*International Refereed Journal of Engineering and Science (IRJES)*
*ISSN (Online) 2319-183X, (Print) 2319-1821*
*Volume 14, Issue 3 (May - June 2025), PP. 31-37*

# Analysis of Parallel and Series Queuing Model with Bulk Arrival and Time Independent Service

Neha Gupta[1], Deepak Gupta[2],Renu Gupta[3]
[1]*Research Scholar in MathematicsMaharishi Markandeshwar (Deemed to be University),Mullana,India*
[2]*Prof & Head, Department of MathematicsMaharishi Markandeshwar (Deemed to be University),Mullana,India*
[3]*Assistant Professor, Govt College Naraingarh, Amabala, India.* [3]*Corresponding author*

***Abstract:*** *In this article, a queuing model with two parallel servers associated to a single server in common is presented.To calculate the various queuing parameters, including queue length, variance, mean waiting time, and probability, statistical tools and generating function approach have been used to ensure that the proposed model is implemented in a variety of real-time problems. A general mathematical expression of the queuing model has been created to guarantee that the suggested model is applied in a range of real-time scenarios. The model that is being provided is simple to comprehend and provides decision-makers with a valuable tool for handling multitasking issues.*
***KEYWORDS:*** *Parallel and series queuing model, Mean queue length,Variance, Bulk Arrival, Average Waiting time.*

---

Date of Submission: 12-05-2025 Date of acceptance: 27-05-2025

---

## I. INTRODUCTION

Many methods that can help us better plan and organize the intricate difficulties we face on a daily basis are demonstrated by operational research.A person can improve their decision-making skills by putting certain strategies (such as queuing theory) into practice. In real-world scenarios, queues form when a system's arrival rate exceeds its capacity to handle it.Queuing theory can be applied to any everyday situation, such as a car pulling up to the gas station, a patient arriving into a doctor's office, or people coming into a bank.Jackson R.R.P.[1954]designed the phase-type service queuing system .In the theory of queues, Maggu[1970] developed the idea of bitendom, which relates to a real-world scenario that arises in a manufacturing concern. Later, several writers expanded on this concept with varying alterations and justifications. KhodadiAbutaleb (1989) made changes to the queue system that Maggu had researchedby presuming that the queue number and the service parameter are linearly proportionate.Singh and Man [4] looked into how a serial queuing system behaved in a steady state when there were impatient customers.Gupta et al.'s examination of parallel and bi-serial channels joined to a solo server was highlighted in their analysis of the queuing model.In 2006, Singh T.P et al. conducted a study on the stablestate of a queue model that consisted of pair of subsystems connected by a common channel and a biserial channel.In their 2007 study, Deepak, Singh T.P., et al. examined a network queue architecture that included a shared server is connected to a parallel and biserialchannels.MittalMeenu investigated the modeling of a queuing network with biserial bulk arrivals connected to a common server. This study by Chien et al. [2023] looks at theoretical dynamic sequencing in a manufacturing system that havebatch procedures and time limits in the process queue.This paper focuses on the creation of a model that comprises three servers $S_1, S_2$ and $S_3$ is the subject of this paper. Unlike previous models, this one has a single server with bulk arrivals rather than two or more servers.

## II. MODEL DESCRIPTION

In the current work model consists of three servers
$S_1, S_2$ and $S_3$inwhich $S_2$and $S_3$are parallel. The system$S_2$ incorporates two parallel servers$S_{21}, S_{22}$ andsystem$S_3$ incorporates two parallel servers $S_{31}$and $S_{32}$.The server $S_1$ is typicallyassociated in tandem with two parallel servers$S_2$ and $S_3$.After service at server $S_1$is completed, the number of visitors $d_1$, arriving at average arrival rate$\lambda_1$and batch size$b_1$, can use the facility at server$S_2$or $S_3$with the possibility$\alpha$ and $\beta$ such that $\alpha + \beta = 1$.The visitor further has two possibility to take service from service channel$S_2$with the possibility $\alpha_1$ from $S_{21}$and $\alpha_2$ from $S_{21}$ such that $\alpha_1 + \alpha_2 = 1$and the customers follows the same process entered in the service channel $S_3$with the possibility $\beta_1$and $\beta_2$ such that $\beta_1 + \beta_2 = 1$and $\alpha\alpha_1 + \alpha\alpha_2 + \beta\beta_1 + \beta\beta_2 = 1$.

