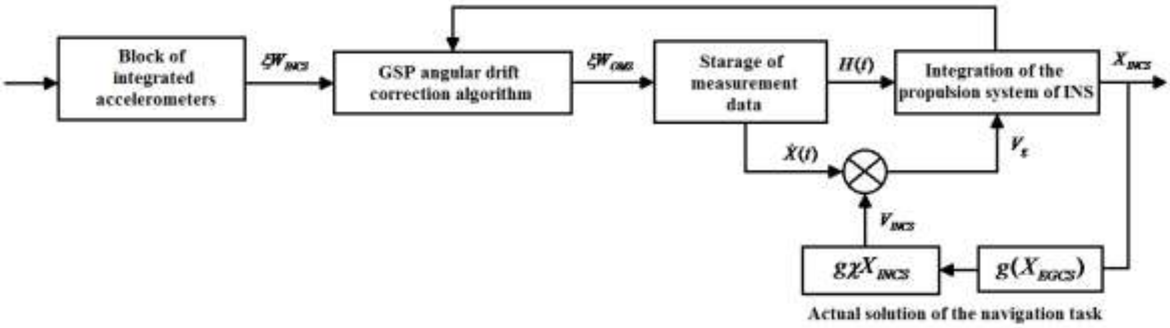

Fig. 4. A functional circuit for INS using GSP

Functionally INS may be considered to be composed of two blocks: a preprocessor and a navigational block [28]. Let us consider the algorithms for each of the blocks.

Preprocessing of navigational information

Measurement data $(\Delta W_{OMS}, \vartheta, \varphi, \gamma)$ coming to the input of preprocessor contains besides a valid signal a noise component, a model of which is shown above. The main objective of this block is to compensate the systematic component of the angular drift of GSP. In fact, the compensation algorithm determines the transition matrix from ONCS to INCS. Since some of the disturbing factors are random, it cannot completely compensate for the initial alignment errors and accelerometers construction errors [18]. The total drift of GSP axes on the segment $\left[t_i, t_{i+1}\right]$ with a drift model will be as follows [15, 31]:

$$\Delta\overline{\Omega}_i = \overline{\omega}\Delta t = \dot{\overline{\varepsilon}}_{CB}\Delta t + M_z \frac{1}{g}\Delta\overline{W}_i,$$

where $\dot{\overline{\varepsilon}}_{CB} = (\hat{\dot{\varepsilon}}_1, \hat{\dot{\varepsilon}}_2, \hat{\dot{\varepsilon}}_3)$ is a vector of mathematical expectations of errors in free drift of GSP axes.

These values are also accumulated in the onboard computer. As a result, after 10 cycles we will obtain $\sum\Delta\overline{\Omega} = \sum_{i=1}^{61}\Delta\overline{\Omega},$ and $\Delta\overline{W} - \sum_{i=1}^{61}\Delta\overline{W}.$

The actual solution of the navigation task comes down to integration of the following system of equations:

$$\left.\begin{array}{l} \dot{\Lambda} = \dfrac{1}{2}[F']\overline{\lambda} \\ \dot{V}_g = g(X), \\ \dot{X} = V_g + W_k \end{array}\right\}$$

where $\dot{\Lambda}$ is quaternion; V_g is a velocity component of the launch vehicle due to the influence of GSP; W_k is the apparent velocity of the launch vehicle.

The first equation describes the dynamics of the quaternion Λ which directly shows transfer of ONCS from INCS, where the matrix E^* is actually a fundamental matrix of the system of differential equations for the components of the quaternion:

$$E^* = \begin{vmatrix} 0 & \sum\Delta\Omega_z & \sum\Delta\Omega_y & \sum\Delta\Omega_x \\ -\sum\Delta\Omega_z & 0 & \sum\Delta\Omega_x & \sum\Delta\Omega_y \\ -\sum\Delta\Omega_y & -\sum\Delta\Omega_x & 0 & \sum\Delta\Omega_z \\ -\sum\Delta\Omega_x & -\sum\Delta\Omega_y & -\sum\Delta\Omega_z & 0 \end{vmatrix}$$

