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Trust Weighted Quantum Inspired Consensus for Probabilistic Linguistic Group Decision Making

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Abstract: *Probabilistic linguistic group decision making combines assessments expressed through linguistic terms and associated probabilities. This paper presents a trust weighted quantum inspired model that separates semantic representation, expert aggregation and consensus adjustment. Normalized Gaussian semantic vectors encode complete probabilistic linguistic term sets as density matrices, and a fixed computational basis measurement produces model induced linguistic probabilities. Expert weights are calculated from incoming direct trust. Individual states are then moved toward an alternative specific group reference through a completely positive trace preserving mixing channel. The smallest common adjustment strength has an explicit expression under a weighted trace distance threshold. This adjustment reduces individual disagreement while preserving the aggregated group state and its ranking. A synthetic power battery supplier example with five experts and four alternatives illustrates the calculation. All four alternatives reach the prescribed deviation threshold of 0.15. The preferred alternative remains first in 97.4% of 1,000 probability perturbation runs, whereas the complete ranking is retained in 88.3%. All tested trust perturbations preserve the baseline ranking. Parameter analysis and a matched classical benchmark identify the representation effects and the limits of these numerical findings.*
Keywords: *group decision making; probabilistic linguistic term sets; quantum inspired model*

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I. Introduction

Group decision making combines the assessments of several experts when no single participant has all the information needed to compare alternatives. Supplier selection provides a useful setting: technical capability, delivery, service and commercial considerations can lead experts to different overall judgments. A decision model must specify how these judgments are represented, how expert influence is assigned, and how disagreement is assessed. These choices affect different parts of the calculation and should be interpreted separately.

Linguistic assessment models include linguistic variables [1], the 2-tuple representation [2] and hesitant fuzzy linguistic term sets [3]. Probabilistic linguistic term sets (PLTSs), introduced by Pang et al. [4], associate probabilities with linguistic terms and provide a compact representation of hesitant qualitative assessments. Subsequent work has developed comparison rules, aggregation procedures and preference relations, including revised operational laws for linguistic and probabilistic linguistic terms [5]. Liao et al. [6] review probabilistic linguistic preference relations, including consistency checking, missing information and decision analysis. Bai et al. [7] extend probabilistic linguistic representations to interval valued probabilities and develop corresponding operations and aggregation methods. This literature offers several ways to process uncertainty; a representation should therefore state which information it takes as input and which assumptions it adds.

DeGroot's opinion pooling model provides a classical account of consensus formation [8]. Social relationships provide another source of variation. Wu and Chiclana [9] combine social trust and consensus information to determine expert importance. Zhou et al. [10] connect trust networks with an ordinal consensus measure and use trust related information in expert weighting and feedback. Zhong et al. [11] study relative influence in trust propagation and an integral formulation of adjustment cost. Minimum-cost consensus models also explicitly represent the cost of changing individual opinions [12]. These approaches show that expert influence, trust propagation and adjustment cost can be modeled in considerable detail. For small groups with a complete trust matrix, a simpler incoming trust rule can also be useful, provided that its subjective origin and its effect on the final weights are explicit.

Quantum inspired decision models offer matrix and amplitude representations for qualitative judgments. More broadly, quantum probability has been developed as a framework for cognition and decision modeling [13], including explanations of departures from classical decision predictions [14]. Li and Hao [15] formulate a quantum probabilistic linguistic framework for battlefield assessment. Liu et al. [16] combine probabilistic

linguistic information, trust and interference among experts. Xu et al. [17] investigate quantum linguistic group consensus based on trust networks, while Xu et al. [18] develop a quantum group consensus model using linguistic Z-numbers for meteorological emergency decisions. These studies motivate the use of quantum mathematical structures in linguistic decision analysis. They do not imply that an observed PLTS uniquely determines a quantum state or that every group decision mechanism requires a quantum description.

A practical integration needs to keep several distinctions clear. Nonorthogonal semantic vectors can express similarity between linguistic terms, but their superposition requires explicit normalization. A measurement must be specified before the resulting probabilities can be interpreted. Furthermore, preserving diagonal entries in one basis does not establish preservation of the original linguistic probabilities. Consensus adjustment also needs a precise reference and a stated feasible family: reducing a numerical disagreement measure is different from demonstrating that experts have accepted the recommended revisions.

This paper develops a compact model for complete PLTS inputs and overall alternative assessments. Its first component is a Gaussian semantic representation with a fixed measurement and a matched classical score. Its second component uses direct incoming trust to determine one expert weight vector for all alternatives. Its third component applies a common mixing channel toward each alternative's group reference. The required strength is calculated directly, and the adjustment magnitude is minimized only within this prescribed family. The aggregate remains fixed, so the role of consensus adjustment is individual coordination rather than a change in the group ranking.

The contribution is an explicit integration of these operations with their interpretation boundaries. The state representation is a modeling choice; the channel is not introduced as a new result in quantum information. A fully reported synthetic example and numerical perturbation experiments show how the computation behaves. The analysis distinguishes changes caused by semantic representation from the invariance imposed by the adjustment rule.

The remainder follows this structure. Section II introduces the required concepts. Section III defines the model. Section IV presents the supplier selection example and a classical comparison. Section V examines parameter sensitivity and input perturbations. Section VI discusses the conclusions and limitations.