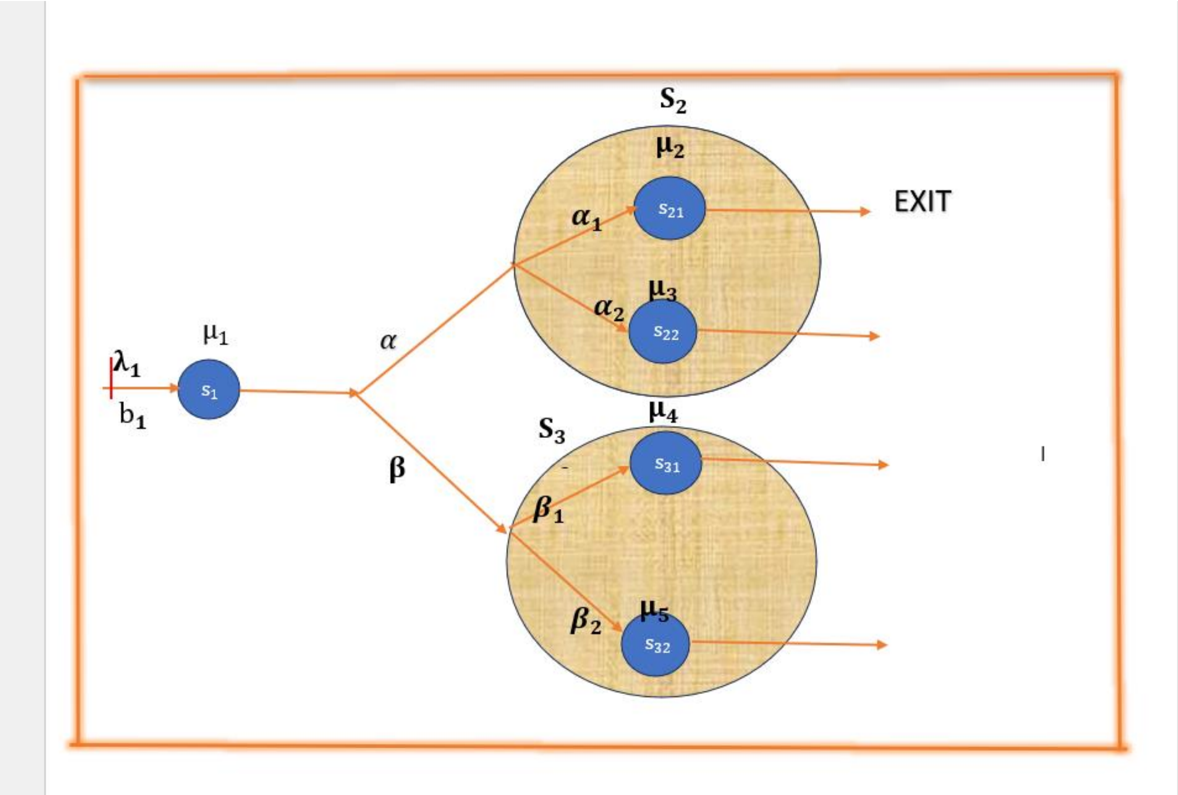


**FIGURE 1 :**QUEUE NETWORK MODEL

| | | | | | |
|---|---|---|---|---|---|
| Number of visitors | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ |
| Service Channels | $S_1$ | $S_{21}$ | $S_{22}$ | $S_{31}$ | $S_{32}$ |
| Service rate | $\mu_1$ | $\mu_2$ | $\mu_3$ | $\mu_4$ | $\mu_5$ |
| Arrival rate | $\lambda_1$ | | | | |
| Possibility of visitors from one service changing to another service channel. | $S_1 \to S_2\alpha$<br>$S_1 \to S_{21}\alpha_1$<br>$S_1 \to S_{22}\alpha_2$ | $S_1 \to S_3\beta$<br>$S_1 \to S_{31}\beta_1$<br>$S_1 \to S_{32}\beta_2$ | $S_{21} \to S_{22}p_{23}$<br>$S_{21} \to S_{22}p_{32}$ | | |
| Batch Size | $b_1$ | | | | |

