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Neutron skin thickness of finite nuclei using finite range effective interaction with dipole properties

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Abstract
The dipole polarizability 'α_D' and isovector giant dipole resonance energy constant (IVGDR) 'D' are analyzed using Droplet Model (DM)in finite range effective interaction for two different splitting of exchange strength parameters $E_{ex}^{l} = E_{ex}/2$ and $E_{ex}^{ul} = E_{ex}/2$ where E_{ex} is the exchange parameter of the interaction. The role of density derivatives of symmetry energy and neutron skin thickness on α_D is studied and it is found that the value of α_D is 24.10 fm^3 and 26.43 fm^3 for E_{ex}^{l} and E_{ex}^{ul} respectively. Also using finite range effective interaction, we have studied the IVGDR energy constant D for ^{208}Pb and found the range of D is 77.6 − 80.6 MeV. In the range of calculated D, the neutron skin thickness S of ^{208}Pb for two different splitting of $E_{ex}{}^{l}$ and $E_{ex}{}^{ul}$ is found almost same and the deviation for two parameter sets is about 0.02 fm.

***Keywords**: Neutron skin thickness; nuclear symmetry energy; droplet model; equation of state;Dipole polarizability; Dipole resonance.*
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I. Introduction

The density-dependent symmetry energy is a key ingredient for the study of many aspects of astrophysics and nuclear physics in both theoretical and experimental fields. But we cannot measure the fundamental quantities like the nuclear symmetry energy directly. So it is very essential to gather some measurable observables to extract information about the fundamental quantity. A useful method discussed in literature [1, 2, 3, 4] to identify the role of the symmetry energy and to provide constraints on its density dependence is based on the study of the correlations between different observables. The isovector giant dipole resonance (IVGDR), dipole polarizability & neutron skin thickness (S) are supposed to be very sensitive indicators which can be used to study the symmetry energy [5].

Several experiments were performed to constrain these various kinds of observables. The Lead Radius Experiment (PREX) was an attempt to find the neutron skin thickness of ^{208}Pb and suggested the value of $S = 0.33^{+0.16}_{-0.18}$ fm [6] which was modified by another experiment, PREX-II to reduce the uncertainty to 0.06 fm [7] and Adhikari *et al.* proposed that the skin thickness of ^{208}Pb is 0.283±0.071 fm [8]. On the other hand, using covariance analysis of SKM functional limits the neutron skin thickness value of ^{208}Pb was $S = 0.156^{+0.025}_{-0.021}$ fm [9]. Using the nuclear droplet model (DM) [10, 11] and finite range effective interaction we calculated the neutron skin thickness S in two different channels and we find 0.11fm< S <0.23fm [12].

The isovector giant dipole resonance (IVGDR) is one type of oscillation mode in which neutrons and protons move collectively relative to each other. The properties of IVGDR depend on the nuclear symmetry energy is a well-established fact [9]. Correlations with different collective excitation modes such as the IVGDR may also put some constraints on the density dependence of the nuclear symmetry energy. D. Behera *et al.* [16] showed that the IVGDR energy constant D can be expressed as D=73.833+0.538$\left[a_{sym}/t\right]^{\frac{1}{2}}$, where

$a_{sym}(A)$ is the symmetry energy coefficient and using this relation they calculate the quantity t=0.202fm which is the distance between the neutron and proton mean surface location, an important parameter of skin thickness S. Therefore, it is very important to study the IVDGR energy constant through our splitting channel and find a relation between skin thickness and D.

Using finite range effective interactions we have successfully explained the various nuclear matter (NM) properties like momentum and density dependence of the mean-field, variation of symmetry energy with nucleon density [17], the neutron-proton-electron-muon matter called npeμ -matter at β-equilibrium, the study of neutron skin thickness of finite nuclei [12, 14], variation in neutron and proton effective mass splitting, neutron star mass-radius relation [18] etc.

