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A New Class of Contra Continuous Functions in Topological Spaces

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ABSTRACT:- *In this paper ,we introduce and investigate the notion of contra πgr-continuous,almost contra πgr-continuous functions and discussed the relationship with other contra continuous functions and obtained their characteristics.*

Keywords:- *Contra πgr-continuous, almost contra πgr-continuous, πgr-locally indiscrete,* $T_{\pi gr}$ *-space.*
AMS Subject Classification:- 54C08,54C10

I. INTRODUCTION

Generalized closed sets in a topological space were introduced by Levine[11] in 1970. N.Palaniappan[13,14] introduced regular generalized closed sets and regular generalized star closed sets.The concept of regular continuous functions was first introduced by Arya.S.P and Gupta.R [1]in the year 1974.Dontchev[2] introduced the notion of contra continuous functions in 1996. Jafari and Noiri[7] introduced contra pre-continuous functions. Ekici.E[4] introduced almost contra pre-continuous functions in 2004.The notion of contra πg-continuity was introduced by Ekici.E [5]in 2008.Jeyanthi.V and Janaki.C[9] introduced the notion of πgr-closed sets in topological spaces in 2012.

In this paper , the notion of contra πgr-continuity which is a stronger form of contra πg-continuity and their characterizations are introduced and investigated. Further,the notion of almost contra πgr-continuity is introduced and its properties are discussed.

II. PRELIMINARIES

In the present paper, the spaces X and Y always mean topological spaces (X,τ) and (Y,σ) respectively.For a subset A of a space , cl(A) and int(A) represent the closure of A and interior of A respectively.

Definition:2.1

A subset A of X is said to be regular open [13]if A=int(cl(A)) and its complement is regular closed. The finite union of regular open set is π-open set[21] and its complement is π-closed set.The union of all regular open sets contained in A is called rint(A)[regular interior of A] and the intersection of regular closed sets containing A is called rcl(A)[regular closure of A]

Definition:2.2

A subset A of X is called

1. gr -closed[12,14] if rcl(A) ⊂U whenever A⊂U and U is open.
2. πgr-closed[9] if rcl(A) ⊂U whenever A⊂U and U is π-open.

Definition:2.3

A function f: (X,τ)→(Y,σ) is called πgr-continuous[9] if $f^{-1}(V)$ is πgr-closed in X for every closed set V in Y.

Definition :2.4

A function f: (X,τ)→(Y,σ) is called

(i)Contra continuous[2] if $f^{-1}(V)$ is closed in X for each open set V of Y.
(ii)Contra πg-continuous[5] if $f^{-1}(V)$ is πg-closed in X for each open set V of Y .
(iii)Contra πgα-continuous [8] if $f^{-1}(V)$ is πgα-closed in X for each open set V of Y.
(iv)Contra πgb-continuous[18] if $f^{-1}(V)$ is πgb-closed in X for each open set V of Y.
(v)Contra π*g-continuous[6] if $f^{-1}(V)$ is π*g-closed in X for each open set V of Y.
(vi)Contra gr-continuous[12] if $f^{-1}(V)$ is gr-closed in X for each open set V of Y.
(vii) RC-continuous[5] if $f^{-1}(V)$ is regular closed in X for each open set V of Y.
(viii)An R-map [5]if $f^{-1}(V)$ is regular closed in X for every regular closed set V of Y.
(ix)Perfectly continuous [4]if $f^{-1}(V)$ is clopen in X for every open set V of Y.
(x)rc-preserving [5]if f(U) is regular closed in Y for every regular closed set U of X.

(xi)A function f: X→Y is called regular set connected [5]if $f^{-1}(V)$ is clopen in X for every
(xii)Contra R-map[5] if $f^{-1}(V)$ is regular closed in X for each regular open set V of Y.
(xiii)Almost continuous[15] if $f^{-1}(V)$ is closed in X for every regular closed set V of Y.

Definition :2.5

A space (X,τ) is called
(i)a πgr-$T_{1/2}$ space [8]if every πgr-closed set is regular closed.
(ii)locally indiscrete[20] if every open subset of X is closed.
(iii)Weakly Hausdorff [17] if each element of X is an intersection of regular closed sets.
(iv)Ultra hausdorff space[19], if for every pair of distinct point x and y in X, there exist clopensets U and V in X containing x and y respectively.
(v)Hyper connected[20] if every open set is dense.