**NOTATIONS**

## III. MATHEMATICAL MODEL FORMULATION

Assume that $P_{d_1,d_2,d_3,d_4,d_5}$represents the joint possibilityof visitors$d_1, d_2, d_3, d_4, d_5$facing the service channels $S_1, S_{21}, S_{22}, S_{31}, S_{31}$,corresponding,with$d_1, d_2, d_3, d_4, d_5 \geq 0$.
The following is the queue network model's differential difference equation in a transient state:
**For $\mathbf{d_1 > b_1, d_2 > 0, d_3 > 0, d_4 > 0, d_5 > 0}$**

$$\begin{aligned} P_{d_1,d_2,d_3,d_4,d_5}(t) &= -(\lambda_1 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)P_{d_1,d_2,d_3,d_4,d_5}(t) + \lambda_1 P_{d_1-b_1,d_2,d_3,d_4,d_5}(t) \\ &+ \mu_1\alpha_1\alpha P_{d_1+1,d_2-1,d_3,d_4,d_5}(t) + \mu_1\alpha_2\alpha P_{d_1+1,d_2,d_3-1,d_4,d_5}(t) + \mu_1\beta_1\beta P_{d_1+1,d_2,d_3,d_4-1,d_5}(t) \\ &+ \mu_1\beta_2\beta P_{d_1+1,d_2,d_3,d_4,d_5-1}(t) + \mu_2 P_{d_1,d_2+1,d_3,d_4,d_5}(t) + \mu_2 p_{23} P_{d_1,d_2+1,d_3-1,d_4,d_5}(t) \\ &+ \mu_3 P_{d_1,d_2,d_3+1,d_4,d_5}(t) + \mu_3 p_{32} P_{d_1,d_2-1,d_3+1,d_4,d_5}(t) + \mu_4 P_{n_1,n_2,d_3,d_4+1,d_5}(t) \\ &+ \mu_5 P_{d_1,d_2,d_3,d_4,d_5+1}(t) \end{aligned}$$

(48) equations are obtained by considering all the possible combination for different values of$d_1, d_2, d_3, d_4, d_5$.
The generating function technique can be defined as:
$H(E,F,G,R,S)=\sum_{d_1=0}^{\infty}\sum_{d_2=0}^{\infty}\sum_{d_3=0}^{\infty}\sum_{d_4=0}^{\infty}\sum_{d_5=0}^{\infty} P_{d_1,d_2,d_3,d_4,d_5} E^{d_1}F^{d_2}G^{d}R^{d_4}S^{d_5}$
In order to solve, we also define the partial generating function as

$$H_{d_2,d_3,d_4,d_5}(E) = \sum_{d_1=0}^{\infty} P_{d_1,d_2,d_3,d_4,d_5} E^{d_1}$$
$$H_{d_3,d_4,d_5}(E,F) = \sum_{d_2=0}^{\infty} H_{d_2,d_3,d_4,d_5}(E) F^{d_2}$$
$$H_{d_4,d_5}(E,F,G) = \sum_{d_3=0}^{\infty} H_{d_3,d_4,d_5}(E,F) G^{d_3}$$
$$H_{d_5}(E,F,G,R) = \sum_{d_4=0}^{\infty} H_{d_4,d_5}(E,F,G) R^{d_4}$$
$$H(E,F,G,R,S) = \sum_{d_5=0}^{\infty} H_{d_5}(E,F,G,R) S^{d_5} \qquad (A')$$

Following the application of the generating function technique and the calculus law to simplify equations (1–48), we obtain

$$H(E,F,G,R,S)=\frac{\begin{array}{c}\mu_1(1-\frac{F\alpha\alpha_1}{E}-\frac{\alpha\alpha_2 G}{E}-\frac{\beta_1\beta R}{E}-\frac{\beta_2\beta S}{E})H_0(F,G,R,S)\\ +\mu_2(1-\frac{S}{F})H_0(E,G,R,S)\\ +\mu_3\left(1-\frac{S}{G}\right)H_0(E,F,R,S)\\ +\mu_4\left(1-\frac{S}{R}\right)H_0(E,F,G,S)\\ +\mu_5(1-\frac{1}{S})H_0(E,F,G,R)\end{array}}{\begin{array}{c}\lambda_1(1-X^{b_1})+\mu_1\left(1-\frac{F\alpha\alpha_1}{E}-\frac{\alpha\alpha_2 G}{E}-\frac{\beta_1\beta R}{E}-\frac{\beta_2\beta S}{E}\right)\\ +\mu_2(1-\frac{S}{F})\\ +\mu_3\left(1-\frac{S}{G}\right)\\ +\mu_4\left(1-\frac{S}{R}\right)\\ +\mu_5(1-\frac{1}{S})\end{array}} \qquad (I)$$