The solution of this equation is the vector $\overline{\lambda}$, which dimensions are 4x1, on which basis the transition matrix to INCS is built:

$$R_\Lambda = \begin{vmatrix} (\lambda_0^2+\lambda_1^2-\lambda_2^2-\lambda_3^2) & 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & 2(\lambda_1\lambda_3+\lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2+\lambda_0\lambda_3) & (\lambda_0^2+\lambda_2^2-\lambda_1^2-\lambda_3^2) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3-\lambda_0\lambda_2) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & (\lambda_0^2+\lambda_3^2-\lambda_1^2-\lambda_2^2) \end{vmatrix}$$

Next, on the basis of the position vector of the launch vehicle calculated in the previous cycle we may calculate $g(X)$. It is necessary to recalculate the position vector X of the launch vehicle in EGCS (the equivalent gyro coordinate system) [6]. This shall be done as follows:

1. The vector X shall be initially transferred to ICS. The transfer is done using standard rotation matrices with the following arguments:

$X_{ICS} = A \times X_{INCS}$,

$A^T = R_y\big(-(90+Az)\big)R_x(\varphi)R_z(\lambda-90)$,

where $Az = 108^0, \varphi = 38^0, \lambda = 56^0$.

2. Transfer from ICS to EGCS shall be made. ICS differs from EGCS by orientation of the axes x and y. So, transfer to EGCS may be made by one turn about the axis z:

$X_{EGCS} = B \times X_{ICS}$,

$B = R_z(S)$,

where $S = S_0 + \omega_\oplus(t-t_0)$ is an angle between the Vernal Equinox direction and the Greenwich meridian [32]; S_0 is an angle between the Vernal Equinox direction at the beginning of the epoch t_0; $\omega_\oplus$ is the angular velocity of rotation of the Earth; t is the current time.

Then using the position vector in EGCS we shall calculate the new value of the acceleration $g(X)$, and then perform the inverse coordinate transformations:

$g_{ICS} = B^T \times g_{EGCS}$,

$g_{ICS} = A^T \times g_{ICS}$.

Knowing the new value of acceleration $g(X)$, we are able to solve the second and the third equations of the system. As a result, we shall define a new position vector for the launch vehicle. Hereafter the algorithm of INS will be repeated.

Mathematical Model of INS of the Launch Vehicle with Strapdown Inertial Navigation System (SINS).

When SINS is used, the source of navigation information will be a block of integrating accelerometers oriented along the body axes of the launch vehicle, and the meter of absolute angular velocity comprising three high-speed gyros (rate gyros) oriented as well along three mutually perpendicular axes [33].

The models of accelerometers and rate gyros described below have fair number of common features, and will be used in particular in the modeling of the processes occurring in the model of the environment. Let's represent accelerometer errors occurring in SINS as follows [28]:

$$\begin{aligned} \Delta n_x &= \mu_{x1} + \mu_{x2} + n_x\mu_{x3} + n_y\mu_{x4} + n_z\mu_{x5}, \\ \Delta n_y &= \mu_{y1} + \mu_{y2} + n_x\mu_{y3} + n_y\mu_{y4} + n_z\mu_{y5}, \\ \Delta n_z &= \mu_{z1} + \mu_{z2} + n_x\mu_{z3} + n_y\mu_{z4} + n_z\mu_{z5}; \end{aligned} \qquad (1)$$

where $\mu_{i1}(i = x, y, z)$ are constant displacements of zero of accelerometers;

$\mu_{i2}(i = x, y, z)$ are accelerometers' measurement noise;

$\mu_{x3}, \mu_{y4}, \mu_{z5}$ are accelerometer scale factor errors;

$\mu_{x4}; \mu_{x5}; \mu_{y3}; \mu_{y5}; \mu_{z3}; \mu_{z4}$ are errors occurring due to nonorthogonality and misalignment of accelerometer response axes;

n_x, n_y, n_z are components of the acceleration vector in the associated coordinate system.