Definition : 2.6

A collection {Ai; i ∈Λ} of open sets in a topological space X is called open cover [16] of a subset B of X if B⊂ ∪ {Ai ; i ∈Λ } holds.

Definition : 2.7

A collection {Ai; i ∈Λ} of πgr-open sets in a topological space X is called πgr-open cover [10] of a subset B of X if B⊂ ∪ {Ai ; i ∈Λ } holds.

Definition:2.8

A space X is called πgr-connected [10] provided that X is not the union of two disjoint non-empty πgr-open sets.

Definition:2.9[5]

Let S be a closed subset of X. The set ∩{U∈τ/S⊂U} is called the kernel of S and is denoted by Ker(S)

III. CONTRA πGR-CONTINUOUS FUNCTION.

Definition:3.1

A function f: (X,τ)→(Y,σ) is called Contra πgr-continuous if $f^{-1}(V)$ is πgr-closed in (X,τ) for each open set V of (Y,σ).

Definition:3.2

A space (X,τ) is called
(i) πgr-locally indiscrete if every πgr-open set is closed.
(ii) $T_{\pi gr}$ -space if every πgr-closed is gr-closed.

Result:3.3

Contra Continuous and contra πgr-continuous are independent concepts.

Example:3.4

a) Let X = {a,b,c,d} = Y, τ = {φ,X,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d}}, σ = {φ ,Y,{ c }}.Let f:X→Y be an identity map. Here the inverse image of the element c in the open set of Y is closed in X but not πgr-closed in X.Hence f is contra continuous and not contra πgr-continuous.
b)Let X={a,b,c,d}=Y, τ ={φ,X,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d}},σ ={φ ,Y, {d}, {a,d}}. Let f:X→Y be an identity map. Here the inverse image of the elements in the open set of Y are πgr-closed in X but not closed in X.Hence f is contra πgr-continuous and not contra continuous.
Hence contra continuity and contra πgr-continuity are independent concepts.

Theorem:3.5

Every RC-continuous function is contra πgr-continuous but not conversely.

Proof: Straight Forward.

Example:3.6

Let X={a,b,c,d}=Y, τ = {φ,X,{a},{c,d},{a,c,d}},σ ={φ,Y,{a},{a,b}} Let f:X→Y be defined by f(a)=b,f(b)=a,f(c)=c,f(d)=d.The inverse image of the element in the open set of Y is πgr-closed in X but not regular closed in X.Hence f is contra πgr- continuous and not RC-continuous.

Theorem:3.7

Every Contra gr-continuous function is contra πgr-continuous but not conversely.

Proof: Follows from the definition.

Example: 3.8

Let X= {a,b,c,d},τ ={φ, X,{c},{d},{c,d},{b,d},{a,c,d},{b,c,d}}, σ ={φ,Y,{a}, {a,d}}. The inverse image of the element {a,d} in the open set of Y is πgr-closed in X but not gr-closed .Hence f is contra πgr- continuous and not contra gr-continuous.

Theorem:3.9

Every contra πgr-continuous function is contra πg- continuous,contra π*g- continuous , contra πgα- continuous and contra πgb- continuous but not conversely.

Proof: Straight Forward.

Example:3.10

a)Let X= {a,b,c,d}, τ ={φ, X, {c},{d},{c,d},{b,d},{a,c,d},{b,c,d }}, σ={φ,Y,{b}}. Here the inverse image of the element {b} in the open set of (Y,σ) is πg-closed in X, but not πgr-closed in X. Hence f is contra πg-continuous and not contra πgr-continuous.