This paper is presented in the following way. First, we represent our interaction form and fix the parameters set using a finite range effective interaction in sec.2. We have derived the dipole polarizability and found the relationship with different symmetry energy parameters in Sec. 2.1. In Sec. 2.2, we have discussed the correlation of dipole resonance with neutron skin thickness. We also compare the neutron skin thickness results in different approaches in sec. 2.3. Finally, we have presented our conclusions in Sec. 3.

II. Interaction and parameter fixation

We have used a simple finite-rage effective interaction in this work in a non-relativistic manner and the potential is defined as [15, 19]

$$V_{eff}(r) = t_0\left(1+x_0 P_\sigma\right)\delta\left(\vec{r}\right) + \frac{t_3}{6}\left(1+x_3 P_\sigma\right)\left[\frac{\rho(\vec{R})}{1+b\rho(\vec{R})}\right]^\gamma \delta\left(\vec{r}\right) + \left(W + B P_\sigma - H P_\tau - M P_\sigma P_\tau\right) f(r) \quad , \tag{1}$$

In which $f(r)$ represents a short-range interaction of Yukawa form with single range parameter Λ, $\rho = \rho_n + \rho_p$ is total nucleon density, t_0, t_3, x_0, x_3, γ, W, B, H, M are adjustable parameters related to the strengths of all the possible combinations of the spin and isospin exchange operators [14], $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$.

The effective interaction is very likewise to the Skyrme-type of interactions except for that the terms t_1 and t_2 are in the latter case have been replaced by the short-range interaction $\left(W + B P_\sigma - H P_\tau - M P_\sigma P_\tau\right) \times f(r)$. The replacement is essential to ensure a description leading to vanishing exchange interaction between a pair of nucleons of very large relative momenta. $P_\sigma = \frac{1}{2}(1+\vec{\sigma}_1.\vec{\sigma}_2)$ and $P_\tau = \frac{1}{2}(1+\vec{\tau}_1.\vec{\tau}_2)$ are the spin and isospin exchange operators respectively. The energy density of the asymmetric nuclear matter (ANM) derived from this effective interaction can be written as [15],

$$\begin{aligned} H(\rho_n,\rho_p) = {} & \frac{\hbar^2}{2m}\left[\int f_n(\vec{k})k^2 d^3k + \int f_p(\vec{k})k^2 d^3k\right] \\ & + \frac{1}{2}\left[\frac{E_0^{l}}{\rho_0} + \frac{E_\gamma^{ul}}{\rho_0^{\gamma+1}}\rho^\gamma\right]\left(\rho_n^2 + \rho_p^2\right) + \left[\frac{E_0^{ul}}{\rho_0} + \frac{E_\gamma^{ul}}{\rho_0^{\gamma+1}}\rho^\gamma\right]\rho_n \rho_p \\ & + \frac{E_{ex}^{l}}{2\rho_0}\iint\left[f_n(k) f_n(k') + f_p(k) f_p(k')\right] g_{ex}\left(\left|\vec{k}-\vec{k'}\right|\right) d^3k\, d^3k' \\ & + \frac{E_{ex}^{ul}}{2\rho_0}\iint\left[f_n(k) f_p(k') + f_p(k) f_n(k')\right] g_{ex}\left(\left|\vec{k}-\vec{k'}\right|\right) d^3k\, d^3k', \end{aligned} \tag{2}$$

where ρ_n and ρ_p are neutron and proton densities respectively and the total nuclear density $\rho = \rho_n + \rho_p$. $f_\tau(\vec{k})(\tau = n, p)$ is the single-particle momentum distribution function normalized to the local density $\rho_\tau = \int f_\tau(\vec{k}) d^3\vec{k}$.