Rate gyros' errors $\Delta\omega_x, \Delta\omega_y, \Delta\omega_z$ may be represented as well as the following rather general model [15]:

$$\begin{aligned}
\Delta\omega_x &= \omega_{x1} + \omega_{x2} + n_x\omega_{x3} + n_y\omega_{x4} + n_z\omega_{x5} + \Omega_\xi\omega_{x6} + \Omega_\eta\omega_{x7} + \Omega_\varsigma\omega_{x8}, \\
\Delta\omega_y &= \omega_{y1} + \omega_{y2} + n_x\omega_{y3} + n_y\omega_{y4} + n_z\omega_{y5} + \Omega_\xi\omega_{y6} + \Omega_\eta\omega_{y7} + \Omega_\varsigma\omega_{y8}, \\
\Delta\omega_z &= \omega_{z1} + \omega_{z2} + n_x\omega_{z3} + n_y\omega_{z4} + n_z\omega_{z5} + \Omega_\xi\omega_{z6} + \Omega_\eta\omega_{z7} + \Omega_\varsigma\omega_{z8};
\end{aligned} \tag{2}$$

where $\omega_{ij}(i = x, y, z;\ j = 1{,}2)$ are drift invariables of rate gyros and their random measurement noises;

$\omega_{ij}(i = x, y, z;\ j = 3{,}4{,}5)$ are specific drift speeds of gyroscopes proportional to g-forces (causes for such dependence may be different for different types of rate gyros, for example, in mechanical gyroscopes such dependence can be explained by imbalance of rate gyros);

$\omega_{x6}, \omega_{y7}, \omega_{z8}$ are scale factor errors of rate gyros;

$\omega_{x7}, \omega_{x8}, \omega_{y6}, \omega_{y8}, \omega_{z6}, \omega_{z7}$ are drifts occurring due to nonorthogonality and misalignment of response axes of rate gyros;

$\Omega_\xi, \Omega_\eta, \Omega_\varsigma$ are projections of absolute angular velocity onto INCS axes.

Noise terms of accelerometer errors $\mu_{i2}(i = x, y, z)$ and rate gyros $\omega_{i2}(i = x, y, z)$ appear as stationary random processes with zero mathematical expectation and the following correlation functions [30, 34-36]:

$$K_n = \sigma_n^2 e^{-h_n|\tau|},$$

$$K_w = \sigma_w^2 e^{-h_w|\tau|};$$

where σ_n, σ_w are root-mean-square deviations (RMSD) of the variables μ_{i2}, ω_{i2} from their mean; h_n, h_w are attenuation coefficients of correlation functions for random errors of accelerometers and gyroscopes respectively.

As knowing, differential equations for generating filters for the mentioned random processes with stationary input signals such as white noise are as follows [37, 38]:

$$\dot{\mu}_{i2} = -h_n\mu_{i2} + \sqrt{2h_n}\sigma_n\varepsilon_{i1},$$

$$\dot{\omega}_{i2} = -h_w\mu_{i2} + \sqrt{2h_w}\sigma_w\varepsilon_{i2};$$

In the above referred models of errors occurring in rate gyros and accelerometers in different parts of the flight trajectory contribution of the individual components can vary greatly [28]. So when considering the movement of the launch vehicle at a speed close to constant one along straight trajectories, constant errors occurring in meters will have major influence [18]. Therefore, in such parts of the trajectory models (1)-(2) may be significantly simplified, which will facilitate solving of tasks set for the onboard measuring system. Furthermore, when time correlation coefficients h_n^{-1} and h_w^{-1} are relatively small when compared to the Schuler period ($T_{Sh} = 5064$ s), processes $\mu_{i2}, \omega_{i2}(i = x, y, z)$ approach to "white noise" with certain intensity. Taking this into consideration, we may present models of gyroscope and accelerometer errors as follows [39]:

$$\Delta n_i = \mu_i + Q_{i1} \cdot \varepsilon_{i1},$$

$$\Delta\omega_i = \upsilon_i + Q_{i2} \cdot \varepsilon_{i2};$$

where μ_i, υ_i are constant errors occurring in meters; Q_{i1}, Q_{i2} are intensities of random errors occurring in meters.

Thus, taking errors of inertial meters into consideration, measurement process of apparent acceleration and angular velocity taken from the accelerometers and rate gyros can be represented as follows. Values of the components of the apparent acceleration vector taken from the accelerometers shall be calculated according to the following equations:

$$\begin{cases} n_{SINS_x} = n_x + \Delta n_x, \\ n_{SINS_y} = n_y + \Delta n_y, \\ n_{SINS_z} = n_z + \Delta n_z. \end{cases}$$

Values of the components of angular velocity vector taken from rate gyros shall be calculated according to the following equations:

$$\begin{cases} \omega_{OMS_x} = \omega_x + \Delta\omega_x, \\ \omega_{OMS_y} = \omega_y + \Delta\omega_y, \\ \omega_{OMS_z} = \omega_z + \Delta\omega_z, \end{cases}$$

where $\Delta n_x, \Delta n_y, \Delta n_z, \Delta\omega_x, \Delta\omega_y, \Delta\omega_z$ are components of error vectors introduced herein above; $n_x, n_y, n_z, \omega_x, \omega_y, \omega_z$ are actual values of measured navigational parameters.