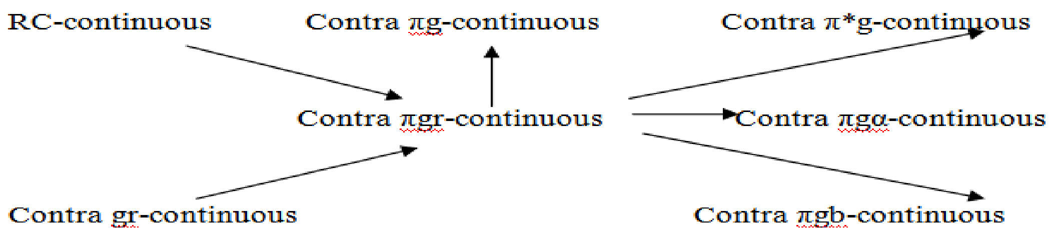
b)Let X= {a,b,c,d}, τ = {φ, X, {c},{d},{c,d},{b,d},{a,c,d},{b,c,d}}, σ={φ,Y,{b}}. Here the inverse image of the element {b} in the open set of (Y,σ) is π*g-closed in X, but not πgr-closed in X. Hence f is contra π*g-continuous and not contra πgr-continuous.

c)Let X={a,b,c,d}=Y, τ = {φ,X,{a},{c,d},{a,c,d}}, σ ={φ,Y,{a},{a,b,c }} . Let f:X→Y be an identity map.The inverse image of the element {a}in the open set (Y,σ) is πgb-closed but not πgr-closed.Hence f is contra πgb-continuous and not contra πgr-continuous.

d)Let X={a,b,c,d}=Y, τ = {φ,X,{a},{b},{a,b},{b,c},{a,b,c}},σ ={φ,Y,{c},{d}, {c, d }} .Let f:X→Y be an identity map.The inverse image of all the elements in Y are πgα-closed but not πgr-closed.Hence f is contra πgα-continuous and not contra πgr-continuous.

Remark:3.11

The above relations are summarized in the following diagram.



Theorem:3.12

Suppose πgrO(X,τ) is closed under arbitrary unions. Then the following are equivalent for a function f: (X,τ)→(Y,σ):

1. f is contra πgr-continuous.
2. For every closed subset F of Y, f^{-1}(F) ∈πgrO(X,τ)
3. For each x∈X and each F∈C(Y,f(x)), there exists a set U∈πGRO(X,x) such that f (U) ⊂ F .

Proof:(1) ⟺(2):Let f is contra πgr-continuous. Then f^{-1}(V) is πgr-closed in X for every open set V of Y.(i.e) f^{-1}(F) is πgr-open in X for every closed set F of Y . Hence f^{-1}∈ πgrO(X).

(2)⇒(1) :Obvious.

(2)⇒(3) :For every closed subset F of Y, f^{-1}(F)∈ πGRO(X). Then for each x∈X and each F∈C(Y,f(x)),there exists a set U ∈πGRO(X,x) such that f(U)⊂F.

(3)⇒(2) : For each x∈X ,F∈C(Y,f(x)), there exists a set U_x ∈πGRO(X,x) such that f(U_x)⊂F. Let F be a closed set of Y and x∈f^{-1}(F). Then f(x)∈F,there exist U∈ πGRO(X,x) such that f(U)⊂F . Therefore, f^{-1}(F)=∪{U_x:x∈f^{-1}(F)} .Hence f^{-1}(F) is πgr-open.

Theorem:3.13

If f:X→Y is contra πgr-continuous and U is open in X. Then f/U : (U,τ) →(Y,σ) is contra πgr-continuous.

Proof: Let V be any closed set in Y .Since f: (X,τ) →(Y,σ) is contra πgr-continuous, f^{-1}(V) is πgr-open in X, $(f/U)^{-1}$(V) = f^{-1}(V)∩U is contra πgr-open in X. Hence f($(f/U)^{-1}$(V)) is πgr-open in U.

Theorem:3.14

If a function f: (X,τ) →(Y,σ) is πgr-continuous and the space (X,τ) is πgr-locally indiscrete ,then f is contra continuous.

Proof: Let V be a open set in (Y,σ). Since f is πgr-continuous, f^{-1}(V) is open in X. Since X is locally πgr-indiscrete ,f^{-1}(V) is closed in X.Hence f is contra continuous.

Theorem:3.15

If a function f:X→Y is contra πgr-continuous, X is a πgr- $T_{1/2}$- space, then f is RC- continuous.

Proof: Let V be open in Y. Since f is contra πgr-continuous, f^{-1}(V) is πgr-closed in X.Since X is a πgr-$T_{1/2}$ -space, f^{-1}(V) is regular-closed in X.Thus for the open set V of Y, f^{-1}(V) is regular closed in X. Hence f is RC-continuous.

