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Semipre Generalized Continuous and Irresolute Mappings in Intuitionistic Fuzzy Topological Spaces

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ABSTRACT: *In this paper we introduce intuitionistic fuzzy semipre generalized continuous mappings and intuitionistic fuzzy semipre generalized irresolute mappings. Some of their properties are studied.*

Keywords: *Intuitionistic fuzzy topology, intuitionistic fuzzy point, intuitionistic fuzzy semipre generalized closed sets, intuitionistic fuzzy semipre generalized continuous mappings and intuitionistic fuzzy semipre generalized irresolute mappings.*

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I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [17] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy semipre generalized continuous mappings and intuitionistic fuzzy semipre generalized irresolute mappings and studied some of their basic properties.

II. PRELIMINARIES

Definition 2.1:[1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x\in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x\in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x)\rangle / x \in X \}$,
(iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$,
(v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1\rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0\rangle / x \in X \}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short)in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$,

(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$,
(iii) $cl(A^c) = (int(A))^c$,
(iv) $int(A^c) = (cl(A))^c$.

Definition 2.5: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy semiclosed set (IFSCS in short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy semiopen set (IFSOS in short) if $A \subseteq cl(int(A))$.

Definition 2.6: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy preclosed set (IFPCS in short) if $cl(int(A)) \subseteq A$,
(ii) intuitionistic fuzzy preopen set (IFPOS in short) if $A \subseteq int(cl(A))$.
Note that every IFOS in (X, τ) is an IFPOS in X.

Definition 2.7: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$,
(ii) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq int(cl(int(A)))$,
(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$,
(iv) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$,
(v) intuitionistic fuzzy β-closed set (IFβCS in short) if $int(cl(int(A))) \subseteq A$,
(vi) intuitionistic fuzzy β-open set (IFβOS in short) if $A \subseteq cl(int(cl(A)))$.

Definition 2.8: [15] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$,
(ii) intuitionistic fuzzy semipre open set (IFSPOS for short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

Definition 2.9: [12] An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy W-closed set (IFWCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy W-open set (IFWOS in short) if A^c is an IFWCS in (X, τ).

Definition 2.10: [6] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an
(i) intuitionistic fuzzy semi generalized closed set(IFSGCS in short) if $scl(A) \subseteq O$ whenever $A \subseteq O$ and O is an IFSOS in (X, τ),
(ii) intuitionistic fuzzy semi generalized open set(IFSGOS in short) if its complement A^c is an IFSGCS in (X, τ).

Definition 2.11: [8] An IFS A is an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre open set (IFGSPOS in short) if A^c is an IFGSPCS in (X, τ).

Definition 2.12: [15] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ).
Every IFCS, IFSCS, IFWCS, IFαCS, IFRCS, IFPCS, IFSGCS, IFSPCS, IFβCS is an IFSPGCS but the converses are not true in general.

Definition 2.13: [14] The complement A^c of an IFSPGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy semipre generalized open set (IFSPGOS for short) in X.

The family of all IFSPGOSs of an IFTS (X, τ) is denoted by IFSPGO(X). Every IFOS, IFSOS, IFWOS, IFαOS, IFROS, IFPOS, IFSGOS, IFSPOS, IFβOS is an IFSPGOS but the converses are not true in general.

Definition 2.14: [10] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $x_{(\alpha, \beta)}$ of X is an IFS of X defined by $x_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x \\ (0, 1) & \text{if } y \neq x \end{cases}$

Definition 2.15: [10] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.16: [16] Let A be an IFS in an IFTS (X, τ). Then
(i) sint(A) = ∪{ G / G is an IFSOS in X and G ⊆A },
(ii) scl(A) = ∩{ K / K is an IFSCS in X and A ⊆K }.
Note that for any IFS A in (X, τ), we have $scl(A^c) = (sint(A))^c$ and $sint(A^c) = (scl(A))^c$.

Definition 2.17: [8] Let A be an IFS in an IFTS (X, τ). Then
(i) spint (A) = ∪{ G / G is an IFSPOS in X and G ⊆A }.
(ii) spcl (A) = ∩{ K / K is an IFSPCS in X and A ⊆K }.
Note that for any IFS A in (X, τ), we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

Result 2.18: [2] For an IFS A in an IFTS(X, τ), we have spcl(A) ⊇ A ∪ (int(cl(int(A)))).