II. Preliminary Knowledge

2.1 Complete probabilistic linguistic assessments

An ordered linguistic set contains labels running from unfavorable to favorable evaluations. A complete PLTS assigns nonnegative probabilities to these labels with total probability one [4]. Terms not used in an assessment can be represented by zero probability, allowing every assessment to be written on a common scale. A probability distribution and a collection of unnormalized rating intensities have different meanings: normalization of intensities constructs a distribution, whereas normalization of incomplete probabilities would redistribute unreported mass.

The present model considers complete distributions on one shared ordered scale. It uses the utilities attached to the labels to summarize final evaluation probabilities. The utility values and their orientation must be identical across experts, alternatives and comparison methods. Attribute specific aggregation and incomplete probability mass are outside the model's scope.

2.2 Density matrices and measurements

A density matrix is a Hermitian positive semidefinite operator with trace one. A normalized state vector gives a pure state through its outer product. A convex combination of valid density matrices is also a valid density matrix and is generally mixed [19]. These mathematical objects have been used in quantum linguistic decision models [15]-[18]. Here they are representations of numerical assessments, with no assumption of a physical quantum system.

Measurement effects are positive operators that sum to the identity. Their trace products with a density matrix are nonnegative probabilities summing to one [19]. Computational basis projectors satisfy these requirements and are used throughout this paper. A measurement probability obtained after semantic encoding is a model output; it need not reproduce the PLTS probability supplied before encoding.

2.3 State disagreement and channel based adjustment

Trace distance measures the difference between density matrices on a scale from zero to one [19], [20]. It vanishes for identical states and can be evaluated from the absolute eigenvalues of their Hermitian difference. In a group model, an average distance to a specified group state provides a numerical disagreement measure. Such a measure concerns the chosen representation and does not by itself establish interpersonal agreement.

A completely positive trace preserving (CPTP) map describes a valid linear transformation of quantum states, including when a state is part of a larger system [19]. The mixing operation used below combines the identity operation with replacement by a fixed reference state. Its fixed reference interpretation is essential: selecting that

reference from the input data is part of the overall decision procedure, rather than evidence that the complete procedure is one fixed linear channel.

III. Methodology

The proposed model uses three separate operations: semantic representation of each complete PLTS, trust weighted aggregation of expert states, and a common mixing adjustment toward an alternative specific group reference. The reference and expert weights remain fixed during adjustment. The following definitions give the entire computation without phase optimization or multipath quantum trust propagation.

3.1 Decision setting and inputs

Let $E = \{e_1, \dots, e_n\}$ and $A = \{A_1, \dots, A_m\}$ denote the experts and alternatives, respectively, where $n \geq 2$ and $m \geq 2$. The ordered linguistic term set is $S = \{s_{-\tau}, \dots, s_\tau\}$, with integer $\tau \geq 1$ and $d = 2\tau + 1$. Larger linguistic indices indicate better performance. Expert e_i evaluates alternative A_j using a complete PLTS [4] with probability vector

$$p_{ij} = (p_{ij,-\tau}, \dots, p_{ij,\tau}), \qquad p_{ij,l} \geq 0, \quad \sum_{l=-\tau}^{\tau} p_{ij,l} = 1. \qquad (1)$$

An unlisted term has probability zero. Input rows must satisfy (1) before encoding. Incomplete probability distributions are outside the present model; their missing mass is not silently redistributed. The default linguistic utility is

$$u_l = \frac{l + \tau}{2\tau}, \qquad l = -\tau, \dots, \tau. \qquad (2)$$

The directed trust matrix is $T = [t_{gi}]_{n \times n}$, where $t_{gi} \in [0,1]$ denotes the trust placed by expert e_g in expert e_i, and $t_{ii} = 1$. Trust assessments are elicited inputs rather than objective measures of expertise.

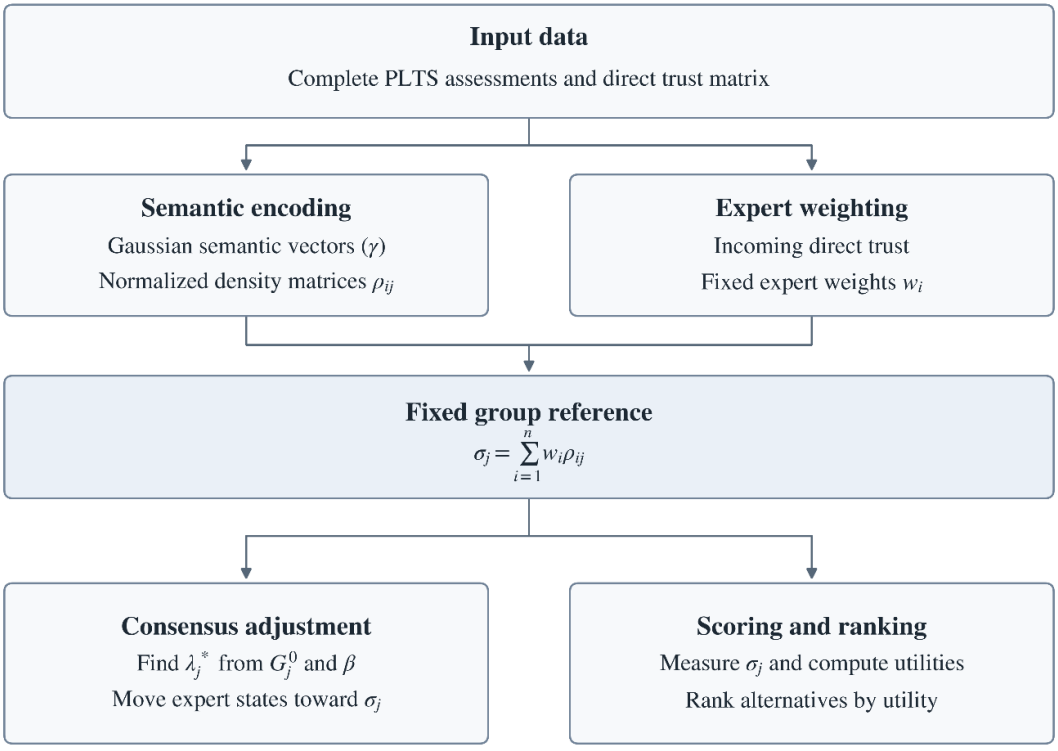


Fig. 1. Flowchart of the proposed model.