Theorem:3.16

I f a function f: (X,τ)→(Y,σ) is contra πgr-continuous, rc-preserving surjection and if X is a πgr - $T_{1/2}$-space, then Y is locally indiscrete.

Proof: Let V be open in Y. Since f is contra πgr-continuous,$f^{-1}(V)$ is πgr- closed in X. Since X is a πgr-$T_{1/2}$ -space ,$f^{-1}(V)$ is regular closed in X. Since f is rc-preserving surjection, $f(f^{-1}(V)) = V$ is regular closed in Y. Thus $cl(V) = cl(int(V)) \subset cl(int(cl(V)))\subset V$.Hence V is closed in Y.Therefore, Y is locally indiscrete.

Theorem:3.17

If a function f: (X,τ)→(Y,σ) is contra πgr-continuous and X is a πgr-space, then f:(X,τ)→(Y,σ) is contra gr-continuous.

Proof: Let V be an open set in Y. Since f is contra πgr-continuous, $f^{-1}(V)$ is πgr-closed in X. Since X is a $T_{\pi gr}$ -space, $f^{-1}(V)$ is gr-closed in X.Thus for every open set V of Y, $f^{-1}(V)$ is gr-closed.Hence f is contra gr-continuous.

Theorem:3.18

Suppose πGRO(X,τ) is closed under arbitrary unions,let f: X→Y be a function and $\{U_i : i\in I=1,2,,,,\}$ be a cover of X such that $U_i\in$ πGRC(X,τ) and regular open for each i∈I.If $f/U_i :(U_i, \tau/U_i)\to(Y,\sigma)$ is contra πgr-continuous for each i∈I, then f is contra πgr-continuous.

Proof: Suppose that F is any closed set of Y. We have $f^{-1}(F) = \cup\{f^{-1}(F) \cap U_i : i\in I\}= \cup\{(f/U_i)^{-1}(F) : i\in I\}$.Since (f/U_i) is contra πgr-continuous for each i∈I, it follows that$(f/U_i)^{-1}(F) \in \pi GRO(U_i)$. $\Rightarrow (f/U_i)^{-1} (F) \in \pi GRO(X)$ and hence f is contra πgr-continuous.

Theorem:3.19

Suppose πGRO(X,τ) is closed under arbitrary unions. If f:X→Y is contra πgr-continuous if Y is regular, then f is πgr-continuous.

Proof: Let x be an arbitrary point of X and V be an open set of Y containing f(x). The regularity of Y implies that there exists an open set W in Y containing f(x) such that cl(W) ⊂V. Since f is contra πgr-continuous, then there exists U ∈πGRO(X,x) such that f(U)⊂cl(W). Then f(U)⊂cl(W)⊂V.Hence f is πgr-continuous.

Theorem:3.20

Suppose that πGRC(X) is closed under arbitrary intersections . Then the following are equivalent for a function f: X→Y.

1)f is contra πgr-continuous.

2)The inverse image of every closed set of Y is πgr-open.

3)For each x∈X and each closed set B in Y with f(x)∈B,there exists a πgr-open set A in X such that x∈A and f(A)⊂B.

4)f(πgr-cl(A))⊂Ker f(A) for every subset A of X.

5)πgr-cl(f^{-1}(B))⊂f^{-1}(Ker(B)) for every subset B of Y.

Proof:

(1)⇒(2) and (2)⇒(1) are obviously true.

(1)⇒(3):Let x∈X and B be a closed set in Y with f(x)∈B. By (1), it follows that $f^{-1}(Y-B) = X- f^{-1}(B)$ is πgr-closed and so $f^{-1}(B)$ is πgr-open.

Take A= $f^{-1}(B)$. We obtain that x∈A and f(A)⊂B

(3)⇒(2):Let B be a closed set in Y with x∈$f^{-1}(B)$. Since f(x)∈B, by (3), there exists a πgr-open set A in X containing x such that f(A) ⊂B. It follows that x∈A⊂$f^{-1}(B)$.Hence $f^{-1}(B)$ is πgr-open.

(2)⇒(1):Obvious.