For S=R=G=F=E=1 and$H(1,1,1,1,1)=1$
Conveniently, we specify: $H_0(F,G,R,S) = H_1$;

$$\left.\begin{array}{l} H_0(E,G,R,S) = H_2 \\ H_0(E,F,R,S) = H_3 \\ H_0(E,F,G,S) = H_4; \\ H_0(E,F,G,R) = H_5; \end{array}\right\} \qquad (1)$$

The equation above simplifies to an indeterminate form when we assume H(E,F,G,R,S)=1 at E=F=G=R=S=1. The following equations can be obtained by differentiating the numerator and denominator w.r.t. E,F,G,R and S and taking E, F, G, R, S  1 one by one, in the equation (1) and using the values of $H_1, H_2, H_3, H_4, H_5$, we get
$\mu_1 H_1 = -\lambda_1 b_1 + \mu_1$(1) ⟶
$\mu_2 H_2 - \mu_1\alpha\alpha_1 H_1 = \mu_2 - \mu_1\alpha\alpha_1$ (2)
$\mu_3 H_3 - \mu_1\alpha\alpha_2 H_1 = \mu_3 - \mu_1\alpha\alpha_2$ (3)
$\mu_4 H_4 - \mu_1\beta\beta_1 H_1 = \mu_4 - \mu_1\beta\beta_1$(4)
$\mu_5 H_5 - \mu_1\beta\beta_2 H_1 = \mu_5 - \mu_1\beta\beta_2$(5)
On solvingthese equations for$H_1, H_2, H_3, H_4, H_5$, weget
$H_1 = 1 - \frac{\lambda_1 b_1}{\mu_1}$
$H_2 = 1 - \frac{\lambda_1 b_1}{\mu_2}\alpha\alpha_1$
$H_3 = 1 - \frac{\lambda_1 b_1}{\mu_3}\alpha\alpha_2$
$H_4 = 1 - \frac{\lambda_1 b_1}{\mu_4}\beta\beta_1$

$H_5 = 1 - \frac{\lambda_1 b_1}{\mu_5}\beta\beta_2$

The joint probability is determined by utilizing the values of $H_1, H_2, H_3, H_4, H_5$

$P_{c_1, c_2, c_3, c_4, c_5} = \rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}\rho_5^{n_5}(1-\rho_5)(1-\rho_4)(1-\rho_3)(1-\rho_2)(1-\rho_1)$

Where

$$\rho_5 = 1 - H_5, \rho_4 = 1 - H_4, \rho_3 = 1 - H_3, \rho_2 = 1 - H_2, \rho_1 = 1 - H_1$$

Additionally, the solution exists under steady state conditions if $\rho_5\rho_4\rho_3\rho_2\rho_1 < 1$

## IV. QUEUE CHARACTERISTICS

The averageline length is provided by

$L = L_{q_1} + L_{q_2} + L_{q_3} + L_{q_4} + L_{q_5}$

Here

$$L_{q_1} = \frac{\rho_1}{1-\rho_1} = \frac{\lambda_1 b_1}{\mu_1 - \lambda_1}$$
$$L_{q_2} = \frac{\rho_2}{1-\rho_2} = \frac{\lambda_1 b_1 \alpha\alpha_1}{\mu_2 - \lambda_1\alpha\alpha_1}$$
$$L_{q_3} = \frac{\rho_3}{1-\rho_3} = \frac{\lambda_1 b_1 \alpha\alpha_2}{\mu_3 - \lambda_1\alpha\alpha_2}$$
$$L_{q_4} = \frac{\rho_4}{1-\rho_4} = \frac{\lambda_1 b_1 \beta\beta_1}{\mu_4 - \lambda_1\beta\beta_1}$$
$$L_{q_5} = \frac{\rho_5}{1-\rho_5} = \frac{\lambda_1 b_1 \beta\beta_2}{\mu_5 - \lambda_1\beta\beta_2}$$

Therefore,

$$L = \frac{\lambda_1 b_1}{\mu_1 - \lambda_1} + \frac{\lambda_1 b_1 \alpha\alpha_1}{\mu_2 - \lambda_1\alpha\alpha_1} + \frac{\lambda_1 b_1 \alpha\alpha_2}{\mu_3 - \lambda_1\alpha\alpha_2} + \frac{\lambda_1 b_1 \beta\beta_1}{\mu_4 - \lambda_1\beta\beta_1} + \frac{\lambda_1 b_1 \beta\beta_2}{\mu_5 - \lambda_1\beta\beta_2}$$