The Algorithm for Solving a Navigation Task when a Strapdown Inertial Navigation System (SINS) is Used. A functional diagram for SINS operation is presented in Fig. 5.

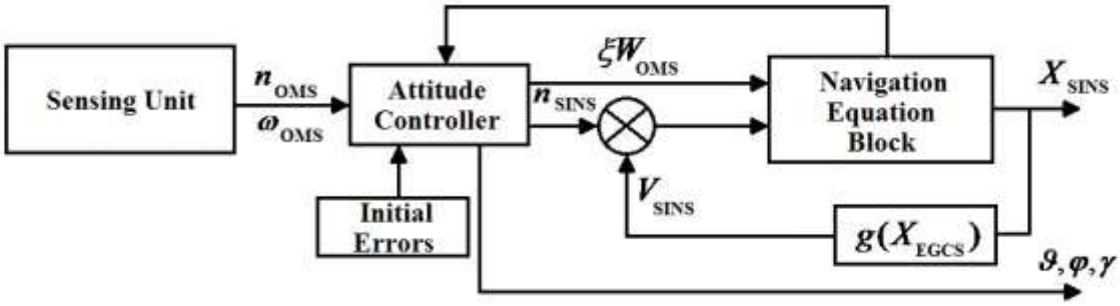

Fig. 5. A functional diagram for SINS operation

SINS start functioning a few seconds before liftoff and stops after cutoff of the last stage (at the end of the powered flight phase). As a rule, the navigation task is solved with the help of SINS installed aboard a launcher with a frequency of 1Hz.

Functionally, the algorithm for solving the navigation task using SINS may be splitted into two blocks:

- an integration block for navigation equations;
- a block for orientation finding.

As we mentioned earlier, a system of the following motion equations is solved once a second:

$$\dot{\Lambda} = \frac{1}{2}[E']\overline{\lambda};$$

$$X = \dot{V};$$

$$\dot{V} = n + g(X);$$

$$n_{IF} = Cn_{BO},$$

where Λ are Rodrigues-Hamilton parameters fully describing the orientation of the launch vehicle; n is a vector of apparent acceleration of the launch vehicle; $g(X)$ is a gravitational vector.

The first three equations of this system are essentially the same as the corresponding equations describing the algorithm for solving the task by using a gyro-stabilized platform.

The last equation, characteristic only for SINS, describes the process of "reprojection" of the measured components of acceleration on measuring axes using matrix C, which algorithm is given below.

Algorithm for determining the orientation of LV

1. Calculate item values for matrix C directional cosines between the reference frame attributed to the launch vehicle and the geographical reference frame:

$$C = \begin{vmatrix} (\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2) & 2(\lambda_1\lambda_2 - \lambda_0\lambda_3) & 2(\lambda_1\lambda_3 + \lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2 + \lambda_0\lambda_3) & (\lambda_0^2 + \lambda_2^2 - \lambda_1^2 - \lambda_3^2) & 2(\lambda_2\lambda_3 - \lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3 - \lambda_0\lambda_2) & 2(\lambda_2\lambda_3 - \lambda_0\lambda_1) & (\lambda_0^2 + \lambda_3^2 - \lambda_1^2 - \lambda_2^2) \end{vmatrix},$$

where $\lambda_i, i = 0,...,3$ are elements of the quaternion Λ.

2. Calculate the orientation parameters of the launch vehicle (pitch, roll and yaw):

$$\psi = \arcsin \frac{-C_{11}}{\sqrt{1-C_{31}^2}};$$

$$\psi = \arcsin \frac{-C_{33}}{\sqrt{1-C_{31}^2}}; \qquad (3)$$

$$\vartheta = \arcsin C_{31}.$$

3. Signals received from the accelerometers are translated into INCS to be used in the first navigation algorithm (the third equation of (3):

$$\begin{vmatrix} n_{INCS_X} \\ n_{INCS_Y} \\ n_{INCS_Z} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} \cdot \begin{vmatrix} n_{OMS_X} \\ n_{OMS_Y} \\ n_{OMS_Z} \end{vmatrix},$$

where $n_{OMS_{X\,i}}$ are projections of the apparent acceleration done by accelerometers.