3.2 Semantic representation of PLTSs

Consider a d dimensional Hilbert space with orthonormal computational basis $\{|k\rangle\}_{k=-\tau}^{\tau}$. For a prespecified parameter $\gamma > 0$, define the normalized semantic vector of term s_l by

$$|v_l\rangle = N_l \sum_{k=-\tau}^{\tau} \exp[-\gamma(k-l)^2]|k\rangle, \qquad (3)$$

$$N_l = \left(\sum_{k=-\tau}^{\tau} \exp\left[-2\gamma(k-l)^2\right] \right)^{-1/2}. \qquad (4)$$

These vectors generally overlap. Smaller γ produces broader semantic representations, whereas larger γ produces more localized representations. Finite endpoints and normalization introduce boundary effects, so their inner products are not assumed to depend only on the index distance. Broad semantic vectors may reduce apparent disagreement by construction; consensus values are therefore interpreted conditional on the selected parameter, whose sensitivity should be examined.

For each expert and alternative, construct

$$|a_{ij}\rangle = \sum_{l=-\tau}^{\tau} \sqrt{p_{ij,l}}\, |v_l\rangle, \qquad |\psi_{ij}\rangle = \frac{|a_{ij}\rangle}{\sqrt{\langle a_{ij}, a_{ij}\rangle}}. \qquad (5)$$

$$\rho_{ij} = |\psi_{ij}\rangle\langle\psi_{ij}|. \qquad (6)$$

All input phases are set to zero. This specifies a coherent representation with no additional phase parameters; the probabilities alone do not establish that this coherence exists in actual expert cognition. The explicit normalization in (5) is required because the semantic vectors are nonorthogonal. Each encoded matrix is positive semidefinite and has unit trace.

We use the computational basis measurement

$$M_l = |l\rangle\langle l|, \qquad q_{ij,l} = \mathrm{Tr}\left(M_l \rho_{ij}\right). \qquad (7)$$

The operators are positive and sum to the identity, giving nonnegative probabilities that sum to one. They form a projective measurement and also a special case of a POVM. The output $q_{ij,l}$ is a model induced linguistic probability and generally differs from the input $p_{ij,l}$. Semantic encoding therefore does not imply exact preservation of the original measured probabilities.

3.3 Expert weights and group references

The incoming trust score and normalized expert weight are

$$r_i = \frac{1}{n} \sum_{g=1}^{n} t_{gi}, \qquad w_i = \frac{r_i}{\sum_{h=1}^{n} r_h}. \qquad (8)$$

Since $t_{ii} = 1$, every r_i is positive and the normalization denominator is nonzero. When no trust information is available, set $w_i = 1/n$. The same weights are used for every alternative and remain fixed during consensus adjustment.

For alternative A_j, define the group reference by the convex mixture

$$\sigma_j = \sum_{i=1}^{n} w_i\, \rho_{ij}. \qquad (9)$$

This is a valid density matrix and is generally mixed. All references and adjustments are defined separately for each alternative; evaluations of different alternatives are not averaged into a single expert state.

3.4 Consensus adjustment

We measure disagreement using the trace distance,

$$D(\rho, \sigma) = \frac{1}{2} \| \rho - \sigma \|_1, \qquad \| X \|_1 = \mathrm{Tr}\sqrt{X^{\dagger} X}. \qquad (10)$$

The initial consensus deviation for alternative A_j is

$$G_j^0 = \sum_{i=1}^{n} w_i\, D\left(\rho_{ij}, \sigma_j\right). \qquad (11)$$

A smaller value indicates less disagreement. Let $\beta \in [0,1]$ be the maximum acceptable deviation. The reference σ_j is held fixed throughout the adjustment.

For a fixed strength $\lambda_j \in [0,1]$, use the channel

$$\Phi_{\lambda_j, j}(X) = \left(1 - \lambda_j\right) X + \lambda_j \mathrm{Tr}(X) \sigma_j. \qquad (12)$$

For a fixed reference and strength, (12) is a CPTP map: it is a convex mixture of the identity channel and a replacement channel [19]. This statement concerns the specified updating operation; the full data dependent workflow is not asserted to be one fixed linear quantum channel. Its action on each expert state is

$$\rho_{ij}^{c} = \left(1 - \lambda_j\right) \rho_{ij} + \lambda_j \sigma_j. \qquad (13)$$

All experts use the same strength for a given alternative. Under this adjustment the consensus deviation is exactly

$$G_j\left(\lambda_j\right) = \left(1 - \lambda_j\right) G_j^0. \qquad (14)$$

Thus the smallest common strength that satisfies the threshold is

$$\lambda_j^* = \begin{cases} 0, & G_j^0 \le \beta, \\ 1 - \beta / G_j^0, & G_j^0 > \beta. \end{cases} \qquad (15)$$

The first branch handles zero initial disagreement without division. A required adjustment reaches the threshold exactly. The weighted adjustment magnitude within this channel family is

$$C_j(\lambda_j) = \sum_{i=1}^{n} w_i \, D\left(\rho_{ij}^c, \rho_{ij}\right) = \lambda_j G_j^0. \qquad (16)$$

Accordingly, (15) also minimizes this magnitude within the specified family using a common strength. It does not establish a global optimum over all possible preference adjustments or quantum channels.