Now, variance of queue

$$V = \frac{\rho_5}{(-\rho_5+1)^2} + \frac{\rho_4}{(-\rho_4+1)^2} + \frac{\rho_3}{(-\rho_3+1)^2} + \frac{\rho_2}{(-\rho_2+1)^2} + \frac{\rho_1}{(-\rho_1+1)^2}$$
$$= \frac{\lambda_1 b_1 \beta\beta_2\mu_5}{(\mu_5 - \lambda_1\beta\beta_2)^2} + \frac{\lambda_1 b_1 \beta\beta_1\mu_4}{(\mu_4 - \lambda_1\beta\beta_1)^2} + \frac{\lambda_1 b_1 \alpha\alpha_2\mu_3}{(\mu_3 - \lambda_1\alpha\alpha_2)^2} + \frac{\lambda_1 b_1 \alpha\alpha_1\mu_2}{(\mu_2 - \lambda_1\alpha\alpha_1)^2} + \frac{\lambda_1 b_1\mu_1}{(\mu_1 - \lambda_1)^2}$$

## V. NUMERICAL ILLUSTRATION

To assess the algorithm's effectiveness, the following calculation is carried out

| Sr.no. | Average service rate | Average arrival rate | Batch size | Probabilities |
|---|---|---|---|---|
| 1 | $\mu_1 = 18$ | $\lambda_1=4$ | $b_1=4$ | $\alpha = 0.6$ |
| 2 | $\mu_2=11$ | | | $\beta = 0.4$ |
| 3 | $\mu_3=20$ | | | $\alpha_1 = 0.3$ |
| 4 | $\mu_4=9$ | | | $\beta_1=0.2$ |
| 5 | $\mu_5=9$ | | | $\alpha_2 = 0.7$ |
| | | | | $\beta_2 = 0.8$ |

Determine the combined probability, average length of the queue, variance of the queue, and average customer wait time.

Solution:

$\rho_1 = \frac{\lambda_1 b_1}{\mu_1} = 0.8$

$\rho_2 = \frac{\lambda_1 b_1}{\mu_2}\alpha\alpha_1 = 0.256$

$\rho_3 = \frac{\lambda_1 b_1}{\mu_3}\alpha\alpha_2 = 0.336$

$$\rho_4 = \frac{\lambda_1 b_1}{\mu_4}\beta\beta_1 = 0.142$$

$\rho_5 = \frac{\lambda_1 b_1}{\mu_5}\beta\beta_2 = 0.568$ where $\alpha\alpha_1 + \alpha\alpha_2 + \beta\beta_1 + \beta\beta_2 = 1$

Therefore , mean line length

$L = L_{q_1} + L_{q_2} + L_{q_3} + L_{q_4} + L_{q_5}$

$$= \frac{\lambda_1 b_1}{\mu_1 - \lambda_1} + \frac{\lambda_1 b_1 \alpha\alpha_1}{\mu_2 - \lambda_1 \alpha\alpha_1} + \frac{\lambda_1 b_1 \alpha\alpha_2}{\mu_3 - \lambda_1 \alpha\alpha_2} + \frac{\lambda_1 b_1 \beta\beta_1}{\mu_4 - \lambda_1 \beta\beta_1} + \frac{\lambda_1 b_1 \beta\beta_2}{\mu_5 - \lambda_1 \beta\beta_2}$$

= 1.14+0.2801+0.3668+0.1474+0.6632

= 2.5975

Now, variance of queue

$$V=\frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2}$$

= 66.11+0.480+0.7636+0.1929+3.043

= 70.57

Customers' average wait time

$E(W)= \frac{L}{\lambda_1} = 0.64$

## VI. BEHAVIOUR ANALYSIS OF THE MODEL

The behavior of the average and partial queue lengths in relation to changes in service and arrival rates will be discussed in this section.

(i) $L, L_{q_1}, L_{q_2}, L_{q_3}, L_{q_4}, L_{q_5}$ behaviors for various values of $\lambda_1$. .

(ii) $L, L_{q_1}, L_{q_2}, L_{q_3}, L_{q_4}, L_{q_5}$ behavior for various values of $b_1$.

(iii) A graphic representation of the system's mean and partial queue lengths for various $\lambda_1$ and $b_1$. values.