Definition 2.19: [14] If every IFSPGCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semipre $T_{1/2}$ ($IFSPT_{1/2}$ for short) space.

Definition 2.20: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous(IF continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every B ∈ σ.

Definition 2.21: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) intuitionistic fuzzy semi continuous(IFS continuous in short) if $f^{-1}(B)$ ∈IFSO(X) for every B ∈ σ,
(ii) intuitionistic fuzzy α- continuous(IFα continuous in short) if $f^{-1}(B)$ ∈IFαO(X) for every B ∈ σ,
(iii) intuitionistic fuzzy pre continuous(IFP continuous in short) if $f^{-1}(B)$ ∈IFPO(X) for every B ∈ σ,
(iv) intuitionistic fuzzy β- continuous(IFβ continuous in short) if $f^{-1}(B)$ ∈IFβO(X) for every B ∈ σ.

Definition 2.22: [13] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intutionistic fuzzy sg-continuous (IFSG continuous for short) mapping if $f^{-1}(B)$ ∈ IFSGO(X) for every B ∈ σ .

Result 2.23:
(i) Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping as well as intuitionistic fuzzy pre continuous mapping. [5]
(ii) Every IF continuous mapping is an IFSG continuous mapping. [7]

Definition 2.24: [16] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy semi-pre continuous (IFSP continuous for short) mapping if $f^{-1}(B)$ ∈ IFSPO(X) for every B ∈ σ .
Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general.

Definition 2.25: [12] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an Intutionistic fuzzy w-continuous (IFW continuous for short) mapping if $f^{-1}(B)$ ∈ IFWO(X) for every B ∈ σ .
Every IF continuous mapping is an IFW continuous mapping and every IFW continuous mapping is an IFSG continuous mapping but converse need not true.

Result 2.26: [14] For any IFS A in (X, τ) where X is an $IFSPT_{1/2}$ space, A ∈ IFSPGO(X) if and only if for every IFP c(α, β) ∈ A, there exists an IFSPGOS B in X such that c(α, β) ∈ B ⊆ A.

III. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED CONTINUOUS MAPPINGS

R. K. Saraf, Govindappa Navalagi and Meena Khanna [9] have introduced fuzzy semipre generalized continuous mappings in fuzzy topology. In this paper we have introduced intuitionistic fuzzy semipre generalized continuous mapping and investigated some of its properties.

Definition 3.1: A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy semipre generalized continuous (IFSPG continuous for short) mappings if $f^{-1}(V)$ is an IFSPGCS in (X, τ) for every IFCS V of (Y, σ).

Example 3.2: Let X= {a, b}, Y= {u, v} and G_1 = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩, G_2 = ⟨y, (0.6, 0.7), (0.4, 0.2)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFSPG continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.4: In Example 3.2, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not an IF continuous mapping. Since $G_2 = \langle y, (0.6, 0.7), (0.4, 0.2)\rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$ is not an IFOS in X.

Theorem 3.5: Every IFS continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.6: In Example 3.2, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not an IFS continuous mapping. Since $G_2 = \langle y, (0.6, 0.7), (0.4, 0.2)\rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$ is not an IFSOS in X.

Theorem 3.7: Every IFP continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.8: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3, 0.2), (0.6, 0.6)\rangle$, $G_2 = \langle y, (0.4, 0.5), (0.6, 0.5)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFSPG continuous mapping but not an IFP continuous mapping.

Theorem 3.9: Every IFSP continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.10: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$, $G_2 = \langle x, (0.2, 0.1), (0.8, 0.9)\rangle$, $G_3 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$, $G_4 = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$ and let $G_5 = \langle y, (0.1, 0.4), (0.9, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$ and $\sigma = \{0_\sim, G_5, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFSPG continuous mapping but not an IFSP continuous mapping.

Theorem 3.11: Every IFβ continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFβCS in X. Since every IFβCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.12: In Example 3.10, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not an IFβ continuous mapping.

Theorem 3.13: Every IFα continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.14: In Example 3.2, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not an IFα continuous mapping.