The algorithm is used for the initial orientation of SINS on Earth. The initial orientation is carried out by method of vector adjustment from measurements of two non-collinear vectors by SINS (accelerometers, gyroscopes) of the vector of absolute angular velocity of launch vehicle rotation, gravity vector g [33, 40].

4. Algorithm for computing the matrix orientation between the basis connected with vehicle-bound coordinate system and INCS:

$$C = \begin{vmatrix} \left(\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2\right) & 2\left(\lambda_1\lambda_2 - \lambda_0\lambda_3\right) & 2\left(\lambda_1\lambda_3 + \lambda_0\lambda_2\right) \\ 2\left(\lambda_1\lambda_2 + \lambda_0\lambda_3\right) & \left(\lambda_0^2 + \lambda_2^2 - \lambda_1^2 - \lambda_3^2\right) & 2\left(\lambda_2\lambda_3 - \lambda_0\lambda_1\right) \\ 2\left(\lambda_1\lambda_3 - \lambda_0\lambda_2\right) & 2\left(\lambda_2\lambda_3 - \lambda_0\lambda_1\right) & \left(\lambda_0^2 + \lambda_3^2 - \lambda_1^2 - \lambda_2^2\right) \end{vmatrix}.$$

5. Algorithm for computing angular orientation parameters of the launch vehicle:

$$\psi = \arcsin \frac{-C_{11}}{\sqrt{1-C_{31}^2}};$$

$$\psi = \arcsin \frac{-C_{33}}{\sqrt{1-C_{31}^2}};$$

$$\vartheta = \arcsin C_{31}.$$

6. The algorithm for conversion of signals obtained with accelerometers in INCS to be used in the first navigation algorithm:

$$\begin{vmatrix} n_{INCS_X} \\ n_{INCS_Y} \\ n_{INCS_Z} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} \cdot \begin{vmatrix} n_{OMS_X} \\ n_{OMS_Y} \\ n_{OMS_Z} \end{vmatrix},$$

where n_{OMS_i} are projections of the apparent acceleration measured by accelerometers.

The results of simulation. Based on the conducted simulations have been obtained depending that presented in Fig.6-9.