**TABLE 1 :**In relation to $\lambda_1$, the system's mean and partial queue lengths

| $\lambda_1$ | $L_{q_1}$ | $L_{q_2}$ | $L_{q_3}$ | $L_{q_4}$ | $L_{q_5}$ | L |
|---|---|---|---|---|---|---|
| **4** | 1.14 | 0.2801 | 0.3668 | 0.1474 | 0.6632 | 2.5975 |
| **4.1** | 9.38 | 0.3633 | 0.5253 | 0.1675 | 1.393 | 11.829 |
| **4.2** | 12.15 | 0.375 | 0.543 | 0.175 | 1.481 | 14.724 |
| **4.3** | 17.51 | 0.3879 | 0.5654 | 0.1771 | 1.56804 | 20.208 |
| **4.4** | 30.25 | 0.4005 | 0.5862 | 0.1820 | 1.659 | 33.077 |

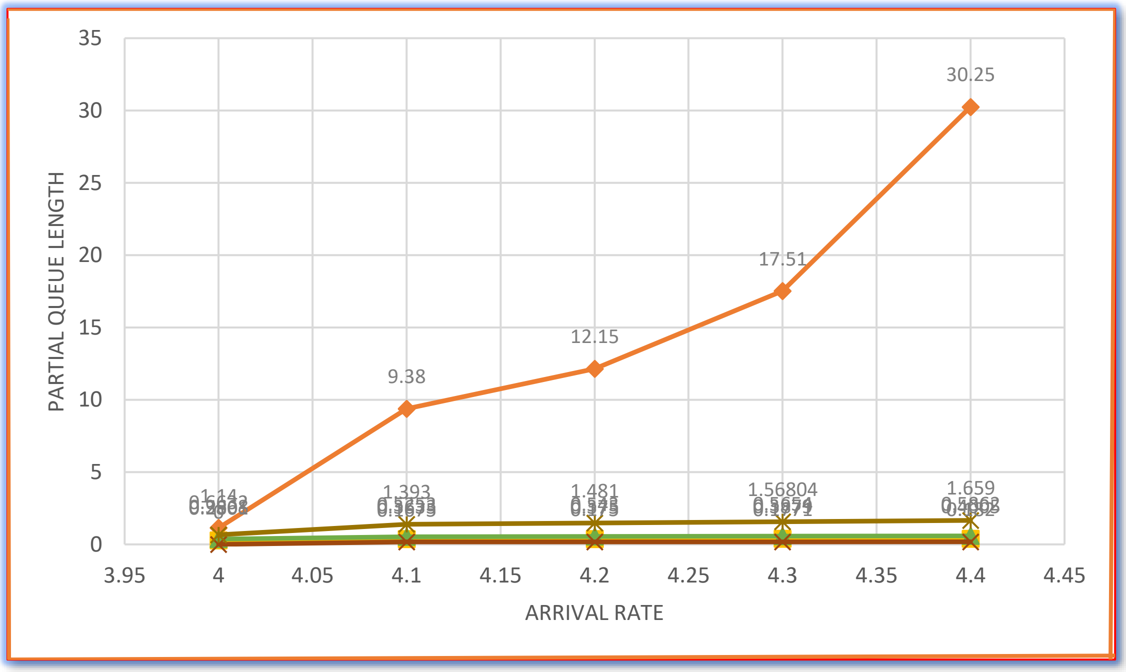


**FIGURE 2:**$L_{q_1}, L_{q_2}, L_{q_3}, L_{q_4}, L_{q_5}$ VS. $\lambda_1$

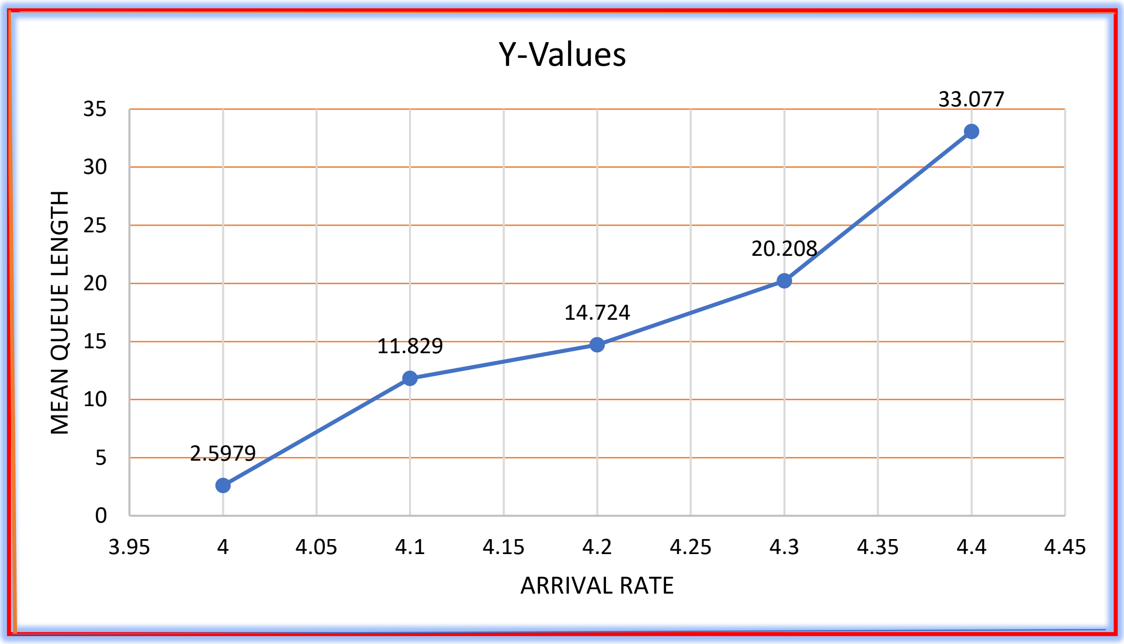


**FIGURE 3:** $L VS. \lambda_1$

| $b_1$ | $L_{q_1}$ | $L_{q_2}$ | $L_{q_3}$ | $L_{q_4}$ | $L_{q_5}$ | L |
|---|---|---|---|---|---|---|
| **2** | 0.497 | 0.108 | 0.144 | 0.054 | 0.269 | 1.072 |
| **3** | 0.992 | 0.172 | 0.233 | 0.084 | 0.466 | 1.947 |
| **4** | 1.976 | 0.243 | 0.336 | 0.116 | 0.736 | 3.407 |
| **5** | 4.88 | 0.324 | 0.459 | 0.149 | 1.127 | 6.939 |