(2) ⇒(4): Let A be any subset of X. Let y∉Ker f(A). Then there exists a closed set F containing y such that f(A)∩F=φ.Hence , we have A∩$f^{-1}(F)$=φ. πgr-cl(A)∩$f^{-1}(F)$=φ. Thus f(πgr-cl(A))⊂F=φ and y∉f(πgr-cl(A)) and hence f(πgr-cl(A))⊂Ker f(A)

(4)⇒(5): Let B be any subset of Y. By (4), f(πgr-cl(f^{-1}(B))⊂Ker B and πgr-cl(f^{-1}(B))⊂f^{-1}(ker B).

(5)⇒(1):Let B be any open set of Y. By (5), πgr-cl(f^{-1}(B))⊂f^{-1}(KerB)=f^{-1}(B)

πgr-cl(f^{-1}(B)) = f^{-1}(B),We obtain f^{-1}(B) is πgr-closed in X.

Hence f is contra πgr-continuous.

IV. ALMOST CONTRA πGR-CONTINUOUS FUNCTIONS.

Definition:4.1

A function f:X→Y is said to be almost contra continuous [4]if $f^{-1}(V)$ is closed in X for each regular open set V of Y.

Definition:4.2

A function f:X→Y is said to be almost contra πgr-continuous if $f^{-1}(V)$ is πgr-closed in X for each regular open set V of Y.

Definition:4.3

A topological space X is said to be πgr- T_1- space if for any pair of distinct points x and y , there exists a πgr-open sets G and H such that x∈G, y∉G and x∉H , y∈H.

Definition:4.4

A topological space X is said to be πgr-T_2-space if for any pair of distinct points x and y , there exists disjoint πgr-open sets G and H such that x∈G and y∈H.

Definition:4.5

A topological space X is said to be πgr-Normal if each pair of disjoint closed sets can be separated by disjoint πgr-open sets.

Definition:4.32

A function f :X→Y is called Weakly πgr-continuous if for each x∈X and each open set V of Y containing f(x), there exists U∈ πgrO(X,x) such that f(U)⊂cl(V).

Definition:4.7

A space X is said to be

1.πgr-compact[10] if every πgr-open cover of X has a finite sub-cover.
2.Nearly compact[16] if every regular open cover has a finite subcover.
3.Nearly lindelof [16] if every regular open cover of X has a countable subcover.
4.S-lindelof [4]if every cover of X by regular closed sets has a countable subcover.
5.S-closed[3] if every regular closed cover of X has a finite subcover.

Definition:4.8

A space X is said to be

1. πgr- lindelof if every πgr-open cover of X has a countable subcover.
2.Mildly πgr-compact if every πgr-clopen cover of X has a finite subcover.
3.Mildly πgr-lindelof if every πgr-clopen cover of X has a countable subcover.
4.Countably πgr-compact if every countable cover of X by πgr-open sets has a finite subcover.

Theorem:4.9

Suppose πgr-open set of X is closed under arbitrary unions. The following statements are equivalent for a function f: X→Y.

(1)f is almost contra πgr- continuous.
(2)$f^{-1}(F)$ ∈πGRO(X,τ) for every F∈RC(Y).
(3)For each x∈X and each regular closed set F in Y containing f(x), there exists a πgr-open set U in X containing x such that f(U) ⊂F.
(4)For each x∈X and each regular open set V in Y not containing f(x), there exists a πgr-closed set K in X not containing x such that $f^{-1}(V)$ ⊂K.
(5)f^{-1}(int (cl(G))∈ πGRC(X,τ) for every open subset G of Y.
(6)f^{-1}(cl(int(F))))∈ πGRO(X,τ) for every closed subset F of Y.

Proof:(1)⇒(2):Let F∈RC(Y,σ). Then Y−F∈RO(Y,σ).Since f is almost contra πgr-continuous,f^{-1}(Y−F) = X− $f^{-1}(F)$ ∈πGRC(X).Hence $f^{-1}(F)$ ∈πGRO(X).

