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Pythagorean Triangle with Area/Perimater as a Special Polygonal Number

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***ABSTRACT**:- Patterns of Pythagorean triangles in each of which the ratio Area/Perimeter is represented by some polygonal number. A few interesting relations among the sides are also given.*

***KEYWORDS**:- Pythagorean triangles, Polygonal number*
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1. NOTATIONS

Special Numbers	***Notations***	***Definitions***
Regular Polygonal number	$t_{m,n}$	$n\left[1+\frac{(n-1)(m-2)}{2}\right]$
Gnomonic number	G_n	$2n-1$
Carol number	$carl_n$	$2^{2n}-2^{n+1}-1$
Star number	S_n	$6n(n-1)+1$
Pronic	P_n	$n(n+1)$

I. INTRODUCTION:

The method of obtaining non-zero integers x, y and z under certain conditions satisfying the relation $x^2+y^2=z^2$ has been a matter of interest to various Mathematicians [1,2,3].In[4-15] special Pythagorean problems are studied. In this communication, we present yet another interesting Pythagorean problem. That is, we search for patterns of Pythagorean triangles where, in each of which the ratio Area/Perimeter is represented by a special polygonal numbers namely, Icosihexagonal, Icosiheptagonal , Icosioctagonal, Icosinonagonal, and Triacontagonal number by the symbols, X,E,Q,N and T respectively. Also, a few relations among the sides are presented.

II.METHOD OF ANALYSIS

The most cited solution of the Pythagorean equation

$$x^2+y^2=z^2 \qquad (1)$$

is represented by

$$x=2rs, y=r^2-s^2, z=r^2+s^2 (r>s>0) \qquad (2)$$

Case 1:

Under our assumption

$$\frac{a}{p}=t_{26,X}$$

leads to the equation

$$s(r-s)=2X(12X-11)$$

This equation is equivalent to the following two systems I and II

$r-s$	s
$12X-11$	$2X$
$2X$	$12X-11$

In what follows , we obtain the values of the generators r, s and hence the corresponding sides of the Pythagorean triangle

Pattern 1:

On evaluation the values of the generators satisfying system I are

$$r = 14X - 11, s = 2X$$

Employing (2), the sides of the Pythagorean triangle are,

$$x = 56X^2 - 44X, y = 192X^2 - 308X + 121, z = 200X^2 - 308X + 121$$

Examples:

X	x	y	z	a	p
1	12	5	13	30	30
2	136	273	305	18564	714
3	372	925	997	172050	2294
4	720	1961	2089	705960	4770
5	1180	3381	3581	1994790	81420
6	1752	5185	5473	4542060	12410

Relations:

1.$7x + 24y - 25z + 121 = 0$

2.$(y + z - 7x - 242)^2 = 11858(z - y)$

3.$3x - y + z$ can be represented by 44 times Decgonal number of rank X

4.$8(y - z) \equiv 3 \pmod{28}$

5.$6(z^2 - x^2)$ is a nasty number.

Pattern 2:

On evaluation the values of the generators satisfying system II are

$$r = 14X - 11, s = 12X - 11$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 336X^2 - 576X + 242, y = 52X^2 - 44X, z = 340X^2 - 572X + 242$$

Examples:

X	x	y	z	a	p
1	6	8	10	24	24
2	442	120	458	26520	1020
3	1550	336	1586	260400	3472
4	3330	656	3394	1092240	7380
5	5782	1080	5882	3122280	12744
6	8906	1608	9050	7160424	19564

Relations:

1.$84x + 132y - 85z + 242 = 0$

2.$(x - 13y + z - 484)^2 = 81796(z - x)$

3.$y = 12t_{18,X} + 4t_{3,X-1}$

4.$13x + y - 13z \equiv 0 \pmod{44}$

5.$3(z - x)$ is a nasty number.