**TABLE 2 :System's mean queue length in relation to $b_1$**

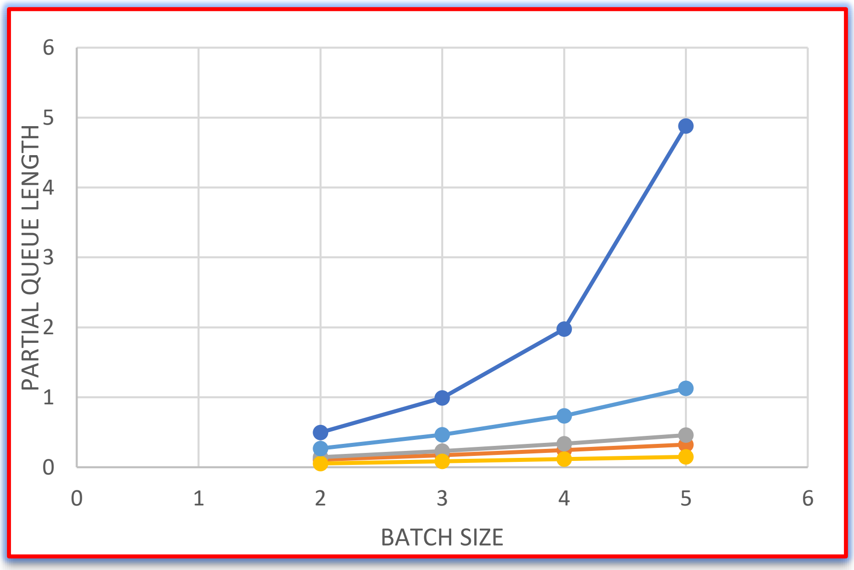


**FIGURE 4 :** $L_{q_1}, L_{q_2}, L_{q_3}, L_{q_4}, L_{q_5}$ VS. $b_1$

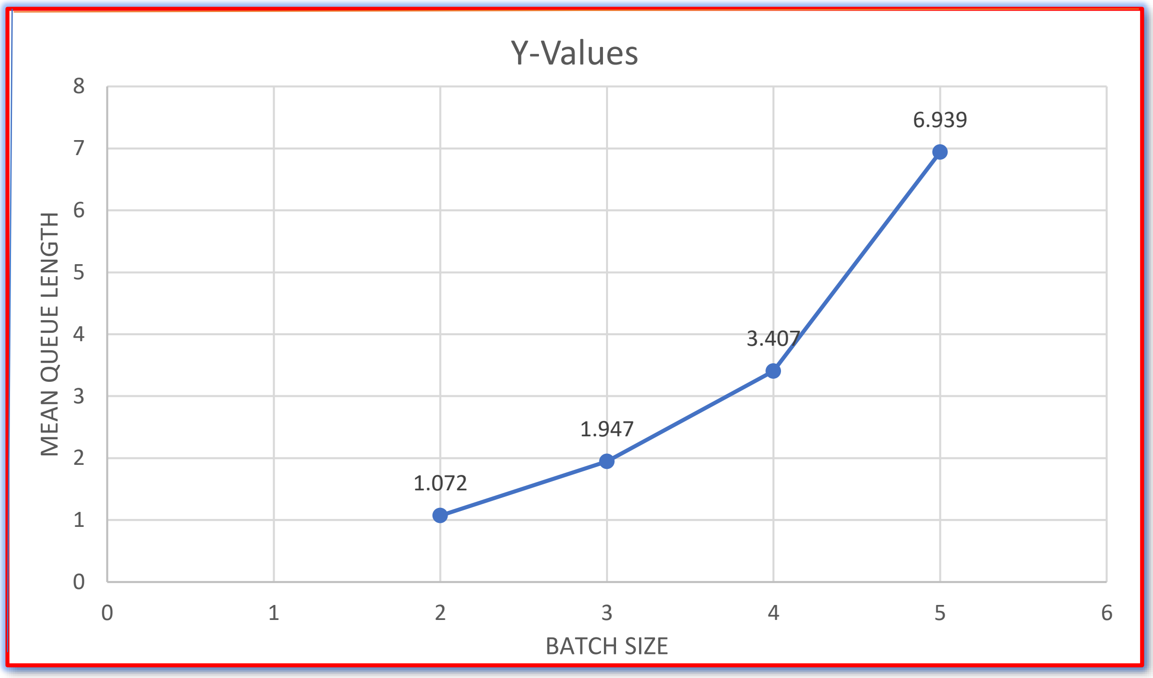


**FIGURE 5:**L VS $b_1$

## VII. RESULT AND DISCUSSION

1.The partial queue length grows gradually and nearly at the same rate as the arrival rate at server $S_1$, as shown by Table 1 and Figure 2.
2.As batch size increases, the mean queue length grows gradually and steadily, as shown by Table 2 and Figures 4 and 5

## VIII. CONCLUSION

We may conclude, after analyzing this complex queue network model with serial and parallel servers, that the queue length grows with arrival rate, but eventually it grows quicker and faster. The length of the line is getting shorter as service rates rise, and eventually it will spread out at a steady pace. This analysis will be extremely beneficial in understanding the system and redesigning it to reduce congestion and raise customer happiness when we apply this model to real-world scenarios.

## REFERENCES

[1]. Jackson, R.R.P., “Queuing system with phase type service”, *O.R.Quat*. Vol.5,109-120(1954).
[2]. Maggu, P.L.,“Phase type service queue with two servers in Biserial”, *J.OP. RES. Soc Japan* Vol.13No.1(1970).