(2)⇒(1):Let V∈RO(Y,σ).Then Y−V∈RC(Y,σ). Since for each F∈RC(Y,σ), $f^{-1}(F)$ ∈ πGRO(X).
⇒f^{-1}(Y−V)= X−$f^{-1}(V)$ ∈πGRO(X)
⇒$f^{-1}(V)$ ∈πGRC(X)
⇒ f is almost contra πgr-continuous.

(2)⇒(3):Let F be any regular closed set in Y containing f(x). $f^{-1}(F)$ ∈πGRO(X,τ), x∈$f^{-1}(F)$.By Taking U= $f^{-1}(F)$, f(U)⊂F.

(3)⇒(2):Let F ∈RC(Y,σ) and x∈$f^{-1}(F)$. From (3), there exists a πgr-open set U in X containing x such that U ⊂ $f^{-1}(F)$. We have $f^{-1}(F)$ = ∪{U: x∈$f^{-1}(F)$}.Thus $f^{-1}(F)$ is πgr-open.

(3)⇒(4):Let X be a regular open set in Y not containing f(x). Then Y−V is a regular closed set containing f(x). By (3), there exists a πgr-open set U in X containing x such that f(U) ⊂Y−V. Hence U⊂f^{-1}(Y−V)⊂X−$f^{-1}(V)$. Then $f^{-1}(V)$ ⊂X−U.

Take K= X−U. We obtain a πgr-closed set K in X not containing x such that $f^{-1}(V)$ ⊂K.

(4)⇒(3)Let F be a regular closed set in Y containing f(x). Then Y−F is a regular open set in Y containing f(x). By (4), there exists a πgr-closed set K in X not containing x such that f^{-1}(Y−F)⊂K, X−$f^{-1}(F)$⊂K. Hence X−K ⊂$f^{-1}(F)$. Hence f(X−K) ⊂F.Take U= X−K, f(U) ⊂F.Then U is a πgr-open set in X containing x such that f(U)⊂F

(1)⇒ (5):L et G be an open subset of Y. Since in(cl(G)) is regular open, then by (1),
f^{-1}(int(cl(G))∈πGRC(X,τ)
⇒ f is almost contra πgr-continuous.

(5)⇒(1):Let V∈RO(Y,σ). Then V is open in X. By (5), f^{-1}(int (cl(V)) ∈πGRC(X,τ)
⇒$f^{-1}(V)$∈πGRC(X,τ)
⇒ f is almost contra πgr-continuous.

(2)⇔(6) is similar to (1) ⇔ (5)