At zero temperature $f(\vec{k})$ is described by a step function $f(\vec{k}) = \frac{g}{(2\pi)^3}\theta(\vec{k}_f - \vec{k})$, where g is the spin-isospin degeneracy factor and $k_f = \left(\frac{3\pi^2}{2}\rho\right)^{\frac{1}{3}}$ is the Fermi momentum. $g_{ex}\left(\left|\vec{k} - \vec{k}'\right|\right)$ is the normalized Fourier transform of the short-range interaction $f(r)$ and for Yukawa form of functional $f(r)$ it is explicitly given as

$$g_{ex}\left(\left|\vec{k} - \vec{k}'\right|\right) = \frac{1}{1 + \frac{\left|\vec{k} - \vec{k}'\right|^2}{\Lambda^2}}.$$

The parameters $E_0^{\,l}, E_0^{\,ul}, E_\gamma^{\,l}, E_\gamma^{\,ul}, E_{ex}^{\,l}$ *and* $E_{ex}^{\,ul}$ are related to the interaction parameters as given in Ref. [20]. Here we express Eq. 2 considering both neutron and proton densities which is slight different from Eq. 35 of Ref. [15].

To calculate the neutron-proton mean-field properties, we require the correct splitting of the parameters like $\left(E_0^{\,l} + E_0^{\,ul}\right)$, $\left(E_\gamma^{\,l} + E_\gamma^{\,ul}\right)$ and $\left(E_{ex}^{\,l} + E_{ex}^{\,ul}\right)$ into two some specific channels for interactions between like and unlike nucleons [15].

$$E_0 = E_0^{\,l} + E_0^{\,ul},\; E_\gamma = E_\gamma^{\,l} + E_\gamma^{\,ul},\; E_{ex} = E_{ex}^{\,l} + E_{ex}^{\,ul}.$$

In the same range, we have divided two different sets of strength parameters for the exchange interaction, and the values of all the parameters are listed in table 1.

$$\text{A1) } E_{ex}^{\,l} = \frac{E_{ex}}{2},\; \text{A2) } E_{ex}^{\,ul} = \frac{E_{ex}}{2}. \tag{3}$$

Table 1: Sets of interaction parameters

Set	$E_{ex}^{\,l}$ (MeV)	$E_{ex}^{\,ul}$ (MeV)	$E_\gamma^{\,l}$ (MeV)	$E_\gamma^{\,ul}$ (MeV)	$E_0^{\,l}$ (MeV)	$E_0^{\,ul}$ (MeV)	t_3 (MeV)	γ	Λ (fm^{-1})
A1	- 129.599	-129.599	127.255	255.866	- 112.5	- 199.7	13270.998	0.181	2.363
A2	- 129.599	-129.599	127.255	255.866	- 23.67	- 288.53			

2. 1 Electric dipole polarizability

Using the droplet model (DM) approach of Myers and Swiatecki [11] the dipole polarizability α_D^{DM} is given by

$$\alpha_D^{DM} = \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{E_{sym}(\rho_0)}\left(1 + \frac{5}{3}\times\frac{9}{4}\frac{E_{sym}(\rho_0)}{Q}A^{-1/3}\right), \tag{4}$$

This equation was first derived by Meyer, Quentein and Jenning [21] . Here $\langle r^2\rangle$ is the mean square radius of the nucleus, A is the nucleus mass number, Q is the surface stiffness coefficient, ρ_0 =0.161 fm^{-3} is the normal nuclear matter density at zero kelvin which indicates the resistance of neutrons against being separated from protons.

Using finite range effective interaction we have calculated the value of dipole polarizability for both the splitings of E_{ex}^{l} and E_{ex}^{ul}. It is found that for $E_{ex}^{l}=\frac{E_{ex}}{2}$, corresponding to the slope of symmetry energy L = 41.38 MeV the value of dipole polarizability α_D=23.01 fm^3 and for $E_{ex}^{ul}=\frac{E_{ex}}{2}$, corresponding to the slope of symmetry energy L = 75.204 MeV the value of dipole polarizability α_D=26.90 fm^3. We can express dipole polarizability in terms of skin thickness as,

$$\alpha_D^{DM}=\frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{E_{sym}(\rho_0)}\left(1+\frac{5}{2}\frac{S}{I\langle r^2\rangle}\right). \qquad (5)$$

The dipole polarizability $10^{-2}\alpha_D E_{sym}(\rho_0)$ is plotted as a function of S for ^{208}Pb (0.12fm < S < 0.34 fm) for finite range effective parameters [24, 25] in **Fig.3.**A linear increasing trend of $10^{-2}\alpha_D E_{sym}(\rho_0)$with S is observed and the result is consistent with other studies [13, 27-28].