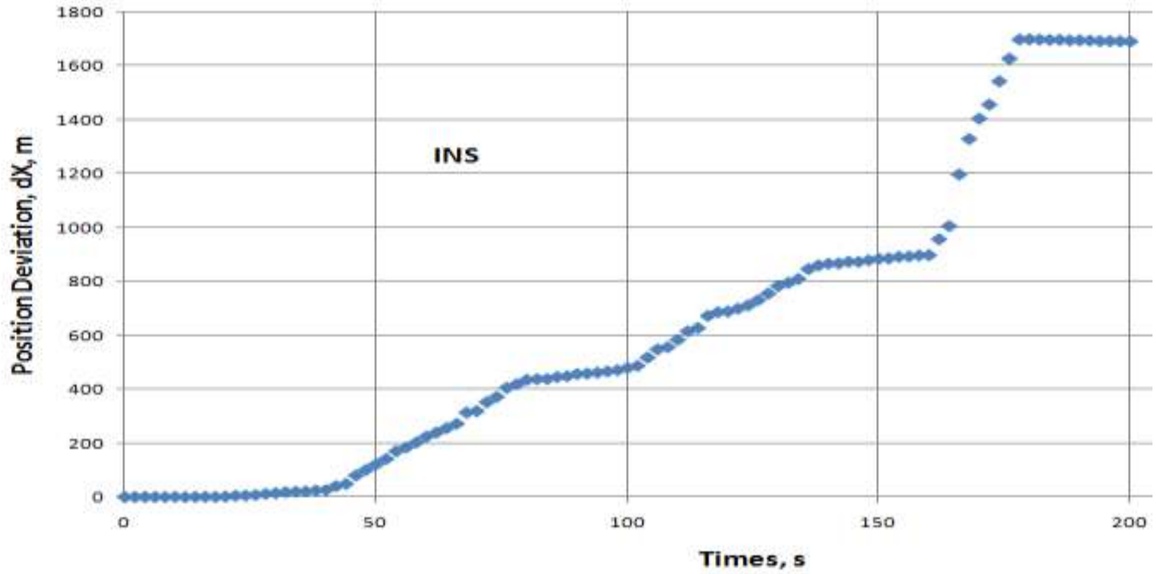

Fig. 6. Estimation error X coordinate

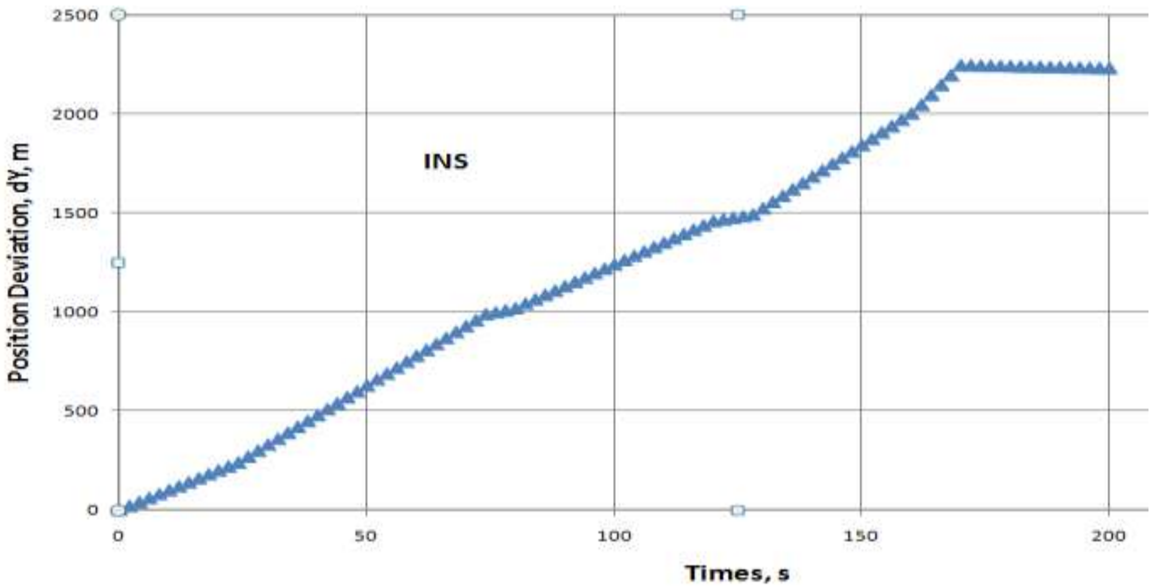

Fig. 7. Estimation error Y coordinate

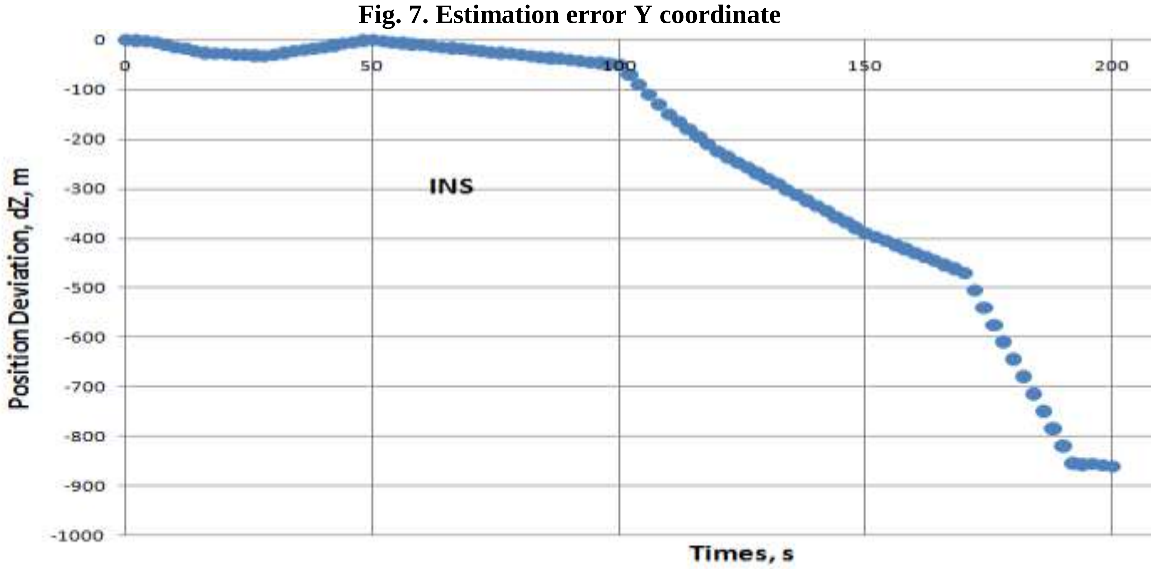

Fig. 8. Estimation error Z coordinate

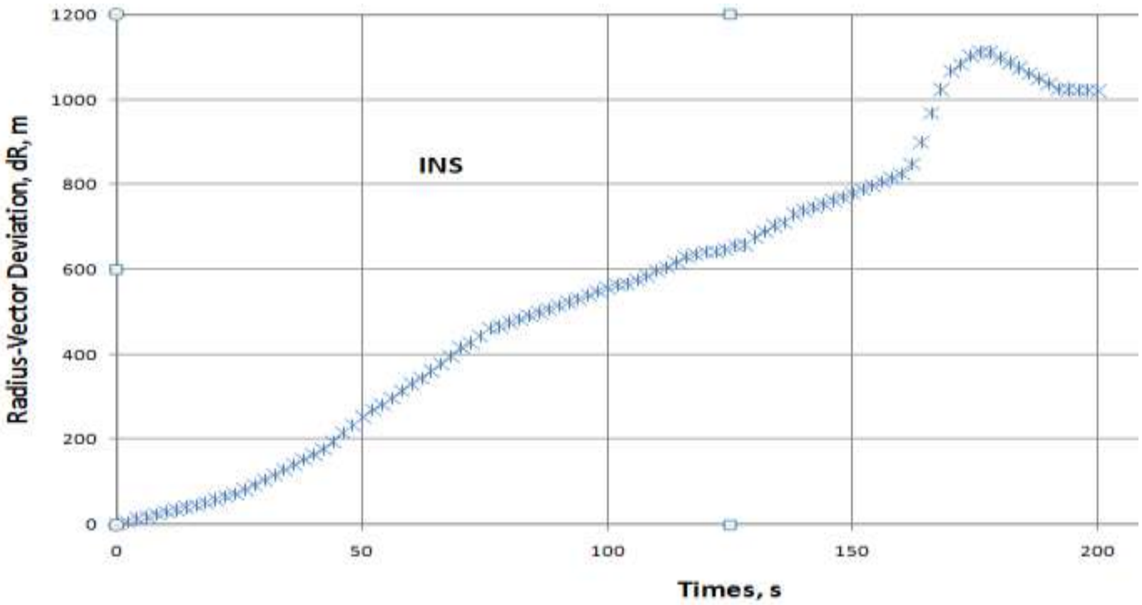

Fig. 9. Estimation error the radius vector

IV. CONCLUSION

1. We developed a system of mathematical models and algorithms providing both modeling of the process of functioning of the integrated navigation system and function simulation of the onboard navigation system of the launch vehicle itself. The system includes:
1) a model of controlled motion of the center of mass within 3 stages of the launch vehicle and relative to the center of mass, taking into account non-centrality of the gravitational field, variations in the density of the atmosphere, wind gusts, thrust distribution, assembly errors and thrust errors;
2) a model of a gyro-stabilized platform, taking into account various components of drift and accelerometer errors;
3) a model of strapdown inertial navigation system taking into account drift of gyro blocks and accelerometers' errors;
4) a model of a constellation of navigation spacecraft using GPS system taking into consideration visibility of the launch vehicle on the launch date;
5) a model of code measurements of GPS-receiver considering zenith errors, hour drift and the internal noise of the receiver;
6) algorithms for solving navigation tasks using GSP, OINS and GPS data;
7) integration algorithms within low and rigidly connected schemes; algorithms for stabilization and control of the 1^{st}, 2^{nd} and 3^{rd} stages of the launch vehicle.
2. Positioning errors occurring during a mission of a launch vehicle using INS monotonically increase in all the coordinates and reach considerable values at the end of the powered flight phase (error along the radius vector is ≈ 3.1 km).

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