Case 2:

Under our assumption

$$\frac{a}{p} = t_{27,E}$$

leads to the equation

$$s(r - s) = E(25E - 23)$$

This equation is equivalent to the following two systems I and II

$r-s$	s
$25E-23$	E
E	$25E-23$

As in the previous case, we obtain the values of the generators r,s and hence the corresponding sides of the Pythagorean triangle

Pattern 3:

On evaluation the values of the generators satisfying system I are

$$r=26E-23, s=E$$

Employing (2) the corresponding Pythagorean triangle is

$$x=52E^2-46E, y=675E^2-1196E+529, z=677E^2-1196E+529$$

Examples:

X	x	y	z	a	p
1	6	8	10	24	24
2	116	837	845	48546	1798
3	330	3016	3034	497640	6380
4	648	6545	6577	2120580	13770
5	1070	11424	11474	6111840	23968
6	1596	17653	17725	14087094	36974

Relations:

1. $52x+675y-677z+1058=0$
2. $(y-26x+z-1058)^2=559682(z-y)$
3. $x-y+z\equiv 4t_{56,E}+4t_{3,E-1}(\mathrm{mod}\,6)$
4. $39\,\mathrm{x}-182\,\mathrm{y}+182\,\mathrm{z}=1196\,\mathrm{t}_{10,E}$
5. $x+3y-3z=92t_{3,E-1}$
6. $6(z^2-x^2)$ is a nasty number.

Pattern 4:

On evaluation the values of the generators satisfying system II are

$$r=26E-23, s=25E-23$$

Employing (2) the corresponding Pythagorean triangle is

$$x=1300E^2-2346E+1058, y=51E^2-46E, z=1301E^2-2346E+1058$$

Examples:

E	x	y	z	a	p
1	12	5	13	30	30
2	1566	112	1570	87696	3248
3	5720	321	5729	918060	11770
4	12474	632	12490	3941784	25596
5	21828	1045	21853	11405130	44726
6	33782	1560	31818	26349960	69160

Relations:

1. $1300x+51y-1301z+1058=0$
2. $(x-51y+z-2116)^2=5503716(z-x)$
3. $x-102y+z\equiv -2E^2(\mathrm{mod}\,23)$
4. $\mathrm{y}-46\,\mathrm{P}_{E-1}\equiv 0(\mathrm{mod}\,5)$

5. $x-216S_E \equiv 2(E+1)(\text{mod } 2^2)$

Case 3:

Under our assumption

$$\frac{a}{p} = t_{28,Q}$$

leads to the equation

$$s(r-s) = 2Q(13Q-12)$$

This equation is equivalent to the following two systems I and II

$r-s$	s
$13Q-12$	$2Q$
$2Q$	$13Q-12$

As in the previous case, we obtain the values of the generators r,s and hence the corresponding sides of the Pythagorean triangle

Pattern 5:

On evaluation the values of the generators satisfying system I are

$$r = 15Q-12, s = 2Q$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 60Q^2 - 48Q, y = 221Q^2 - 360Q + 144, z = 229Q^2 - 360Q + 144$$

Examples:

Q	x	y	z	a	p
1	12	5	13	30	30
2	144	308	340	22176	762
3	396	1053	1125	208494	2574
4	768	2240	2368	860160	5376
5	1260	3869	4069	2437470	9198
6	1872	5940	6228	5559840	14040

Relations:

1. $60x + 221y - 229z + 1152 = 0$

2. $(6(z-y) - 45x - 1728)^2 = 583200(z-y)$

3. $2x + 3y - 3z = 192t_{3,Q-1}$

4. $15z - 15y - 2x \equiv 0(\text{modQ})$

5. $50y - 48z - t_{52,Q} \equiv 0(\text{mod } 96)$

6. $2\left[x - 96t_{3,Q-1}\right]$ is a nasty number.

Pattern 6:

On evaluation the values of the generators satisfying system II are

$$r = 15Q-12, s = 13Q-12$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 390Q^2 - 672Q + 288, y = 56Q^2 - 48Q, z = 394Q^2 - 672Q + 288$$

Examples:

Q	x	y	z	a	p
1	6	8	10	24	24
2	504	128	520	32256	1152
3	1782	360	1818	320760	3960
4	3840	704	3904	1351680	8448

5	6678	1160	6778	3873240	14616
6	10296	1728	10440	8895744	22464

Relations:

1. $390x + 56y - 394z + 1152 = 0$
2. $(x - 14y + z - 576)^2 = 112896(z - x)$
3. $14x + y - 14z \equiv 0 \pmod{48}$
4. $z^2 - x^2 = 64Q^2 t_{16,Q}$
5. y can be represented by 8 times hexadecagonal number of rank Q

Case 4:

Under our assumption

$$\frac{a}{p} = t_{29,N}$$

leads to the equation

$$s(r - s) = N(27N - 25)$$

This equation is equivalent to the following two systems I and II

$r - s$	s
$27N - 25$	N
N	$27N - 25$

As in the previous case, we obtain the values of the generators r, s and hence the corresponding sides of the Pythagorean triangle

Pattern 7:

On evaluation the values of the generators satisfying system I are

$$r = 28N - 25, s = N$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 56N^2 - 50N, y = 783N^2 - 1400N + 625, z = 785N^2 - 1400N + 625$$

Examples:

N	x	y	z	a	p
1	6	8	10	24	24
2	124	957	965	59334	2046
3	354	3472	3490	61454	7316
4	696	7553	7585	2628444	15834
5	1150	13200	13250	7590000	27600
6	1716	20413	20485	17514354	42614

Relations:

1. $56x + 783y - 785z + 1250 = 0$
2. $(y - 28x + z - 1250)^2 = 980000(z - y)$
3. $x + 28y - 28z \equiv 0 \pmod{50}$
4. $6x - 7y + 7z = 100t_{16,N}$
5. $785y - 783z + 1375G_N + 125 = 0$
6. $x(2^n) - 25 \equiv 25carl_n \pmod{31}$

Pattern 8:

On evaluation the values of the generators satisfying system II are

$$r = 28N - 25, s = 27N - 25$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 1512N^2 - 2750N + 1250, y = 55N^2 - 50N, z = 1513N^2 - 2750 + 1250$$

Examples:

N	x	y	z	a	p
1	12	5	13	30	30
2	1798	120	1802	107880	3720
3	6608	345	6617	1139882	13570
4	14442	680	14458	4910280	29580
5	25300	1125	25325	142324250	51750
6	39182	1680	39218	32912880	80080

Relations:

1. $1512x + 55y - 1513z + 1250 = 0$
2. $(x - 110y + z - 2500)^2 = 7562500(z - x)$
3. $55x + y - 55z \equiv 0 \pmod 5$
4. $z - 1250 \equiv 135t_{24,N} + t_{58,N} \pmod{1373}$
5. y can be represented by 5 times Icositetragonal number of rank N .

Case 5:

Under our assumption

$$\frac{a}{p} = t_{30,T}$$

leads to the equation

$$s(r - s) = 2T(14T - 13)$$

This equation is equivalent to the following two systems I and II

$r - s$	s
$14T - 13$	$2T$
$2T$	$14T - 13$

As in the previous case, we obtain the values of the generators r, s and hence the corresponding sides of the Pythagorean triangle

Pattern 9:

On evaluation the values of the generators satisfying system I are

$$r = 16T - 13, s = 2T$$

Employing (2) the corresponding Pythagorean triangle is

$$x = 56T^2 - 52T, y = 192T^2 - 364T + 169, z = 200T^2 - 364T + 169$$

Examples:

T	x	y	z	a	p
1	12	5	13	30	30
2	152	345	377	26220	874
3	420	1189	1261	249690	2870
4	816	2537	2665	1035096	6018
5	1340	4389	4589	2940630	10318
6	1992	6745	7033	6718020	15770

Relations:

1. $14x + 48y - 50z + 338 = 0$
2. $(y - 7x + z - 338)^2 = 16562(z - y)$
3. $7x - 3y + 3z = 52t_{18,T}$

$4.2x + y - z = 208t_{3,T-1}$

$5.x + 32y - 31z \equiv 0(\text{mod}13)$

Pattern10:

On evaluation the values of the generators satisfying system II are

$r = 16T - 13, s = 14T - 13$

Employing (2) the corresponding Pythagorean triangle is

$$x = 448T^2 - 780T + 338, y = 60T^2 - 52T, z = 452T^2 - 780T + 338$$

Examples:

T	x	y	z	a	p
1	6	8	10	24	24
2	570	136	586	38760	1292
3	2030	384	2066	389760	4480
4	4386	752	4450	1649136	9588
5	7638	1240	7738	4735560	16616
6	11786	1848	11930	10890264	25564

Relations:

$1.112x + 15y - 113z + 338 = 0$

$2.4y - 5x + 5z = 52t_{22,T}$

$3.(x - 15y + z - 780)^2 = 151200(z - x)$

$4.226x + 15y - 225z + 338 = 0$

$5.6(z^2 - x^2)$ is a nasty number.

CONCLUSION

To conclude one may search for other patterns of Pythagorean triangles under consideration.

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