It is found that for $E_{ex}^{l}=\frac{E_{ex}}{2}$, the value of dipole polarizability α_D=23.10 fm^3 corresponding to the skin thickness S =0.13 fm and for $E_{ex}^{l}=\frac{E_{ex}}{2}$, the value of dipole polarizability α_D=26.10 fm^3 corresponding to the skin thickness S =0.21 fm for $E_{ex}^{ul}=\frac{E_{ex}}{2}$. It is further observed that the values of α_D for both the splitting of our interaction obtained from the correlation between dipole polarizability with the slope of symmetry energy L as well as with the neutron skin thickness S consistent with each other. The values of both the splitting are listed in table 2 with their corresponding values of L and S.

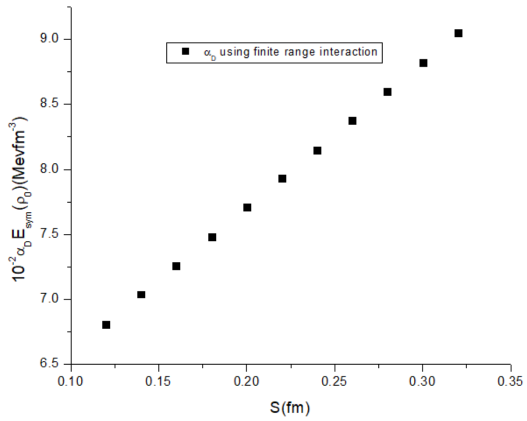

Fig.1

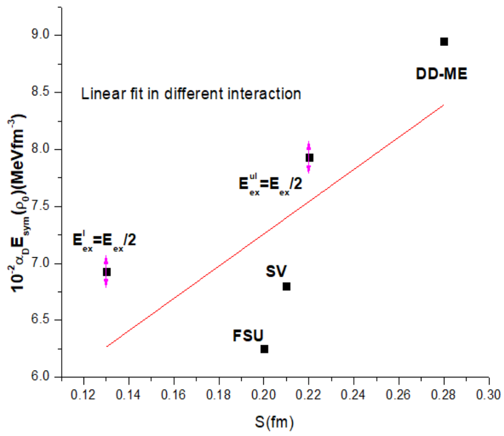

Fig.2

Fig.1. Study of $10^{-2}\alpha_D E_{sym}(\rho_0)$as a function of neutron skin thickness S with the using finite range effective interaction.

Fig.2. Comparison of dipole polarizability corresponding to neutron skin thickness S for $E_{ex}{}^{l}=E_{ex}/2$, $E_{ex}{}^{ul}=E_{ex}/2$, SV, FSU, and DD-ME and their linear fit in the same graph.

The values of dipole polarizability $10^{-2}\alpha_D E_{sym}(\rho_0)$obtained by using our interaction corresponding to skin thickness S for two different strength parameters are represented in fig.2. Our results are compared with the results obtained from other interactions like DDME, FSU, and SV, which gives the linear fit equation $10^{-2}\alpha_D E_{sym}(\rho_0)$=(14.17 ± 8.135) S +(4.426 ± 1.736) MeV fm^3 with correlation coefficient, in this case, is r =0.85.The values of dipole polarizability using the linear fit equation:

11.72 fm^3< α_D <30.18 fm^3 for $E_{ex}{}^{l} = E_{ex}/2$ and 13.41 fm^3< α_D <36.69 fm^3 for $E_{ex}{}^{ul} = E_{ex}/2$.