[3]. Singh, Man, “Steady state behaviour of serial queuing processes with impatient customers", *Math, Operations forsch. U. statist*. Ser. 15(2),289-298, (1984).
[4]. KhodadiAbutaleb, “On two heterogeneous parallel service linked in series with a common server underparametric services constraint”, *PAMS* VOL.XXIX No.1 (1989).
[5]. Kumar Vinod, Singh T.P. and K. Rajinder, “Steady state behaviour of a queue model comprised of twosubsystems with biserial channels linked with common channel”, *Reflection des ERA*. Vol. 1 issue 2, May2006, pp135-152 (2006).
[6]. Gupta Deepak, Singh T.P., Rajinderkumar, “Analysis of a network queue model comprised of biserial andparallel channel linked with a common server”, *Ultra Science* Vol. 19(2)M, 407-418 (2007).
[7]. Sharma, S., & Sharma, D. G. S, “On linkage of a flowshop scheduling model including job block criteria with a parallel biserial queue network”, *Computer Engineering and Intelligent System*, 3(2), 17-28, (2012).
[8]. MittalMeenu,R.Gupta,“Modelling of biserial bulk queue network linked with common server”.*International Journal of Mathematics Trends and Technology*,vol.56,no.6,pp.430- 436,(2018).
[9]. Chien, W. C., Chou, Y. L., & Wu, C. H,“Stochastic Scheduling for Batch Processes With Downstream Queue Time Constraints”. *IEEE Transactions on Semiconductor Manufacturing*(2023).