Theorem 3.15: Every IFW continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFW continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFWCS in X. Since every IFWCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.16: In Example 3.2, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not an IFW continuous mapping.

Theorem 3.17: Every IFSG continuous mapping is an IFSPG continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSGCS in X. Since every IFSGCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.18: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$, $G_2 = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$, $G_3 = \langle x, (0.4, 0.3), (0.6, 0.4)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFSPG continuous mapping but not an IFSG continuous mapping.

Theorem 3.19: Every IFSPG continuous mapping is an IFGSP continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSPG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPGCS in X. Since every IFSPGCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

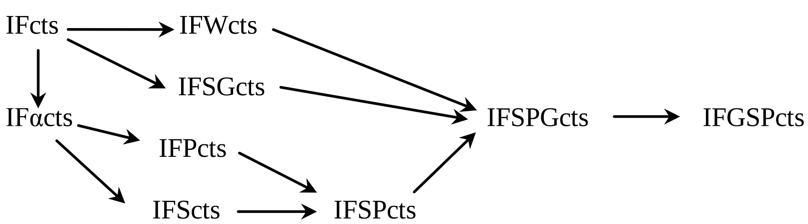
Example 3.20: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3, 0.6), (0.7, 0.4)\rangle$, $G_2 = \langle y, (0.3, 0.4), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGSP continuous mapping but not an IFSPG continuous mapping.

Theorem 3.21: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping where $f^{-1}(V)$ is an IFRCS in X for every IFCS in Y. Then f is an IFSPG continuous mapping but not conversely.

Proof: Let A be an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSPGCS, $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 3.22: In Example 3.2, $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping but not a mapping defined in Theorem 3.21.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous.



The reverse implications are not true in general in the above diagram.

Theorem 3.23: If $f : (X, \tau) \to (Y, \sigma)$ is an IFSPG continuous mapping, then for each IFP $c(\alpha, \beta)$ of X and each $A \in \sigma$ such that $f(c(\alpha, \beta)) \in A$. Then there exists an IFSPGOS B of X such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.

Proof: Let $c(\alpha, \beta)$ be an IFP of X and $A \in \sigma$ such that $f(c(\alpha, \beta)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFSPGOS in X such that $c(\alpha, \beta) \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.24: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSPG continuous mapping. Then f is an IFSP continuous mapping if X is an $IFSPT_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPGCS in X, by hypothesis. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IFSP continuous mapping.

Theorem 3.25: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSPG continuous mapping and $g: (Y, \sigma) \to (Z, \eta)$ is an IF continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IFSPG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IFSPG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFSPGCS in X. Hence $g \circ f$ is an IFSPG continuous mapping.

Theorem 3.26: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ space:
(i) f is an IFSPG continuous mapping,
(ii) $f^{-1}(B)$ is an IFSPGOS in X for each IFOS B in Y,
(iii) for every IFP $c(\alpha, \beta)$ in X and for every IFOS B in Y such that $f(c(\alpha, \beta)) \in B$, there exists an IFSPGOS A in X such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

Proof: (i) $\Leftrightarrow$ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.
(ii) $\Rightarrow$ (iii) Let B be any IFOS in Y and let $c(\alpha, \beta) \in X$. Given $f(c(\alpha, \beta)) \in B$. By hypothesis $f^{-1}(B)$ is an IFSPGOS in X. Take $A= f^{-1}(B)$. Now $c(\alpha, \beta) \in f^{-1}(f(c(\alpha, \beta))$. Therefore $f^{-1}(f(c(\alpha, \beta)) \in f^{-1}(B) = A$. This implies $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.
(iii) $\Rightarrow$ (i) Let A be an IFCS in Y. Then its complement, say $B = A^c$, is an IFOS in Y. Let $c(\alpha, \beta) \in C$ and $f(C) \subseteq B$. Now $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Thus $c(\alpha, \beta) \in f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFSPGOS in X by Result 2.26. That is $f^{-1}(A^c)$ is an IFSPGOS in X and hence $f^{-1}(A)$ is an IFSPGCS in X. Thus f is an IFSPG continuous mapping.