The values of dipole polarizability α_D of ^{208}Pb for our interaction obtained by using the correlation with the slope of symmetry energy ‘ L ’ at saturation density as well as with the skin thickness ‘ S ’ are listed in table 2.

Table 2: Values of the slope of symmetry energy L, skin thickness S and dipole polarizability α_D of ^{208}Pb

Interaction	L (MeV)	α_D (fm^3)	S (^{208}Pb)
$E_{ex}{}^{l} = \frac{E_{ex}}{2}$	41.38	23.01	0.13 fm
$E_{ex}{}^{ul} = \frac{E_{ex}}{2}$	75.204	26.9	0.21 fm
Gogny Model	43	20.1	0.168 fm

2.2.Correlation of neutron skin “ S ” with dipole resonance

The dipole response of a heavy nucleus to an externally applied electric field is mostly dominated by the giant dipole resonance (GDR) of width 2 − 4 MeV [16, 26, 27]. Due to excess neutrons in heavy nuclei like ^{208}Pb, the E_1 value is in the range between 9 to 11 MeV [29]. It is well known that nuclear symmetry energy dominates the properties of IVGDR to a great extent [9, 26, 29, 30].

There are some correlations of nuclear symmetry energy parameters with pygmy dipole resonance [31, 32, 33], isovector giant dipole resonance [13]and dipole polarizability [13, 34, 35] as seen in this paper also.

The IVGDR energy constant D is defined as [37, 39]

$$D = D_\infty / \sqrt{1 + 3E_{sym}(\rho_0) A^{-1/3} / Q}\,, \qquad (6)$$

where $D_\infty = \sqrt{8\hbar^2 E_{sym}(\rho_0) m r_0^2}$.

As the E_{sym}/Q depends on neutron skin thickness S [40-42] we can find out a relation between the IVGDR energy constant D with S.

We can rewrite Eq. (15) as $D = \sqrt{\frac{8\hbar^2}{m r_0^2}} \left(\frac{3}{A^{1/3} Q} \right)^{-1/2} \left[1 + \frac{A^{1/3} Q}{1 + 3E_{sym}(\rho_0)} \right]$. (7)

Expanding the square-bracketed term in powers of $\frac{A^{1/3} Q}{3E_{sym}(\rho_0)}$ and retaining up to the 1st order we get

$$D \approx \sqrt{\frac{8\hbar^2}{m r_0^2}} \left(\frac{3}{A^{1/3} Q} \right)^{-1/2} \left[1 - \frac{A^{1/3} Q}{6E_{sym}(\rho_0)} \right] \qquad (8)$$

Which may be written as

$$D = B_A \left(\frac{a_{sym}(A)}{t} \right)^{1/2} \left[1 - \frac{A^{1/3} Q}{6E_{sym}(\rho_0)} \right]. \qquad (9)$$

$$D = B_A \left(\frac{a_{sym}(A)}{t} \right)^{1/2} \left[1 - \frac{3}{8} \frac{A^{1/3} E_{sym}(\rho_0)}{\left(L - \frac{K_{sym}}{12} \right)} \right] \qquad (10)$$

We substitute Q in terms of t and define a constant $B_A = \sqrt{\frac{4\hbar^2}{mr_0^2} A^{1/3}(I - I_c)}$ for a given nucleus. In eq. (15), the leading term is proportional to $\left(\frac{a_{sym}(A)}{t}\right)^{1/2}$.

IVGDR energy constant 'D' is plotted as a function of slope of symmetry energy 'L' for ^{208}Pb in figure 3 and as a function of skin thickness S of ^{208}Pb in figure 6 by using finite range effective interaction for both the splitting of $E_{ex}{}^{l}$ and $E_{ex}{}^{ul}$. In both cases, we used $E_{sym}(\rho_0)$=30 MeV and ρ_0=0.161 fm^{-3}.