Theorem:4.10
Every contra πgr- continuous function is almost contra πgr-continuous but not conversely.
Proof: Straight forward.
Example:4.11
Let X = {a,b,c,d}, τ= {φ, X,{a},{c,d},{a,c,d}}, πgr-closed set={φ, X ,{b} {a,b},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d}}.Let Y= {a,b,c,d}, σ={ φ,Y,{a}, {a,b}}. Let f be an identity map.The inverse image of open set in Y is not πgr-closed in X. But the inverse image of regular open set in Y is πgr-closed in X.Hence f is almost contra πgr-continuous and not contra πgr-continuous .
Theorem:4.12
Every regular set connected function is almost contra πgr-continuous but not conversely.
Example:4.13
Let X = {a,b,c}, τ= {φ, X,{a},{b},{a,b},{a,c}} τ^c = { φ, X, {b,c}, {a,c}, {c},{b}}, πgr-closed set={φ, X ,{a},{b},{c},{a,b},{a,c},{b,c}}.Let Y= {a,b,c}, σ={ φ,Y,{a},{b},{a,b}}. Let f be an identity map.The inverse image of regular open set {a} is not clopen in X. But the inverse image of open set in Y is πgr-closed in X.Hence f is almost contra πgr-continuous and not regular set connected.
Theorem:4.14
Let f:X→Y, g:Y→Z be two functions. Then the following properties hold.
a)If f is almost contra πgr-continuous and g is regular set connected, then gof: X→Z is almost contra πgr-continuous and almost πgr-continuous.
b) If f is almost contra πgr-continuous and g is perfectly continuous, then gof: X→Z is πgr-continuous and contra πgr-continuous.
c) If f is contra πgr-continuous and g is regular set connected , then gof: X→Z is πgr-continuous and almost πgr-continuous.
Proof:
a)Let V∈RO(Z). Since g is regular set connected, $g^{-1}(V)$ is clopen in Y. Since f is almost contra πgr-continuous,$f^{-1}[g^{-1}(V)] = (gof)^{-1}(V)$ is πgr-open and πgr-closed.Therefore, (gof) is almost contra πgr-continuous and almost πgr-continuous
b) Let V be open in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y.Since f is almost contra πgr-continuous, $f^{-1}[g^{-1}(V)] = (gof)^{-1}(V)$ is πgr-open and πgr-closed.Hence gof is contra πgr-continuous and πgr-continuous.
c) Let V∈RO(Z). Since g is regular set connected, $g^{-1}(V)$ is clopen in Y. Since f is a contra πgr-continuous,$f^{-1}[g^{-1}(V)] = (gof)^{-1}(V)$ is πgr-closed in X.Therefore, (gof) is πgr-continuous and almost πgr-continuous.
Theorem:4.15
If f:X→Y is an almost contra πgr-continuous, injection and Y is weakly hausdorff, then X is πgr-T_1.
Proof: Suppose Y is weakly hausdorff. For any distinct points x and y in X, there exists V and W regular closed sets in Y such that f(x)∈V,f(y)∉V,f(y)∈W and f(x)∉W. Since f is almost contra πgr-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are πgr-open subsets of X such that x ∈$f^{-1}(V)$, y∉$f^{-1}(V)$, y∈$f^{-1}(W)$ and x∉$f^{-1}(W)$. This shows that X is πgr-T_1.
Corollary:4.16
If f:X→Y is a contra πgr-continuous injection and Y is weakly hausdorff, then X is πgr-$T_{1.}$
Proof: Since every contra πgr-continuous function is almost contra πgr-continuous, the result of this corollary follows by using the above theorem.
Theorem:4.17
If f:X→Y is an almost contra πgr-continuous injective function from space X to a ultra Hausdorff space Y, then X is πgr-T_2.
Proof: Let x and y be any two distinct points in X. Since f is injective, f(x)≠f(y) and Y is Ultra Hausdorff space , there exists disjoint clopen sets U and V of Y containing f(x) and f(y) respectively. Then x∈$f^{-1}(U)$, y∈$f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint πgr-open sets in X. Therefore, X is πgr-T_2.
Theorem:4.18
If f:X→Y is an almost contra πgr-continuous injection and Y is Ultra Normal. Then X is πgr-normal.
Proof: Let G and H be disjoint closed subsets of X. Since f is closed and injective,f(E) and f(F) are disjoint closed sets in Y. Since Y is Ultra Normal, there exists disjoint clopen sets U and V in Y such that f(G)⊂U and f(H)⊂V.Hence G⊂$f^{-1}(U)$, H⊂$f^{-1}(V)$. Since f is an almost contra πgr-continuous injective function, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint πgr-open sets in X.Hence X is πgr- Normal.
Theorem:4.20
If f:X→Y is an almost contra πgr-continuous surjection and X is πgr-connected space, then Y is connected.

Proof: Let $f:X\rightarrow Y$ be an almost contra πgr-continuous surjection and X is πgr-connected space. Suppose Y is not connected space,then there exists disjoint open sets U and V such that $Y=U\cup V$. Therefore, U and V are clopen in Y. Since f is almost contra πgr-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are πgr-open sets in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint πgr-open sets and $X=f^{-1}(U)\cup f^{-1}(V)$. This is a contradiction to the fact that X is πgr-connected space. Therefore, Y is connected.

Theorem:4.21

If X is πgr-Ultra connected and $f:X\rightarrow Y$ is an almost contra πgr-continuous surjective, then Y is hyper connected.

Proof: Let X be a πgr-Ultra connected and $f: X\rightarrow Y$ is an almost contra πgr-continuous surjection. Suppose Y is not hyper connected. Then there exists an open set V such that V is not dense in Y. Therefore, there exists non-empty regular open subsets $B_1 = int(cl(V))$ and $B_2= Y-cl(V)$ in Y. Since f is an almost contra πgr-continuous surjection, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint πgr-closed sets in X.This is a contradiction to the fact that X is πgr-ultra connected. Therefore, Y is hyper connected.