The adjustment may change an expert's measured linguistic probabilities. It represents a mathematical recommendation for moving toward the group reference; expert acceptance requires a separate practical process. Because the weights and reference remain fixed, the group aggregate satisfies

$$\sum_{i=1}^{n} w_i \, \rho_{ij}^c = \sigma_j. \qquad (17)$$

The purpose of this step is to reduce individual disagreement. It preserves the aggregated group state and therefore leaves the group ranking unchanged.

3.5 Group scoring and implementation

The final group probabilities and utility score are

$$\bar{q}_{j,l} = \mathrm{Tr}\left(M_l \sigma_j\right) = \sum_{i=1}^{n} w_i \, \mathrm{Tr}\left(M_l \rho_{ij}^c\right), \qquad (18)$$

$$U_j = \sum_{l=-\tau}^{\tau} u_l \, \bar{q}_{j,l}. \qquad (19)$$

Alternatives are ranked by decreasing U_j; equal scores receive a tied rank. The same utility vector is used throughout the numerical analyses. If another increasing utility vector is elicited, it must replace (2) consistently throughout the analysis.

For comparison, a classical benchmark uses the same input probabilities, expert weights and utility values:

$$U_j^{\mathrm{cl}} = \sum_{i=1}^{n} w_i \sum_{l=-\tau}^{\tau} u_l \, p_{ij,l}. \qquad (20)$$

Differences between (19) and (20) describe the effect of the semantic representation, without establishing greater decision accuracy. As γ tends to infinity, the semantic vectors approach the computational basis and the score in (19) approaches (20). This is a statement about the scores; the encoded density matrices need not become diagonal.

Implementation requires the normalized PLTS inputs, trust matrix or equal weight option, semantic parameter γ and consensus threshold β. Compute the semantic vectors and expert states, calculate the fixed weights and alternative specific references, evaluate G_j^0, apply (15), and obtain the final scores from (18)-(19). The adjustment strength is calculated directly, with no phase optimization or bisection search. Numerical reporting should include the selected parameters, initial and final consensus deviations, adjustment strengths, group probabilities and scores.

IV. Case Study

4.1 Numerical setting and data construction

We consider a synthetic power battery supplier selection problem with four hypothetical candidates, A1-A4, and five experts, E1-E5. The experts represent technical, procurement, production, after-sales and general management perspectives. Each assessment concerns a candidate's overall benefit oriented performance. The example does not contain attribute level measurements or verified supplier records; it illustrates the model rather than validates a real procurement decision.

The seven labels are Extremely Poor, Poor, Slightly Poor, Medium, Slightly Good, Good and Extremely Good, corresponding to indices -3 through 3. Their utilities are $0,1/6,2/6,3/6,4/6,5/6,1$. To construct the test distributions, Table 1 reports nonnegative synthetic assessment intensities $b_{ij,l}$ and their row sums. The complete PLTS inputs are obtained by

$$p_{ij,l} = \frac{b_{ij,l}}{s_{ij}}, \qquad s_{ij} = \sum_{l=-3}^{3} b_{ij,l} > 0. \qquad (21)$$

The listed intensities are not presented as probabilities. Their sums range from 0.20 to 1.30, and 11 of the 20 rows have sums different from one. For example, the two intensities 0.10 in the E4-A2 row become probabilities 0.50 and 0.50. Equation (21) defines synthetic complete distributions; it does not estimate missing mass in an elicited incomplete PLTS. All calculations use the exact listed intensities before rounding reported results.

Table 1. Synthetic assessment intensities and row sums

Expert	**Alt.**	s_{-3}	s_{-2}	s_{-1}	s_0	s_1	s_2	s_3	**Sum**
E1	A1	0.00	0.20	0.00	0.20	0.20	0.10	0.20	0.90
E1	A2	0.10	0.10	0.30	0.20	0.20	0.10	0.10	1.10
E1	A3	0.05	0.10	0.10	0.15	0.20	0.20	0.20	1.00
E1	A4	0.20	0.20	0.15	0.05	0.20	0.10	0.10	1.00
E2	A1	0.30	0.10	0.05	0.00	0.10	0.10	0.00	0.65
E2	A2	0.20	0.15	0.20	0.10	0.20	0.10	0.20	1.15
E2	A3	0.10	0.25	0.20	0.30	0.10	0.20	0.10	1.25
E2	A4	0.10	0.00	0.30	0.15	0.10	0.10	0.20	0.95
E3	A1	0.10	0.10	0.10	0.40	0.10	0.10	0.10	1.00
E3	A2	0.30	0.00	0.10	0.20	0.10	0.20	0.40	1.30
E3	A3	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.70
E3	A4	0.00	0.05	0.15	0.30	0.25	0.15	0.10	1.00
E4	A1	0.20	1.00	0.00	0.10	0.00	0.00	0.00	1.30
E4	A2	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.20
E4	A3	0.20	0.00	0.00	0.10	1.00	0.00	0.00	1.30
E4	A4	0.00	0.00	0.10	0.20	0.30	0.20	0.20	1.00
E5	A1	0.10	0.10	0.20	0.20	0.10	0.10	0.20	1.00
E5	A2	0.20	0.10	0.10	0.20	0.20	0.10	0.10	1.00
E5	A3	0.10	0.15	0.15	0.20	0.20	0.10	0.10	1.00
E5	A4	0.05	0.10	0.20	0.20	0.20	0.15	0.10	1.00

The assumed direct trust matrix is given in Table 2. It is symmetric in this numerical example, although the model permits directed trust. Diagonal entries equal one. No additional trust propagation or row normalization is applied.