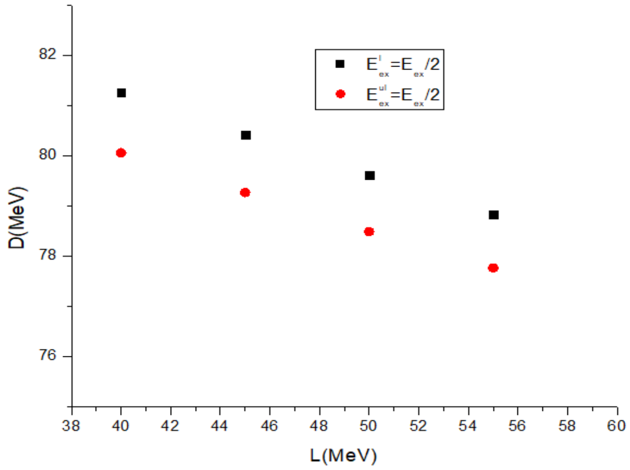

Fig. 3.The variation of IVGDR energy constant D of ^{208}Pb is shown as a function of the density slope parameter L for both $E_{ex}{}^{l} = E_{ex}/2$ & $E_{ex}^{ul} = E_{ex}/2$.

From figure 3 we saw a decreasing nature of D with the increased value of slope parameter for both the sets of parameter $E_{ex}{}^{l} = E_{ex}/2$ and $E_{ex}^{ul} = E_{ex}/2$. We obtained the IVGDR energy constant for ^{208}Pb in the range 77.6 MeV− 80.6 MeV by using the Q values calculated in the present work, which is in close agreement with the experimental value $D_{\exp} \approx$ 80 MeV for heavy nuclei.

Neutron skin thickness also plays an important role to study the quantity related to symmetry energy and its derivatives. So it is an important task to observe the IVGDR energy constant D of ^{208}Pb concerning S by using the splitting parameters of our finite range effective interaction.

IVGDR energy constant 'D' is varied as a function of S from 0.16 fm to 0.24 fm in figure 4 and a decreasing nature of 'D' is observed with the increase of 'S' for both $E_{ex}{}^{l} = E_{ex}/2$ & $E_{ex}^{ul} = E_{ex}/2$ and these results were also found consistent with another study [16]. From the experimental result of IVGDR energy constant D, the skin thickness 'S' of Pb^{208} is found to be 0.16 fm for $E_{ex}{}^{l} = E_{ex}/2$ & 0.18 fm for $E_{ex}^{ul} = E_{ex}/2$. Here we observed that the value of skin thickness for two different splitting of $E_{ex}{}^{l}$ and $E_{ex}{}^{ul}$ is almost same and the deviation values of 'S' for two parameter sets is about 0.02 fm.

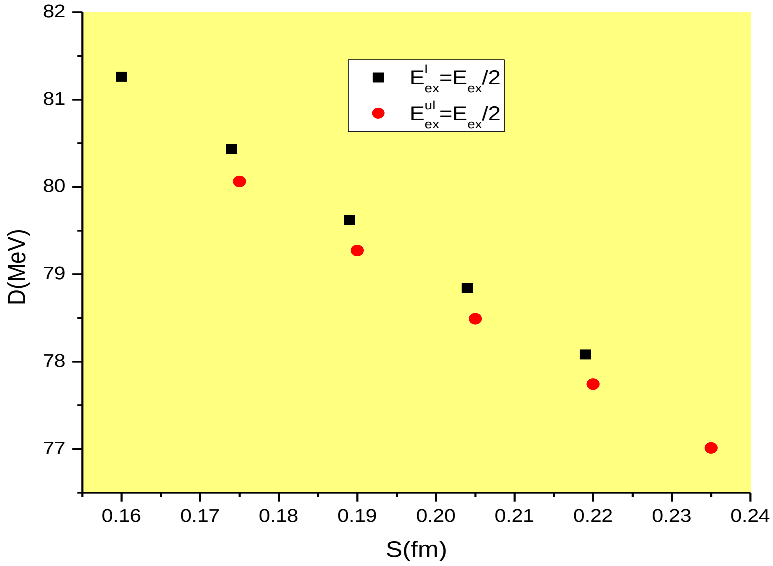

Fig. 4.the plot of IVGDR energy constant D of [208]Pb is as a function of neutron skin thickness S for both $E_{ex}{}^{l} = E_{ex}/2$ & $E_{ex}^{ul} = E_{ex}/2$.

