

Mathematical Model On Bread Baking

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ABSTRACT: *Apart from bread being staple food in European countries, Middle East, North Africa, various communities in Nigeria and most African countries rely on bread as breakfast. Bread baking requires many ingredients but major ones are flour, sugar and yeast. In this work, we propose and study a mathematical model on the bread baking and make analysis on which factor plays the most role. The model reveals that bread can be made in the absence of sugar.*

Keywords: *Bread, Flour, Sugar, Yeast, bake, Mathematical model.*

INTRODUCTION

Bread is the staple food in European. European-derived cultures such as the Americas, Middle East and North Africa as opposed to East Asia whose staple is rice. (P.Gelinas and Carole m; 2006). In many homes especially Nigeria, bread is used as breakfast. Bread is usually made from wheat-flour dough that is cultured with yeast allowed to rise and finally baked in an oven (Peter Reinhart; 2001). Bread is also made from the flour of other wheat species such as durum, spelt, emmer, rye, barley, maize etc. The major required products in bread making are flour, sugar, yeast. Flour is a product made from grain that has been ground to a powdery consistency. Flour provides the primary structure to the final baked bread. Wheat flour is most commonly used for breads. Sugar is a sweet substance in form of white or brown crystals use in bread making especially in Nigeria. Yeast ferments carbohydrates in flour, including any sugar, producing carbon dioxide. In this paper, we propose and study a mathematical model on the baking of bread by considering flour and sugar as major factors. The model reveals that bread can be made in the absence of sugar or when both are available. Mathematical modeling is the art of transforming a situation from its real world to mathematical world where it is more conveniently studied and then back. Model simply means mathematical expression of a real life situation or problem.

MODEL ASSUMPTIONS

In building the model we make the following assumptions on sugar and flour as main components/ingredients of bread baking.

- (i) The rate of production of bread depends on the availability of flour and sugar.
- (ii) Supply of flour depends on availability of sugar.
- (iii) In the absence of flour no bread.

Model Building

Parameters:

- $f(t)$ – the number of flour at time t.
- $s(t)$ – the number of sugar at time t.
- α - rate of flour supply in the absence of sugar.
- Ψ - rate of sugar supply in the absence of flour.
- β - reduction rate of flour supply in the absence of sugar.

Based on the assumptions made, we have the following Ordinary Differential Equations (ODEs) governing the supply of flour and sugar.

$$\left. \begin{aligned} f'(t) &= \alpha f \\ s'(t) &= \Psi s \end{aligned} \right\} \text{-----(1)}$$

When we consider ii and ii we finally obtain

$$\left. \begin{aligned} f'(t) &= \alpha f - \beta fs; s = 0 \\ s'(t) &= \beta fs - \Psi s; f = 0 \end{aligned} \right\} \text{----- (2)}$$

Model Analysis

Here we apply qualitative approach rather than analytic because of non-linear nature of the DE.

At equilibrium point the LHS of equation (2) must be zero, implies

$$\left. \begin{aligned} \alpha f - \beta f s &= 0 \\ \beta f s - \Psi s &= 0 \end{aligned} \right\} \text{----- (3)}$$

From equation 3

$$f(\alpha - \beta s) = 0$$

$$\Rightarrow \text{either } f = 0 \text{ or } \alpha - \beta s = 0$$

$$\Rightarrow s = \alpha / \beta$$

$$\text{Also } s(\beta f - \Psi) = 0$$

$$\Rightarrow \text{either } s = 0 \text{ or } \beta f - \Psi = 0$$

$$\Rightarrow f = \Psi / \beta$$

Suppose f°, s° are the equilibrium points then, we have

$$f^\circ, s^\circ = (\alpha / \beta, \Psi / \beta)$$

We also apply Jacobian approach.

$$\text{Consider } \left. \begin{aligned} f'(t) &= \alpha f - \beta f s = J_1(f, s) \\ S'(t) &= \beta f s - \Psi s = J_2(f, s) \end{aligned} \right\} \text{----- (4)}$$

Suppose f°, s° are the equilibrium points

Take Jacobian matrix of equation (4), we have

$$J(f^\circ, s^\circ) = \begin{bmatrix} \alpha - \beta s & -\beta f \\ \beta & \beta f - \Psi \end{bmatrix}$$

$$J(0,0) = \begin{bmatrix} \alpha & 0 \\ \beta & -\Psi \end{bmatrix}$$

Find the eigenvalues λ_1 and λ_2 .

$$J^\circ - \lambda = \begin{bmatrix} \alpha - \lambda & 0 \\ \beta & -(\Psi - \lambda) \end{bmatrix}$$

Characteristics equation

$$(\alpha - \lambda)(-\Psi - \lambda) - \beta(0)$$

$$\text{either } \alpha - \lambda = 0 \Rightarrow \lambda_1 = \alpha$$

$$\text{or } -\Psi - \lambda = 0 \Rightarrow \lambda_2 = -\Psi$$

The eigenvalues are $\lambda = \alpha$ and $\lambda_2 = -\Psi$.

Since λ_2 is negative, it satisfies assumption that no flour no bread.

Discussion

From our analysis λ_1 is positive and λ_2 is negative. The two eigenvalues could be negative, positive, zero or combine and it implies the production stability could be stable or unstable depending on the values of the model parameter at a given time. Here we find out that flour supply is positive and sugar supply is negative implies bread can be baked without sugar. The negativity of λ_2 satisfies the condition necessary for stability.

Conclusion

We have proposed and studied a mathematical model that describes bread baking on considering two factors. The result suggests that with or without sugar, bread can be baked.

Reference

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