Table 2. Assumed direct trust matrix

Trustor	E1	E2	E3	E4	E5
E1	1.00	0.60	0.70	0.50	0.90
E2	0.60	1.00	0.50	0.40	0.90
E3	0.70	0.50	1.00	0.60	0.90
E4	0.50	0.40	0.60	1.00	0.80
E5	0.90	0.90	0.90	0.80	1.00

4.2 Parameters and expert weights

The baseline parameters are γ=0.60 and β=0.150. The semantic parameter is selected for illustration rather than fitted to a desired ranking. Section V varies it over a wider range. The same equal interval utility vector is used for the proposed score and the classical benchmark.

Incoming trust and fixed expert weights are reported in Table 3. E5 has the largest weight because its incoming trust score is highest. The common self-trust entries ensure positive weights and also moderate their differences. The weights do not depend on the alternative or on the subsequent consensus adjustment.

Table 3. Incoming trust scores and normalized expert weights

Expert	Incoming trust score	Fixed weight
E1	0.740000	0.198925
E2	0.680000	0.182796
E3	0.740000	0.198925
E4	0.660000	0.177419
E5	0.900000	0.241935

4.3 Consensus adjustment results

The normalized PLTS inputs are encoded using (3)-(6), and each group reference is formed using (9). Table 4 reports the initial disagreement, selected common strength, final disagreement and weighted adjustment magnitude.

Table 4. Consensus adjustment with a threshold of 0.150

Alt.	Initial deviation	Strength	Final deviation	Adjustment magnitude
A1	0.310319	0.516626	0.150000	0.160319
A2	0.164088	0.085856	0.150000	0.014088
A3	0.174516	0.140478	0.150000	0.024516
A4	0.178501	0.159668	0.150000	0.028501

All four initial deviations exceed the threshold, so each alternative requires a positive adjustment. A1 has the greatest initial disagreement, 0.310319, and the largest adjustment strength, 0.516626. The other strengths range from 0.085856 to 0.159668. In each case the final deviation is 0.150000 within floating point precision.

The magnitude for A1 is 0.160319, while the other magnitudes are considerably smaller. These are weighted trace distances between original encoded states and recommended adjusted states; they are not monetary costs or observed expert effort. The reported strengths are the smallest values within the common mixing family, with no claim of optimality over unrestricted feedback mechanisms.

4.4 Final probabilities and comparison

Table 5 gives the final group measurement probabilities. Each row sums to one before rounding. Since the common adjustment preserves the group reference, the table is also the group measurement distribution before consensus adjustment.

Table 5. Group probabilities under the computational basis measurement

Alt.	s_{-3}	s_{-2}	s_{-1}	s_0	s_1	s_2	s_3
A1	0.14420	0.20865	0.14899	0.14938	0.14719	0.12677	0.07481
A2	0.09257	0.15956	0.14774	0.16778	0.19535	0.14787	0.08913
A3	0.07613	0.12365	0.14509	0.19601	0.23137	0.15216	0.07559
A4	0.04782	0.09582	0.15756	0.20694	0.21206	0.17984	0.09996

Table 6 compares the resulting utility scores with the matched classical calculation in (20). Both use the same normalized PLTS inputs, trust weights and utility vector.

Table 6. Proposed and classical utility scores

Alternative	**Proposed score**	**Classical score**	**Difference**	**Rank in both**
A1	0.437713	0.431187	0.006525	4
A2	0.502315	0.489340	0.012976	3
A3	0.523610	0.527073	-0.003464	2
A4	0.563157	0.560158	0.002998	1

The resulting order is A4≻A3≻A2≻A1. A4 has the largest score under both representations. The semantic representation changes the scores but does not change the order in this example. The largest absolute score difference is 0.012976 for A2. The A4-A3 gap changes from 0.033085 in the classical calculation to 0.039547 in the proposed calculation, whereas the A3-A2 gap decreases from 0.037734 to 0.021294.

These observations show that the representation can affect evaluation margins even when the ranking agrees. They do not establish which score is closer to a true supplier performance level. No external performance outcome is available in this synthetic example.

V. Sensitivity and Robustness Analysis

5.1 Parameter sensitivity

5.1.1 Semantic parameter

The semantic parameter is varied from 0.10 to 2.00 in increments of 0.10, keeping the original normalized inputs, trust matrix, utilities and threshold fixed. Figure 2 reports scores, initial disagreement and required adjustment strengths. Dashed horizontal lines in the score panel show the classical values.

The ranking A4≻A3≻A2≻A1 is retained at all 20 tested values. A1's score ranges from 0.433149 to 0.458047, A2's from 0.496854 to 0.502444, A3's from 0.508383 to 0.525447, and A4's from 0.547960 to 0.563700. The score curves are continuous but need not be monotone.