Finally, we studied the IVGDR energy constant D in [208]Pb with respect to $\left[a_{sym}/t\right]^{\frac{1}{2}}$ by using finite range effective interaction and is shown in figure 5and an increasing trend of 'D' is found with respect to $\left[a_{sym}/t\right]^{\frac{1}{2}}$. From the linear fit in the same graph and we found a linear fit equation D=(61.05+1.68 $\left[a_{sym}/t\right]^{\frac{1}{2}}$). Using $a_{sym}(A)$, with an IVGDR energy constant of 80 MeV, we obtained t =0.19 fm for $E_{ex}^{l} = E_{ex}/2$ and t =0.17 fm for $E_{ex}^{ul} = E_{ex}/2$.

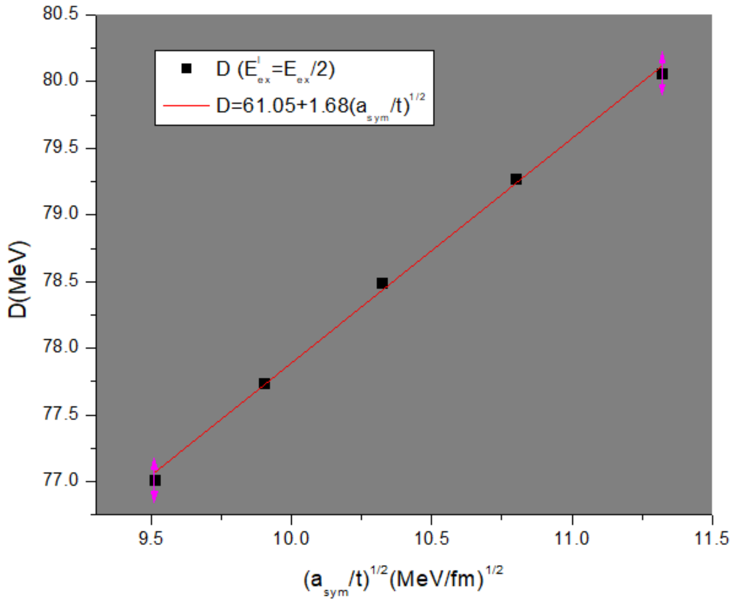

Fig. 5:the variation of IVGDR energy constant D with the quantity $\left[a_{sym}/t \right]^{\frac{1}{2}}$ using finite range effective interaction and in the same graph we plotted the linear fit for D.

Using the experimental values of IVGDR energy constant D in our interaction we calculated the values of the skin thickness S of ^{208}Pb as well as quantity t in table 3.

Table 3: Values of skin thickness S as well as quantity t by using experimental value of D

Interaction	$D_{\exp}$ (MeV)	t (fm)	S (fm)
$E_{ex}{}^{l} = \frac{E_{ex}}{2}$	80	0.19	0.16
$E_{ex}{}^{ul} = \frac{E_{ex}}{2}$	80	0.17	0.18

2.3. Comparison of neutron skin thickness

Using finite range effective interaction and DM we calculated neutron skin thickness [12] of ^{208}Pb and we found the value of neutron skin thickness, S = 0.13 fm for $E^l_{ex} = E_{ex}/2$, S =0.21 fm for $E^{ul}_{ex} = E_{ex}/2$ and the deviation between two results is 0.08 fm.