Theorem 3.27: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IFSPT_{1/2}$ space:
(i) f is an IFSPG continuous mapping,
(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFSPGOS in X.
(iii) $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) $\Leftrightarrow$ (ii) is obvious true by Theorem 3.26.
(ii) $\Rightarrow$ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFSPGOS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(int(B))$ is an IFSPOS in X. Therefore $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$.
(iii) $\Rightarrow$ (i) Let B be an IFOS in Y. By hypothesis $f^{-1}(B)= f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IFβOS in X. Therefore it is an IFSPGOS in X and hence f is an IFSPG continuous mapping, by Theorem 3.26.

Theorem 3.28: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ space:
(i) f is an IFSPG continuous mapping,
(ii) $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(spcl(B))$ for each IFCS B in Y,
(iii) $f^{-1}(spint(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for each IFOS B of Y,
(iv) $f(int(cl(int(A)))) \subseteq cl(f(A))$ for each IFS A of X.

Proof: (i) $\Rightarrow$ (ii) Let B be any IFCS in Y. Then $f^{-1}(B)$ is an IFSPGCS in X. Since X is $IFSPT_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS. Therefore $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(spcl(B))$.
(ii) $\Rightarrow$ (iii) can be easily proved by taking complement in (ii).
(iii) $\Rightarrow$ (iv) Let $A \in X$. Taking $B = f(A)$ we have $A \subseteq f^{-1}(B)$. Here $int(f(A))= int(B)$ is an IFOS in Y. Then (iii) implies that $f^{-1}(spint(int(B))) \subseteq cl(int(cl(f^{-1}(int(B))))) \subseteq cl(int(cl(f^{-1}(B))))$. Now $(cl(int(cl(A^c))))^c \subseteq (cl(int(cl(f^{-1}(B^c)))))^c \subseteq (f^{-1}(spint(int(B))))^c$. Therefore $int(cl(int(A))) \subseteq f^{-1}(spcl(cl(B)))$. Now $f(int(cl(int(A)))) \subseteq f(f^{-1}(spcl(cl(B)))) \subseteq cl(B) = cl(f(A))$.
(iv) $\Rightarrow$ (i) Let B be any IFCS in Y. Then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(int(f^{-1}(B))))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B) = B$. Now $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(f(int(cl(int(f^{-1}(B)))))) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFβCS and hence it is an IFSPGCS in X. Thus f is an IFSPG continuous mapping.

Theorem 3.29: Let f : (X, τ) → (Y, σ) be an IFSPG continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS in X and hence it is an IFSPGOS in X. Therefore f is an IFSPG continuous mapping, by Theorem 3.26.

Theorem 3.30: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IFSPT_{1/2}$ space:
(i) f is an IFSPG continuous mapping,
(ii) $f^{-1}(B)$ is an IFSPGCS in X for every IFCS B in Y,
(iii) $int(cl(int(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: (i) ⟺ (ii) is obvious form the Definition 3.1.
(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFSPGCS in X. Since X is an $IFSPT_{1/2}$ space, $f^{-1}(cl(A))$ is an IFSPCS. Therefore $int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$. Now $int(cl(int(f^{-1}(A)))) \subseteq int(cl(int(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$.
(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $int(cl(int(f^{-1}(A)))) \subseteq f^{-1}(cl(A))= f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFβCS in X and hence it is an IFSPGCS. Thus f is an IFSPG continuous mapping.

IV. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED IRRESOLUTE MAPPINGS

R.K. Saraf [9] introduced fuzzy SPG-irresolute mappings in fuzzy topological spaces. In this paper we have introduced intuitionistic fuzzy semipre generalized irresolute mappings and studied some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) be an intuitionistic fuzzy semipre generalized irresolute (IFSPG irresolute) mapping if $f^{-1}(V)$ is an IFSPGCS in (X, τ) for every IFSPGCS V of (Y, σ).

Theorem 4.2: If f : (X, τ) → (Y, σ) be an IFSPG irresolute mapping, then f is an IFSPG continuous mapping but not conversely.
Proof: Let f be an IFSPG irresolute mapping. Let V be any IFCS in Y. Then V is an IFSPGCS and by hypothesis $f^{-1}(V)$ is an IFSPGCS in X. Hence f is an IFSPG continuous mapping.

Example 4.3: Let X = {a, b}, Y= {u, v} and $G_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$, $G_2 = \langle y, (0.5, 0.3), (0.5, 0.5)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFSPG continuous mapping but not IFSPG irresolute mapping.