Initial disagreement increases on this grid for all four alternatives. For example, A1 changes from 0.178359 at γ=0.10 to 0.423893 at γ=2.00. Its required strength increases from 0.159002 to 0.646137. At the smallest tested parameter, A2-A4 already have deviations below 0.15 and require no adjustment.

A smaller parameter broadens the semantic vectors and can bring different assessments closer in the representation space. The resulting lower disagreement is partly imposed by the encoding. It should therefore be interpreted conditional on the semantic assumption, rather than as evidence that the experts themselves became more similar. A larger parameter reduces overlap and approaches the matched classical score limit. This experiment illustrates parameter dependence for the present data, without establishing a universal monotonic law.

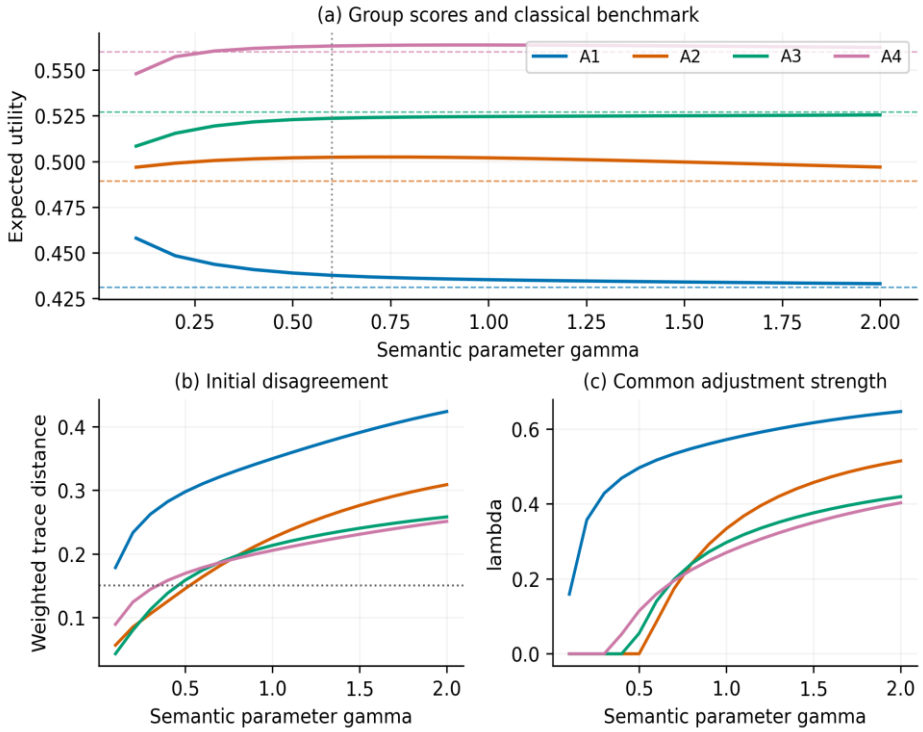


Fig. 2. Sensitivity to the semantic parameter.

5.1.2 Semantic parameter

The threshold is varied from 0.00 to 0.30 in increments of 0.01, with γ=0.60. Figure 3 reports final disagreement and adjustment strength.

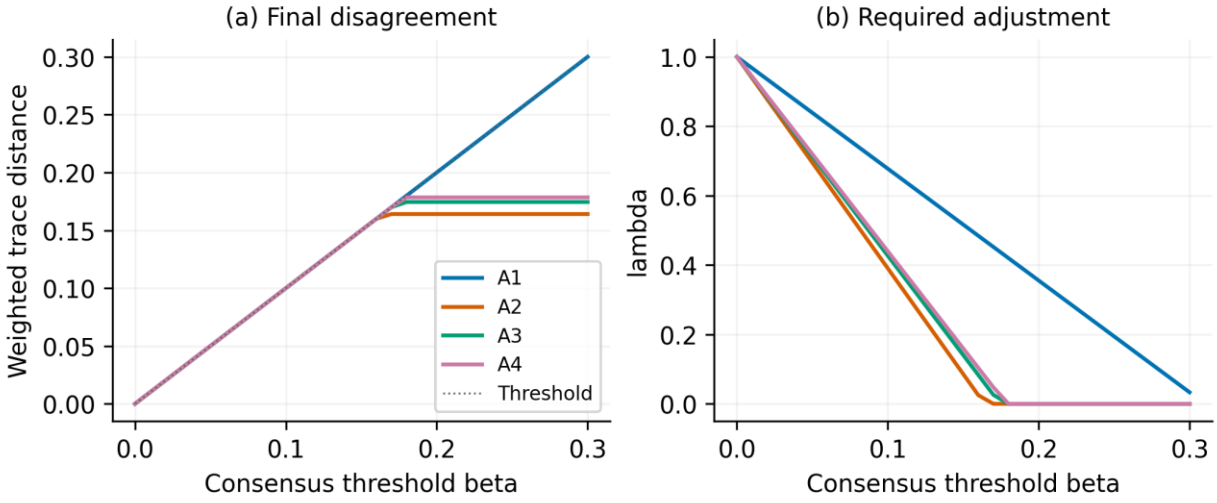


Fig. 3. Sensitivity to the consensus threshold.

For each alternative, final disagreement equals $\min(G_j^0, \beta)$. If the threshold is below the initial deviation, the selected strength decreases as the threshold is relaxed; once the initial deviation is acceptable, the strength is zero. At a threshold of zero, all expert states are replaced by the corresponding group reference. At the baseline threshold of 0.15, the strengths agree with Table 4.