Theorem:4.22

If a function $f: X\rightarrow Y$ is an almost contra πgr-continuous, then f is weakly πgr-continuous function.

Proof: Let $x\in X$ and V be an open set in Y containing f(x). Then cl(V) is regular closed in Y containing f(x). Since f is an almost contra πgr-continuous function for every regular closed set $f^{-1}(V)$ is πgr-open in X.Hence $f^{-1}(cl(V))$ is πgr-open set in X containing x.Set $U= f^{-1}(cl(V))$, then $f(U)\subset f(f^{-1}(cl(V))\subset cl(V)$.This shows that f is weakly πgr-continuous function.

Theorem:4.23

Let $f: X\rightarrow Y$ be an almost contra πgr-continuous surjection. Then the following properties hold:

1. If X is πgr-compact, then Y is S-closed.
2. If X is countably πgr-closed, then Y is countably S-closed.
3. If X is πgr-lindelof, then Y is S-lindelof.

Proof:

1)Let $\{V_\alpha:\alpha\in I\}$ be any regular closed cover of Y. Since f is almost contra πgr-continuous,$\{f^{-1}\{V_\alpha\}:\alpha\in I\}$ is πgr-open cover of X. Since X is πgr-compact, there exists a finite subset I_o of I such that $X=\cup\{f^{-1}\{V_\alpha\}:\alpha\in I_o\}$.Since f is surjective, $Y=\cup\{V_\alpha:\alpha\in I_o\}$ is finite sub cover of Y. Therefore, Y is S-closed.

2) Let $\{V_\alpha:\alpha\in I\}$ be any countable regular closed cover of Y. Since f is almost contra πgr-continuous,$\{f^{-1}\{V_\alpha\}:\alpha\in I\}$ is countable πgr-open cover of X. Since X is countably πgr-compact, there exists a finite subset I_o of I such that $X=\cup\{f^{-1}\{V_\alpha\}:\alpha\in I_o\}$. Since f is surjective $Y=\cup\{V_\alpha:\alpha\in I_o\}$is finite subcover for Y. Therefore, Y is countably S-closed.

3)Let $\{V_\alpha:\alpha\in I\}$be any regular closed cover of Y. Since f is almost contra πgr-continuous, $\{f^{-1}\{V_\alpha\}:\alpha\in I\}$ is πgr-open cover of X. Since X is πgr-lindelof, there exists a countable subset I_o of I such that $X=\cup\{f^{-1}\{V_\alpha\}:\alpha\in I_o\}$.Since f is surjective, $Y=\cup\{V_\alpha:\alpha\in I_o\}$ is finite sub cover of Y.Therefore Y is S-lindelof.

Theorem:4.24

Let $f: X\rightarrow Y$ be an almost contra πgr-continuous and almost continuous surjection. Then the following properties hold.

(1)If X is mildly πgr-closed, then Y is nearly compact.

(2)If X is mildly countably πgr-compact, then Y is nearly countably compact.

(3)If X is mildly πgr-lindelof, then Y is nearly lindelof.

Proof:

1) Let $\{V_\alpha:\alpha\in I\}$be any open cover of Y. Since f is almost contra πgr-continuous and almost πgrcontinuous function, $\{f^{-1}\{V_\alpha\}:\alpha\in I\}$ is πgr-clopen cover of X.Since X is mildly πgr-compact, there exists a finite subset I_o of I such that $X=\cup\{f^{-1}\{V_\alpha\}:\alpha\in I_o\}$ Since f is surjective, $Y==\cup\{V_\alpha:\alpha\in I_o\}$ is finite subcover for Y. Therefore, Y is nearly compact.

2) Similar to that of (1).

3)Let $\{V_\alpha:\alpha\in I\}$be any regular open cover of Y. Since f is almost contra πgr-continuous and almost πgr-continuous function, $\{f^{-1}\{V_\alpha\}:\alpha\in I\}$ is πgr-closed cover of X.Since X is mildly πgr-lindelof, there exists a countable subset I_o of I such that $X=\cup\{f^{-1}\{V_\alpha\}:\alpha\in I_o\}$. Since f is surjective, $Y==\cup\{V_\alpha:\alpha\in I_o\}$ is finite subcover for Y. Therefore, Y is nearly lindelof.

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