Theorem 4.4: Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be IFSPG irresolute mapping. Then g ∘ f : (X, τ) →(Z, η) is an IFSPG irresolute mapping.

Proof: Let V be an IFSPGCS in Z. Then $g^{-1}(V)$ is an IFSPGCS in Y. Since f is an IFSPG irresolute, $f^{-1}(g^{-1}(V))$ is an IFSPGCS in X, by hypothesis. Hence g ∘ f is an IFSPG irresolute mapping.

Theorem 4.5: Let f : (X, τ) → (Y, σ) be an IFSPG irresolute mapping and g : (Y, σ) → (Z, η) be IFSPG continuous mapping, then g ∘ f : (X, τ) →(Z, η) is an IFSPG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFSPGCS in Y. Since f is an IFSPG irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFSPGCS in X. Hence g ∘ f is an IFSPG continuous mapping.

Theorem 4.6: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IFSPT_{1/2}$ space:
(i) f is an IFSPG irresolute mapping,
(ii) $f^{-1}(B)$ is an IFSPGOS in X for each IFSPGOS B in Y,
(iii) $f^{-1}(spint(B)) \subseteq spint(f^{-1}(B))$ for each IFS B of Y,
(iv) $spcl(f^{-1}(B)) \subseteq f^{-1}(spcl(B))$ for each IFS B of Y.

Proof: (i) ⟺ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) ⇒ (iii) Let B be any IFS in Y and spint(B) ⊆ B. Also f^{-1}(spint(B)) ⊆ f^{-1}(B). Since spint(B) is an IFSPOS in Y, it is an IFSPGOS in Y. Therefore f^{-1}(spint(B)) is an IFSPGOS in X, by hypothesis. Since X is an $IFSPT_{1/2}$ space, f^{-1}(spint(B)) is an IFSPOS in X. Hence f^{-1}(spint(B)) = spint(f^{-1}(spint(B))) ⊆ spint(f^{-1}(B)).
(iii) ⇒ (iv) is obvious by taking complement in (iii).
(iv) ⇒ (i) Let B be an IFSPGCS in Y. Since Y is an $IFSPT_{1/2}$ space, B is an IFSPCS in Y and spcl(B) = B. Hence f^{-1}(B) = f^{-1}(spcl(B)) ⊇ spcl(f^{-1}(B)), by hypothesis. But f^{-1}(B) ⊆ spcl(f^{-1}(B)). Therefore spcl(f^{-1}(B)) = f^{-1}(B). This implies f^{-1}(B) is an IFSPCS and hence it is an IFSPGCS in X. Thus f is an IFSPG irresolute mapping.

Theorem 4.7: Let f : (X, τ) → (Y, σ) be a an IFSPG irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆ spint(f^{-1}(cl(int(cl(B))))) for every IFSPGOS B in Y, if X and Y are $IFSPT_{1/2}$ spaces.

Proof: Let B be an IFSPGOS in Y. Then by hypothesis f^{-1}(B) is an IFSPGOS in X. Since X is an $IFSPT_{1/2}$ space, f^{-1}(B) is an IFSPOS in X. Therefore spint(f^{-1}(B)) = f^{-1}(B). since Y is an $IFSPT_{1/2}$ space, B is an IFSPOS in Y and B ⊆ cl(int(cl(B))). Now f^{-1}(B) = spint(f^{-1}(B)) implies, f^{-1}(B) ⊆ spint(f^{-1}(cl(int(cl(B))))).

Theorem 4.8: Let f : (X, τ) → (Y, σ) be a an IFSPG irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆ spint(cl(int(cl(f^{-1}(B))))) for every IFSPGOS B in Y, if X and Y are $IFSPT_{1/2}$ spaces.

Proof: Let B be an IFSPGOS in Y. Then by hypothesis f^{-1}(B) is an IFSPGOS in X. Since X is an $IFSPT_{1/2}$ space, f^{-1}(B) is an IFSPOS in X. Therefore spint(f^{-1}(B)) = f^{-1}(B) ⊆ cl(int(cl(f^{-1}(B)))). Hence f^{-1}(B) ⊆ spint(cl(int(cl(f^{-1}(B))))).

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