The utility scores and ranking are exactly invariant to the threshold in this model. This is a structural consequence of (17), rather than an experimental finding of superior robustness. The threshold controls individual coordination effort while the fixed aggregate determines the decision result.

5.2 Input perturbation experiments

Two Monte Carlo experiments use 1,000 repetitions each. The baseline semantic parameter, threshold and utilities remain fixed. For every repetition, all quantities that depend on the perturbed input, including group

references and adjustment strengths, are recomputed. Experiments use double precision calculations in NumPy 2.3.5 and its default PCG64 generator, with seeds 20261025 for PLTS perturbations and 20261026 for trust perturbations. Probability noise is drawn as arrays ordered by expert, alternative and term; trust noise is drawn as a full 5 by 5 array with the diagonal ignored.

5.2.1 PLTS probability perturbations

Independent zero mean Gaussian noise with standard deviation 0.03 is added to every component of the normalized baseline distributions. The resulting values are clipped below at zero and normalized row by row:

$$\tilde{p}_{ij,l} = \frac{\max(0, p_{ij,l} + \epsilon_{ij,l})}{\sum_{h=-3}^{3} \max(0, p_{ij,h} + \epsilon_{ij,h})}, \qquad \epsilon_{ij,l} \sim N(0, 0.03^2). \qquad (22)$$

If an entire clipped row is zero, the baseline row is retained; this fallback is not needed in the reported runs. Perturbations are applied to the normalized probabilities rather than the raw intensities in Table 1. Figure 4 shows the resulting utility distributions, with black diamonds indicating baseline scores.

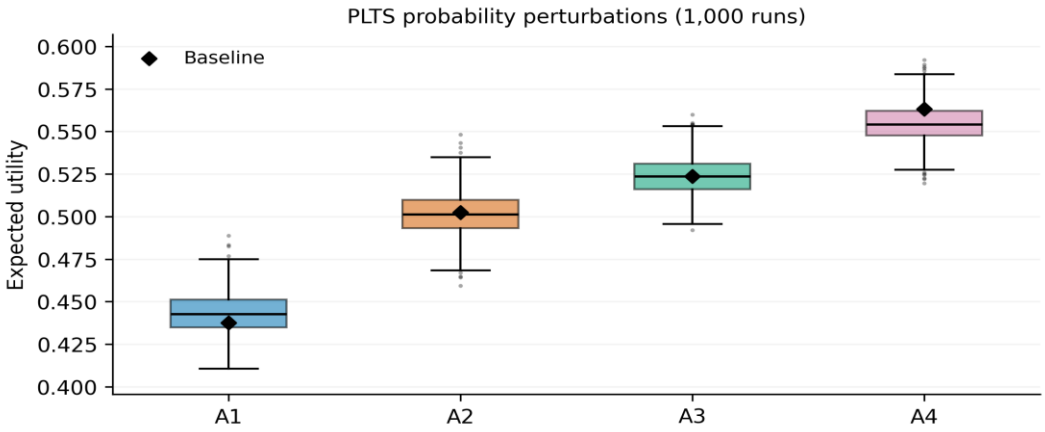


Fig. 4. Utility distributions under PLTS probability perturbations.

The mean scores are 0.443080, 0.501376, 0.523561 and 0.554812 for A1-A4. Table VII reports their sample standard deviations and empirical 2.5th and 97.5th percentiles. Clipping and normalization do not preserve the mean of the original noise process, and the semantic encoding is nonlinear. Thus mean perturbed scores need not coincide with baseline values.

A4 remains first in 974 of the 1,000 runs. A3 is first in 25 runs and A2 in one. The complete baseline order is retained in 883 runs. Ninety runs exchange the positions of A2 and A3 while retaining A4 first, showing that the smaller middle score gap is more vulnerable than the baseline order alone suggests. The mean Spearman correlation with the baseline ranking is 0.9762, with sample standard deviation 0.0666.

5.2.2 Trust matrix perturbations

For the second experiment the normalized PLTS inputs are fixed. Independent Gaussian noise with standard deviation 0.05 is added to each off-diagonal trust entry, followed by clipping to the unit interval. Diagonal entries remain one:

$$\tilde{t}_{gi} = \min\{1, \max(0, t_{gi} + \zeta_{gi})\} \quad (g \neq i), \qquad \tilde{t}_{ii} = 1. \qquad (23)$$

The off-diagonal draws are independent, so a perturbed matrix need not remain symmetric. This is allowed by the directed trust definition. Expert weights are recalculated for every matrix. Figure 5 uses the same vertical scale as Figure 6 to make the score dispersion visible on a common scale.