In this paper, the neutron skin thickness S is calculated from two dipole properties via electric dipole polarizability and dipole resonance vector. In our calculations, we have used simple finite range effective interaction of Yukawa form with different splitting channels.

Using dipole polarizability, we found the neutron skin thickness 'S' of ^{208}Pbis0.17 fm for $E^l_{ex} = E_{ex}/2$ and 0.25 fm for $E^{ul}_{ex} = E_{ex}/2$ respectively. The deviation of results is within 0.08 fm for two different splitting of E^l_{ex} and E^{ul}_{ex}.

On the other hand, using IVGDR we obtained the neutron skin thickness of ^{208}Pbis 0.16 fm for $E^l_{ex} = E_{ex}/2$ and 0.18 fm for $E^{ul}_{ex} = E_{ex}/2$ respectively. The deviation is observed to be very small for both types of splitting, within 0.02fm.

So the study of neutron skin thickness can rectify the result and we enlisted our results in Table 4. The results are in good agreement with the range of PREX results [6].

Table 4: Neutron skin thickness comparison.

Methods or technique	S (fm) of Pb^{208}		Deviation of S in two different splitting.
	$E_{ex}^{l}=E_{ex}/2$	$E_{ex}^{ul}=E_{ex}/2$	
Droplet Model	0.13	0.21	0.08 fm
Dipole Polarizability	0.17	0.25	0.08 fm
IVDGR energy constant	0.16	0.18	0.02 Fm

III. Conclusion

In this paper, we have studied analytically the role of density-dependent EOS on electric dipole polarizability and IVGDR constant using a simple density-dependent finite range effective interaction having Yukawa form. The calculations are carried out with the Yukawa form of exchange interaction having the same range but with different strengths for interactions between two like and unlike nucleons, namely set A_1 and A_2. First, we have studied the variation of $10^{-2}\alpha_D\, E_{sym}(\rho_0)$ with neutron skin thickness S of ^{208}Pb and found the value of α_D = 24.10 fm^3 for $E_{ex}^{l}=E_{ex}/2$ and which is smaller compared to $E_{ex}^{ul}=E_{ex}/2$ for which α_D = 26.43 fm^3. In this case, a linear fit was also drawn and found $10^{-2}\alpha_D\, E_{sym}(\rho_0)$=(14.17 ± 8.135) S +(4.426 ± 1.736) MeV.

A decreasing nature of IVGDR constant is observed with the increased value of slope parameter for both the sets of parameters.We obtained the IVGDR energy constant for ^{208}Pb in the range 77.6 MeV− 80.6 MeV by using the Q values calculated in the present work, which is in close agreement with the experimental value D_{exp} ≈ 80 MeV for heavy nuclei. We also observed a similar variation of D with S for both cases anddecreasing nature of IVGDR constant is observed with the increased value of slope parameter for both the sets of parameters.From the experimental result of IVGDR energy constant D, we also calculated the neutron skin thickness S = 0.16 fm for $E_{ex}^{l}=E_{ex}/2$ & S = 0.18 fm for $E_{ex}^{ul}=E_{ex}/2$ and a very small deviation in the values of skin thickness is observed for both the splitting.

In our study, we have obtained a linear fit D = (61.05+1.68 $\left[\frac{a_{sym}}{t}\right]^{1/2}$). Using a_{sym} values in our two sets of parameters and IVGDR energy constant of 80 MeV, we found t =0.19 fm for $E_{ex}^{l}=E_{ex}/2$ which is also an important factor to study neutron skin thickness. From the results,it is clear that the numerical values of dipole polarizability, IVDGR energy constant D depend not only on the form of interaction but on the density dependence of EOS and isospin asymmetry there as well.

Finally, we have compared neutron skin thickness in different approaches. The deviation of neutron skin thickness of two different splitting channels is small in the case of dipole resonance study. From all these observations it can be concluded that though there is some uncertainty in results using these data we can modify different symmetry energy parameters and there is a way to cross-check the values of certain finite nuclei parameters like neutron skin thickness.

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