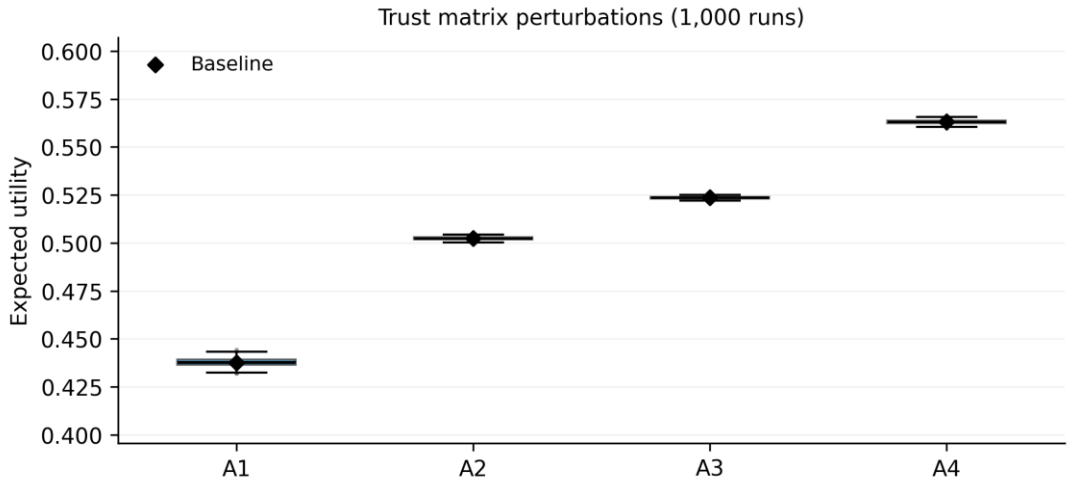


Fig. 4. Utility distributions under trust matrix perturbations.

The score standard deviations range from 0.000555 for A3 to 0.001963 for A1. Every repetition retains the baseline order and therefore also retains A4 first. The Spearman correlation is 1.0000 in each run. These observations describe the assumed matrix and the selected noise magnitude. They do not imply that other trust configurations or larger perturbations would preserve the ranking.

Table 7. Score summaries across the perturbation experiments

Noise source	Alt.	Mean score	SD	P2.5	P97.5
PLTS	A1	0.443080	0.011798	0.420815	0.466124
PLTS	A2	0.501376	0.012407	0.476731	0.523650
PLTS	A3	0.523561	0.011249	0.500601	0.546237
PLTS	A4	0.554812	0.010875	0.532639	0.575987
Trust	A1	0.437818	0.001963	0.433951	0.441532
Trust	A2	0.502308	0.000797	0.500736	0.503852
Trust	A3	0.523620	0.000555	0.522611	0.524727
Trust	A4	0.563077	0.000943	0.561305	0.564949

SD denotes the sample standard deviation across 1,000 runs. P2.5 and P97.5 are empirical score percentiles, not confidence limits for the mean.

Table 8. Ranking retention across the perturbation experiments

Noise source	Runs	Mean correlation	Correlation SD	Full ranking retained	A4 retained first
PLTS	1000	0.9762	0.0666	88.3%	97.4%
Trust	1000	1.0000	0.0000	100.0%	100.0%

5.3 Discussion and practical limits

The proposed model is suitable as a compact numerical procedure when complete linguistic distributions, a shared utility scale and an interpretable trust weighting rule are available. A decision analyst can inspect how semantic broadening affects scores and can calculate the common individual adjustment needed for a stated distance threshold. Keeping the reference fixed makes the relation between consensus adjustment and group aggregation transparent.

Several limitations follow from the design. The pure state representation with zero phases is an assumed embedding of the probability vector. The semantic parameter has no empirical calibration here. Moreover, with a fixed computational basis measurement, the scoring stage can be implemented as nonlinear amplitude

smoothing; the study establishes no quantum computational advantage. Trust weights reflect assumed social confidence and can favor socially trusted experts without guaranteeing that their judgments are accurate.

The consensus rule applies one strength to all experts for a given alternative. It does not represent heterogeneous revision costs, refusal to adjust or a bargaining process. Its minimum magnitude property applies within that common mixing family. Since it preserves the weighted aggregate, it cannot correct a poor initial group benchmark. More flexible feedback mechanisms, including cost and trust models such as [10] and [11], would require additional assumptions and a different optimization problem.

Finally, the synthetic setting contains only five experts, four alternatives and complete distributions. No claim is made about the preferred supplier in an actual market. Attribute level assessments, missing probability mass, empirical semantic calibration and expert acceptance are appropriate subjects for further investigation.

VI. Conclusion

This paper presents a trust weighted quantum inspired procedure for complete probabilistic linguistic group decision making. Gaussian semantic vectors and explicit state normalization define the representation, incoming trust determines fixed expert weights, and a specified computational basis measurement connects the group state to linguistic utility scores. A single common mixing channel moves individual states toward each alternative's group reference.

The selected adjustment strength has a direct expression under a weighted trace distance threshold. Its adjustment magnitude is minimal within the prescribed common mixing family. The aggregate remains unchanged, which clarifies that the adjustment coordinates individual representations while the ranking follows from the original weighted group reference. These statements do not imply unrestricted global optimality, preservation of every original linguistic probability or an observed model of human cognition.

In the synthetic supplier example, all four initial deviations exceed 0.15 and are reduced to that threshold. Both the proposed representation and the matched classical benchmark rank the alternatives as A4, A3, A2 and A1, with different score margins. This ranking is retained throughout the tested semantic parameter grid. Probability perturbations retain A4 first in 97.4% of runs and retain the entire order in 88.3%; all tested trust perturbations retain the order. The occasional changes under probability noise limit any claim of unconditional ranking stability.

Future work can use elicited semantic similarities to calibrate the representation, study incomplete distributions and attribute level aggregation, and model heterogeneous or voluntary expert revisions. Validation with independent decision outcomes would be needed before making claims about predictive accuracy or practical superiority.

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Declaration of interest statement
The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted. We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work.

Author contributions
Xiuli Geng: Writing – original draft, Visualization, Resources, Methodology, Investigation, Funding acquisition, Conceptualization.
Yuanxin Xiong: Writing–original draft, Software, Methodology, Formal analysis, Data curation, Conceptualization.

Data availability
The initial data for the case study has been given in the paper and appendix.